Modelling Linguistic Hedges by L-fuzzy Modifiers

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Abstract

In this paper we discuss the representation of linguistic terms by means of L-fuzzy sets, i.e. fuzzy sets taking membership degrees in a complete lattice L. We develop a framework for the modelling of linguistic hedges acting on these terms, by means of L-fuzzy modifiers taking images under L-fuzzy relations. This framework not only allows for a unified approach to the representation of hedges, but it also endows them with a clear semantics. To our knowledge it is the first modelling scheme for hedges acting on terms represented by L-fuzzy sets.

1 Introduction

The success of fuzzy set theory in applications such as expert systems, approximate reasoning systems, database systems, decision making systems, etc. is rooted in its ability to model vague linguistic information in an easy-to-deal-with mathematical representation that is yet very expressive and close to human intuition. In fuzzy set theoretical frameworks linguistic terms such as large, more or less old, very warm etc. are usually modelled by means of fuzzy sets taking membership degrees in the unit interval $[0,1]$ ([1]). During the last decades many representations for hedges such as very, rather, more or less,... acting on these kinds of fuzzy sets were developed (see [2] for an overview).

The construction of a fuzzy set corresponding to a linguistic term has a large degree of freedom: we can choose from many possible parametrized general shape functions (see [3]) and moreover the parameters can often be arbitrarily chosen within some small domain, all this with negligible or no impact on the functionality of the systems which use this model of linguistic terms. The exact numerical values of the membership degrees are usually not justifiable nor important; only the values 0, 0.5 and 1 have a special intuitive meaning, while the other ones are mostly only shots. Often this is explained away by saying that the only really important thing is that the membership degrees induce a graded ordering of "belonging to" on the set of objects, which is the power of fuzzy set theory.

Indeed the mapping of elements of the universe to the interval $[0,1]$ implies a crisp, linear ordering of these elements. However there exists a great deal of incomparable information in the real world, among which linguistic terms which do not correspond to a linear ordering on the universe. These are not only the very subjective and difficult to define terms that describe matters near to the human soul, such as beloved, sad, hopeful etc.. We can even find an example in physics: in the HSV (hue, saturation, value) color model which is close to the human color perception we cannot prefer nor hue neither saturation in order to decide what is "more red". Moreover we cannot assign to two different but non comparable blushes the same membership degree, for what would happen when a third one occurs which is obviously "more red" than the first one, but non comparable to the second one?

It is clear that $[0, 1]$-valued fuzzy set theory is inadequate to deal with non-comparable information. The key to the solution however was present from the very start of fuzzy set theory. In fact, in 1965 in his founding paper Zadeh [4] included the footnote:

> *"In a more general setting, the range of the membership function can be taken to be a suitable partially ordered set P".*

Although the unit interval always was and still is considered to be a very natural and transparent set of membership degrees [5], from the beginning some attention has been paid to other partially ordered sets as well. In fact in 1967 Goguen [6] formally introduced the notion of an L-fuzzy set with a membership function taking values in a lattice L. The concept of an L-fuzzy set is still studied albeit mostly on a theoretical level. Since a lot of information in the real world is incomparable, we believe that there is still a huge gap on the market of applications, to be filled by systems able to represent this kind of information. In this paper we start walking in this direction by providing a framework for the representation of linguistic hedges acting on linguistic terms modelled by L-fuzzy sets.

After presenting important notions based on L-fuzzy set theory, we will discuss the representation of linguistic terms by means of L-fuzzy sets. We will especially pay attention to the representation of linguistic hedges by developing a framework based on L-fuzzy relational calculus. To demonstrate the power of this framework, we will illustrate it by means of a very down to earth example.

2 Notions from L-fuzzy Set Theory

As a steady house can only be built on solid fundaments, in this section we will pay attention to the building blocks needed to develop the representation of modified linguistic terms by L-fuzzy sets. For completeness we start by recalling the definition of a lattice.

Definition 1 (Lattice) *An algebraic structure $(L, \vee, \wedge)$ consisting of a nonvoid set L and two binary operations $\vee$ and $\wedge$ on L is called a lattice iff for all a, b and c in L:*

(L.1) $a \vee a = a$ *and* $a \wedge a = a$
(L.2) $a \vee b = b \vee a$ *and* $a \wedge b = b \wedge a$
(L.3) $a \vee (b \vee c) = (a \vee b) \vee c$ *and* $a \wedge (b \wedge c) = (a \wedge b) \wedge c$
(L.4) $a \vee (a \wedge b) = a$ *and* $a \wedge (a \vee b) = a$

$(L, \vee, \wedge)$ is usually abbreviated to L, tacitly assuming the presence of the join-operation $\vee$ and the meet-operation $\wedge$. Every lattice L is a partially ordered set with the ordering defined by $a \leq b \Leftrightarrow a \vee b = b$, for all a and b in L. Throughout the remaining of this paper, let L denote a complete lattice, i.e. the infimum and supremum of arbitrary subsets of L exists. The smallest and the greatest element of L will be denoted by l and u respectively.

A generalization of logical operators from the real unit interval to partially ordered sets was realised in [7]. For further studies regarding t-norms on lattices we also refer to [8] and [9]. We will comply with the following definitions:

Definition 2 (Triangular norm) *A triangular norm (t-norm for short) $\mathcal{T}$ on L is an associative, commutative and increasing $L^2 - L$ mapping $\mathcal{T}$ satisfying the boundary condition $\mathcal{T}(a, u) = a$, for all a in L.*

As an example of such a t-norm we mention $\mathcal{T}_\wedge$ defined by $\mathcal{T}_\wedge(a, b) = a \wedge b$, for all a and b in L.

Definition 3 (Implication) *Let $\mathcal{T}$ be a t-norm on L. The $L^2 - L$ mapping $\mathcal{I}$ defined by $\mathcal{I}(a, b) = \sup\{\lambda | \lambda \in L \text{ and } \mathcal{T}(\lambda, a) \leq b\}$, for all a and b in L, is called the implication induced by $\mathcal{T}$.*

Note that the definition of implication is a generalization of the definition of residual implication on $[0, 1]$. Furthermore for all b in L: $\mathcal{I}(u, b) = b$. Throughout this paper let $\mathcal{T}$ and $\mathcal{I}$ denote a t-norm and the implication induced by it on L respectively. Using these logical operators, we can now define some basic concepts regarding L-fuzzy sets.

Definition 4 (L-fuzzy set) *An L-fuzzy set A on X is a mapping from X to L, also called the membership function of A. For all x in X, $A(x)$ is called the membership degree of x in A. The class of all L-fuzzy sets on X is denoted by $\mathcal{L}(X)$.*

A $[0, 1]$-fuzzy set is commonly called a fuzzy set.

Definition 5 (T-intersection) *For A and B L-fuzzy sets on X, the $\mathcal{T}$-intersection of A and B is the L-fuzzy set on X defined by, for x in X:*

$$(A \cap_\mathcal{T} B)(x) = \mathcal{T}(A(x), B(x))$$

Furthermore we will write $\mathcal{I}(A, B)$ to denote the pointwise extension of $\mathcal{I}$ to fuzzy sets, i.e. $\mathcal{I}(A, B)(x) = \mathcal{I}(A(x), B(x))$, for all x in X.

Definition 6 (Inclusion) *For A and B L-fuzzy sets on X, we say that A is included in B iff*

$$(\forall x \in X)(A(x) \leq B(x))$$

We denote this by $A \subseteq B$.

Likewise the notion of foreset (see [10]) of a $[0, 1]$-fuzzy relation can be generalized to the L-fuzzy case.

Definition 7 (L-fuzzy relation) *An L-fuzzy relation R on X is an L-fuzzy set on $X \times X$.*

Definition 8 (Foreset) *Let R be an L-fuzzy relation on X. For all y in X the R-foreset of y is the L-fuzzy set on X defined by $(Ry)(x) = R(x, y)$, for x in X.*

Finally we generalize the direct and the superdirect image (see [3]) of a fuzzy set under a fuzzy relation.

Definition 9 (Images of L-fuzzy sets under L-fuzzy relations) *For R an L-fuzzy relation on X, the direct and the superdirect image under R of an L-fuzzy set A on X are* *the L-fuzzy sets $R^{\clubsuit}(A)$ and $R^{\heartsuit}(A)$ on X defined by, for y in X:*

$$R^{\clubsuit}(A)(y) = \sup_{x \in X} \mathcal{T}(R(x,y), A(x))$$

$$R^{\heartsuit}(A)(y) = \inf_{x \in X} \mathcal{I}(R(x,y), A(x))$$

Note that $R^{\clubsuit}$ and $R^{\heartsuit}$ are in fact $\mathcal{L}(X) - \mathcal{L}(X)$ mappings transforming one L-fuzzy set into another. Later on we will show that — for a suitable L-fuzzy relation R — they can be used to model linguistic hedges. For now it is worthwhile to notice that one can easily derive

$$R^{\clubsuit}(A)(y) = \sup_{x \in X} (Ry \cap_{\mathcal{T}} A)(x)$$
$$R^{\heartsuit}(A)(y) = \inf_{x \in X} \mathcal{I}(Ry, A)(x)$$

The first expression can be interpreted as the degree to which Ry and A overlap, whereas the latter can be seen as the degree of inclusion of Ry in A. Note that if $Ry \subseteq A$, then $R^{\heartsuit}(A)(y)$ will be 1. In case the relation R is reflexive, i.e. $R(x,x) = u$ for all x in X, the following property will be particularly of interest:

Proposition 1 (Entailment) *If R is a reflexive L-fuzzy relation on X then for all A in $\mathcal{L}(X)$:*

$$R^{\heartsuit}(A) \subseteq A \subseteq R^{\clubsuit}(A)$$

Proof. For all y in X:

$$\begin{aligned} R^{\heartsuit}(A)(y) &\leq \mathcal{I}(R(y,y), A(y)) \\ &\leq \mathcal{I}(u, A(y)) \\ &\leq A(y) \\ &\leq \mathcal{T}(u, A(y)) \\ &\leq \mathcal{T}(R(y,y), A(y)) \\ &\leq R^{\clubsuit}(A)(y) \end{aligned}$$

According to Definition 6 this proves the proposition.

3 Representing Linguistic Terms

In fuzzy set theoretical contexts "linguistic term" usually refers to an *atomic term* (an adjective), a *logically composed term* (a logical composition of terms using not, and and or) or a *modified term* (a term generated by applying a linguistic hedge, i.e. a linguistic modifier, to a term). Furthermore every term is represented by a fuzzy set A on a universe X, characterized by a $X - [0,1]$ mapping — for simplicity also denoted by A — which is called the membership function. Hence for every x in X, $A(x)$ is the membership degree of x in the fuzzy set A, a degree which can vary between 0 and 1.

This graded approach makes fuzzy set theory extremely suitable to model linguistic terms, which are often inherently vague. As indicated in the introduction however the representation of a term by a $[0,1]$-fuzzy set forces a total ordering on the objects of the universe, which might not be wanted if the universe contains incomparable objects.

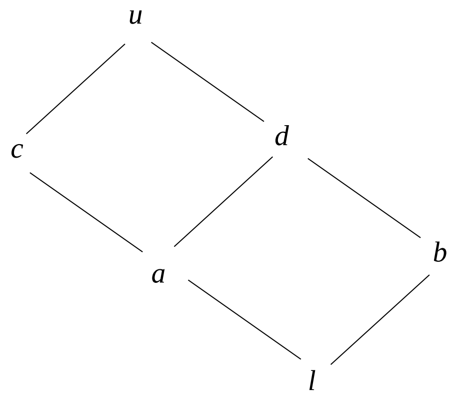

Figure 1: Lattice $L = \{l, a, b, c, d, u\}$

We will illustrate this by means of an example, fixing the problem by representing the linguistic term by an L-fuzzy set (L being a complete lattice that contains incomparable elements).

Example 1 *Ones favorite "ingredients" for a dessert might be chocolate, vanilla-ice and marzipan. Depending on the absence or the presence of one or more of these ingredients, a dessert can be called delicious to some lower or higher degree. Since one likes all of the three ingredients, it is clear that adding one of them makes the dessert plate more delicious. E.g. a vanilla-ice topped with a bit of chocolate is considered to be more delicious than a plain vanilla-ice. Adding some marzipan makes it even more delicious. On the other hand it could be hardly impossible to say whether a vanilla-ice & chocolate dessert is less, or more delicious than a vanilla-ice & marzipan dessert. Note that we can not solve this by stating that they are delicious to the same degree, because the ordering discussed in the paragraph above would then imply that vanilla-ice & marzipan is more delicious than chocolate, although we have no actual ground to assume this is true.*

Due to the incomparability of the deliciousness of some of the dessert plates X, $[0,1]$-fuzzy set theory is inadequate to model the term delicious *in the universe of desserts. We propose to represent this term by means of an L-fuzzy set A on X, using membership degrees from the lattice with Hasse-diagram depicted in Figure 1. For convenience we will use the abbreviations C (chocolate), V (vanilla-ice) and M (marzipan). Furthermore a concatenation of ingredient symbols refers to a combined plate (e.g. CV refers to a chocolate & vanilla-ice dessert):*

$$A = \{(V, a), (C, b), (CV, d), (VM, c), (CVM, u)\}$$

Note that the universe of the dessert plates is given by $X = \{C, V, M, CV, VM, CVM\}$.

4 Representing Linguistic Hedges

In this section we will explain how the L-fuzzy modifiers $R^{\clubsuit}$ and $R^{\heartsuit}$ can model the weakening hedge more or less and the intensifying hedge very respectively.

Definition 10 (L-fuzzy modifier) *An L-fuzzy modifier on X is an $\mathcal{L}(X) - \mathcal{L}(X)$-mapping.*

During the last three decades many $[0,1]$-fuzzy modifiers are proposed for the representation of linguistic hedges acting on terms represented by $[0,1]$-fuzzy sets (see [2] for an overview). Generalizing them to L-fuzzy modifiers is in general anything but straightforward. Denoting the degree to which an object x has a property P by $P(x)$, the most popular representations dictate that very $P(x) = P(x)^2$ (powering modifiers [11]) or that very $P(x) = P(x-\alpha)$ ($x \in \mathbb{R}, \alpha \in \mathbb{R}$, shifting modifiers [12]). Since their is no $.^2$ operation available on arbitrary lattices, we cannot extend the use of powering fuzzy modifiers to L-fuzzy sets. The shifting operators on the other hand assume a linear ordering on the universe of objects - otherwise it would not be possible to perform a shift to the left or to the right. Since the starting idea of this paper is the use of L-fuzzy sets for the representation of linguistic terms which do not necessarily imply a linear ordering on the universe, we can forget the shifting modifiers as well.

The framework of $[0,1]$-fuzzy modifiers presented in [13, 14] however is entirely based on $[0,1]$-fuzzy relational calculus. Since this calculus can be nicely extended to L-fuzzy relations as shown in Section 2, a generalization to L-fuzzy modifiers is very natural. In fact according to Definition 10, $R^\clubsuit$ and $R^\heartsuit$ as defined in Definition 9 are L-fuzzy modifiers on X. Furthermore we can preserve the clear semantics indicated in [14], which is based on taking the context into account. In the case of the weakening modifier more or less and the intensifying modifier very, the context of an object y of X will consist of all the objects that *resemble* to y. To model the context we will therefore use an L-fuzzy relation R on X that models approximate equality on X. Note that Ry, i.e. the R-foreset of y, can then be interpreted as the L-fuzzy set of all objects that resemble to y. In the remainder of this section we will explain the details of this procedure and illustrate them.

4.1 Weakening Hedge: More Or Less

We could say that somebody is more or less adult "if he resembles to an adult". Likewise a park is more or less large "if it resembles to a large park". In general: y is more or less A if y resembles to an x that is A. Hence we can say that y is more or less A if the intersection of A and Ry is not empty. Or to state it more fuzzy: y is more or less A to the degree to which Ry and A overlap, i.e.

$$\text{more or less } A(y) = R^\clubsuit(A)(y)$$

The semantics of this representation become even more clear if A is a crisp singleton, i.e. $A(z) = u$ for some z in X and $A(x) = l$ for all other x in X. We also denote this by $A = \{z\}$. In this case for all y in X, more or less $A(\text{y})$ is equal to $R(z,y)$. In other words y is more or less $\{z\}$ to the degree to which z resembles to y.

It is natural to assume that the approximate equality relation R is reflexive. Therefore Proposition 1 ensures that

$$A \subseteq \text{more or less } A$$

meaning that every object that is A is also more or less A to the same or to a higher degree. This property is called semantic entailment [12] and is often assumed when representing

$\mathcal{T}_\wedge$	l	a	b	c	d	u
l	l	l	l	l	l	l
a	l	a	l	a	a	a
b	l	l	b	l	b	b
c	l	a	l	c	a	c
d	l	a	b	a	d	d
u	l	a	b	c	d	u

$\mathcal{I}$	l	a	b	c	d	u
l	u	u	u	u	u	u
a	b	u	b	u	u	u
b	c	c	u	c	u	u
c	b	d	b	u	d	u
d	l	c	b	c	u	u
u	l	a	b	c	d	u

Table 1: t-norm and implication on $L = \{l, a, b, c, d, u\}$

modified terms by $[0, 1]$-fuzzy sets ([3, 11, 15]).

4.2 Intensifying Hedge: Very

Unlike more or less, it sounds unnatural to apply very to crisp concepts. However very is applied very often in natural language to vague terms to intensify their meaning. For the representation of very in an L-fuzzy framework, we suggest an analogous scheme as presented above for more or less.

If all men resembling in height to Alberik are tall, then Alberik must be very tall. Likewise a woman is very beautiful "if all women resembling to her are beautiful". In general: y is very A if all x resembling to y are A. Hence y is very A if Ry is included in A. To state it more fuzzy: y is very A to the degree to which Ry is included in A, i.e.

$$\text{very } A(y) = R^{\heartsuit}(A)(y)$$

Under the natural assumption that R is reflexive, the semantic entailment holds:

$$\text{very } A \subseteq A$$

To illustrate the techniques presented in this paper, we will show how they can be used to model the linguistic terms more or less delicious and very delicious, starting from the representation of delicious in Example 1.

Example 2 *To construct the L-fuzzy modifiers $R^{\clubsuit}$ and $R^{\heartsuit}$ we need three important ingredients, namely a t-norm and an implication on L on one hand, and a L-fuzzy relation R that models approximate equality on the other. Table 1 represents the t-norm $\mathcal{T}_\wedge$ and the implication $\mathcal{I}$ induced by it on the lattice L depicted in Figure 1.*

Table 2 represents a reflexive and symmetrical L-fuzzy relation R that models approximate equality on X. Every dessert plate is considered to be approximately equal to itself to the highest degree u (reflexivity). Furthermore if two dessert plates are not exactly equal but adding one ingredient to one of them results in the other dessert plate, they are still considered to be approximately equal to degree c. E.g. $R(C, CV) = c$, $R(CVM, VM) = c$ etc. Otherwise they are considered as not approximately equal, i.e. approximately equal to the lowest degree l.

Now the membership degrees of all dessert plates in the L-fuzzy sets $R^{\clubsuit}(A)$ and $R^{\heartsuit}(A)$, where A represents the term delicious *as in Example 1, can be determined. As an example*

R	V	C	CV	VM	CVM
V	u	l	c	c	l
C	l	u	c	l	l
CV	c	c	u	l	c
VM	c	l	l	u	c
CVM	l	l	c	c	u

Table 2: Relation R modelling approximate equality on X.

we compute:

$$\begin{aligned} R^{\clubsuit}(A)(V) = \; & \sup(\mathcal{T}_\wedge(R(V,V),A(V)),\mathcal{T}_\wedge(R(C,V),A(C)),\mathcal{T}_\wedge(R(CV,V),A(CV)),\\ & \mathcal{T}_\wedge(R(VM,V),A(VM)),\mathcal{T}_\wedge(R(CVM,V),A(CVM)))\\ = \; & \sup(\mathcal{T}_\wedge(u,a),\mathcal{T}_\wedge(l,b),\mathcal{T}_\wedge(c,d),\mathcal{T}_\wedge(c,c),\mathcal{T}_\wedge(l,u))\\ = \; & \sup(a,l,a,c,l) = c \end{aligned}$$

$$\begin{aligned} R^{\heartsuit}(A)(CV) = \; & \inf(\mathcal{I}(R(V,CV),A(V)),\mathcal{I}(R(C,CV),A(C)),\mathcal{I}(R(CV,CV),A(CV)),\\ & \mathcal{I}(R(VM,CV),A(VM)),\mathcal{I}(R(CVM,CV),A(CVM)))\\ = \; & \inf(\mathcal{I}(c,a),\mathcal{I}(c,b),\mathcal{I}(u,d),\mathcal{I}(l,c),\mathcal{I}(c,u))\\ = \; & \inf(d,b,d,u,u) = b \end{aligned}$$

Computing all membership degrees, we obtain

$$\textsf{more or less } A = R^{\clubsuit}(A) = \{(V,c),(C,d),(CV,u),(VM,c),(CVM,u)\}$$

$$\textsf{very } A = R^{\heartsuit}(A) = \{(V,a),(C,b),(CV,b),(VM,a),(CVM,d)\}$$

In the example the semantic entailment is clear: $R^{\clubsuit}$ — used to model the weakening hedge more or less — keeps or increases the original membership degrees, while $R^{\heartsuit}$ — used to represent the intensifying hedge very — corresponds to a reduction.

5 Conclusion

In this paper we have indicated the existence of incomparable information in the real world. The need to model this kind of information can be met using L-fuzzy set theory, representing linguistic terms by means of L-fuzzy sets. We have developed L-fuzzy modifiers based on L-fuzzy relations for the modelling of linguistic hedges in this framework, we have explained their semantical underpinnings and we have illustrated that they can yield intuitively satisfying results. The choice of suitable lattices and relations modelling approximate equality, as well as the development of applications using these techniques will be the subject of further research.

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