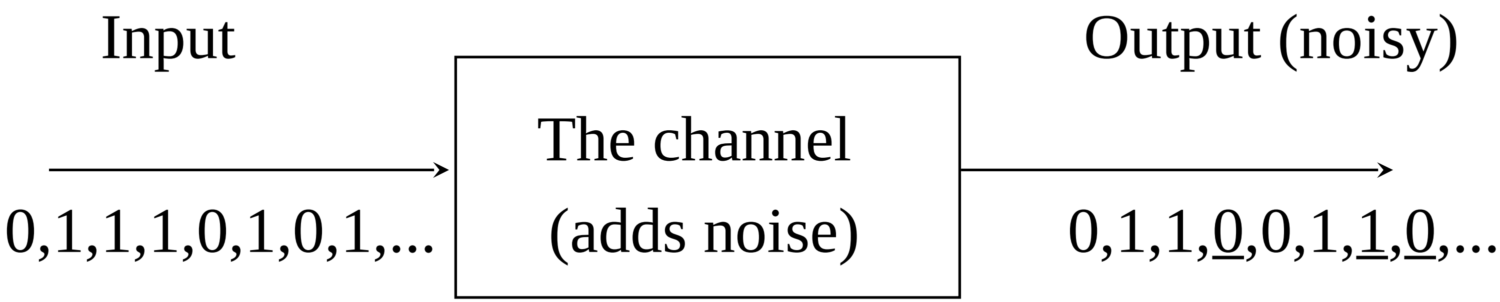
Language Modeling
(and the Noisy Channel)
Part 2 of Intro NLP

The Noisy Channel

- Prototypical case:



- Model: probability of error (noise):
- Example: p(0|1) = .3 p(1|1) = .7 p(1|0) = .4 p(0|0) = .6
- <u>The Task</u>:

known: the noisy output; want to know: the input (***<u>decoding</u>***)

Noisy Channel Applications

- OCR
  - straightforward: text → print (adds noise), scan → image
- Handwriting recognition
  - text → neurons, muscles (“noise”), scan/digitize → image
- Speech recognition (dictation, commands, etc.)
  - text → conversion to acoustic signal (“noise”) → acoustic waves
- Machine Translation
  - text in target language → translation (“noise”) → source language
- Also: Part of Speech Tagging
  - sequence of tags → selection of word forms → text

Noisy Channel: The Golden Rule of ...

OCR, ASR, HR, MT, ...

- Recall:

  $p(A|B) = p(B|A)\ p(A) / p(B)$ (Bayes formula)

  $A_{best} = \text{argmax}_A\ p(B|A)\ p(A)$ (The Golden Rule)

- $p(B|A)$: the acoustic/image/translation/lexical model
  - application-specific name
  - will explore later
- $p(A)$: ***the language model***

The Perfect Language Model

- Sequence of word forms [forget about tagging for the moment]
- Notation: $A \sim W = (w_1,w_2,w_3,...,w_d)$
- The big (modeling) question:

$$p(W) = ?$$

- Well, we know (Bayes/chain rule $\rightarrow$):

$$p(W) = p(w_1,w_2,w_3,...,w_d) =$$

$$= p(w_1) \times p(w_2|w_1) \times p(w_3|w_1,w_2) \times ... \times p(w_d|w_1,w_2,...,w_{d-1})$$

- Not practical (even short W $\rightarrow$ too many parameters)

Markov Chain

- Unlimited memory (cf. previous foil):
  - for w_i, we know <u>all</u> its predecessors $w_1,w_2,w_3,\ldots,w_{i-1}$
- Limited memory:
  - we disregard “too old” predecessors
  - remember only *k* previous words: $w_{i-k},w_{i-k+1},\ldots,w_{i-1}$
  - called “k^{th} order Markov approximation”
- + stationary character (no change over time):

$$p(W) \cong \Pi_{i=1..d} p(w_i|w_{i-k},w_{i-k+1},\ldots,w_{i-1}),\ d = |W|$$

n-gram Language Models

- $(n\text{-}1)^{th}$ order Markov approximation → n-gram LM:

$$\boxed{p(W) =_{df} \Pi_{i=1..d}\, p(\overbrace{w_i}^{\text{prediction}}|\overbrace{w_{i-n+1},w_{i-n+2},\ldots,w_{i-1}}^{\text{history}}) \quad \mathbf{!}}$$

- In particular (assume vocabulary |V| = 60k):
  - **0-gram LM: uniform model, $p(w) = 1/|V|$, 1 parameter**
  - **1-gram LM: unigram model, $p(w)$, 6×10^4 parameters**
  - **2-gram LM: bigram model, $p(w_i|w_{i-1})$ 3.6×10^9 parameters**
  - **3-gram LM: trigram model, $p(w_i|w_{i-2},w_{i-1})$ 2.16×10^{14} parameters**

Maximum Likelihood Estimate

- MLE: Relative Frequency...
  - ...best predicts the data at hand (the "training data")
- Trigrams from Training Data T:
  - count sequences of three words in T: $c_3(w_{i-2},w_{i-1},w_i)$

    [NB: notation: just saying that the three words follow each other]
  - count sequences of two words in T: $c_2(w_{i-1},w_i)$:
    - **either use $c_2(y,z) = \Sigma_w\, c_3(y,z,w)$**
    - **or count differently at the beginning (& end) of data!**

$$\boxed{\mathbf{p(w_i|w_{i-2},w_{i-1}) =_{est.} c_3(w_{i-2},w_{i-1},w_i) \,/\, c_2(w_{i-2},w_{i-1})} \;\; !}$$

LM: an Example

- Training data:

  <s> <s> He can buy the can of soda.

  - Unigram: $p_1(\text{He}) = p_1(\text{buy}) = p_1(\text{the}) = p_1(\text{of}) = p_1(\text{soda}) = p_1(.) = .125$

    $p_1(\text{can}) = .25$
  - Bigram: $p_2(\text{He}|\text{<s>}) = 1$, $p_2(\text{can}|\text{He}) = 1$, $p_2(\text{buy}|\text{can}) = .5$,

    $p_2(\text{of}|\text{can}) = .5$, $p_2(\text{the}|\text{buy}) = 1$,...
  - Trigram: $p_3(\text{He}|\text{<s>},\text{<s>}) = 1$, $p_3(\text{can}|\text{<s>},\text{He}) = 1$,

    $p_3(\text{buy}|\text{He},\text{can}) = 1$, $p_3(\text{of}|\text{the},\text{can}) = 1$, ..., $p_3(.|\text{of},\text{soda}) = 1$.
  - Entropy: $H(p_1) = 2.75$, $H(p_2) = .25$, $H(p_3) = 0$ $\leftarrow$ Great?!

LM: an Example (The Problem)

- Cross-entropy:
- S = <s> <s> It was the greatest buy of all.
- Even $H_S(p_1)$ fails (= $H_S(p_2) = H_S(p_3) = \infty$), because:
  - all unigrams but p_1(the), p_1(buy), p_1(of) and p_1(.) are 0.
  - all bigram probabilities are 0.
  - all trigram probabilities are 0.
- We want: to make all (theoretically possible*) probabilities non-zero.

*in fact, <u>all</u>: remember our graph from day 1?

LM Smoothing
(And the EM Algorithm)

Why do we need Nonzero Probs?

- To avoid infinite Cross Entropy:
  - happens when an event is found in test data which has not been seen in training data

    H(p) = ∞: prevents comparing data with > 0 "errors"
- To make the system more robust
  - low count estimates:
    - **they typically happen for "detailed" but relatively rare appearances**
  - high count estimates: reliable but less "detailed"

Eliminating the Zero Probabilities: Smoothing

- Get new p'(w) (same Ω): almost p(w) but no zeros
- Discount w for (some) p(w) > 0: new p'(w) < p(w)

$$\Sigma_{w \in \text{discounted}} (p(w) - p'(w)) = D$$

- Distribute D to all w; p(w) = 0: new p'(w) > p(w)
  - possibly also to other w with low p(w)
- For some w (possibly): p'(w) = p(w)
- Make sure $\Sigma_{w \in \Omega}$ p'(w) = 1
- There are many ways of ***smoothing***

Smoothing by Adding 1

- Simplest but not really usable:
  - Predicting words w from a vocabulary V, training data T:

    $p'(w|h) = (c(h,w) + 1) \;/\; (c(h) + |V|)$

    - **for non-conditional distributions: $p'(w) = (c(w) + 1) / (|T| + |V|)$**
  - Problem if $|V| > c(h)$ (as is often the case; even $\gg c(h)$!)
- Example: Training data: <s> what is it what is small ? $|T| = 8$
  - **V = { what, is, it, small, ?, <s>, flying, birds, are, a, bird, . }, |V| = 12**
  - **p(it)=.125, p(what)=.25, p(.)=0 p(what is it?) = $.25^2 \times .125^2 \cong$ .001**

    p(it is flying.) = $.125 \times .25 \times 0^2 = 0$
  - **p'(it) =.1, p'(what) =.15, p'(.)=.05 p'(what is it?) = $.15^2 \times .1^2 \cong$ .0002**

    p'(it is flying.) = $.1 \times .15 \times .05^2 \cong$ .00004

Adding *less than* 1

- Equally simple:
  - Predicting words w from a vocabulary V, training data T:

    $p'(w|h) = (c(h,w) + \lambda) \;/\; (c(h) + \lambda|V|),\ \lambda < 1$

    - **for non-conditional distributions: $p'(w) = (c(w) + \lambda) / (|T| + \lambda|V|)$**
- Example: Training data: <s> what is it what is small ? |T| = 8
    - **V = { what, is, it, small, ?, <s>, flying, birds, are, a, bird, . }, |V| = 12**
    - **p(it)=.125, p(what)=.25, p(.)=0 p(what is it?) = $.25^2 \times .125^2 \cong$ .001**

      p(it is flying.) = $.125 \times .25 \times 0^2 = 0$
    - **Use $\lambda = .1$:**
    - **p'(it)$\cong$ .12, p'(what)$\cong$ .23, p'(.)$\cong$ .01 p'(what is it?) = $.23^2 \times .12^2 \cong .0007$**

      p'(it is flying.) = $.12 \times .23 \times .01^2 \cong .000003$

Smoothing by Combination: Linear Interpolation

- Combine what?
  - **distributions of various level of detail vs. reliability**
- n-gram models:
  - **use (n-1)gram, (n-2)gram, ..., uniform**

reliability

detail

- Simplest possible combination:
  - sum of probabilities, normalize:
    - **p(0|0) = .8, p(1|0) = .2, p(0|1) = 1, p(1|1) = 0, p(0) = .4, p(1) = .6:**
    - **p’(0|0) = .6, p’(1|0) = .4, p’(0|1) = .7, p’(1|1) = .3**

Typical n-gram LM Smoothing

- Weight in less detailed distributions using $\lambda=(\lambda_0,\lambda_1,\lambda_2,\lambda_3)$:

  $p'_\lambda(w_i| w_{i-2},w_{i-1}) = \lambda_3\ p_3(w_i| w_{i-2},w_{i-1}) +$

  $\lambda_2\ p_2(w_i| w_{i-1}) + \lambda_1\ p_1(w_i) + \lambda_0\ /|V|$

- Normalize:

  $\lambda_i > 0,\ \Sigma_{i=0..n} \lambda_i = 1$ is sufficient ($\lambda_0 = 1 - \Sigma_{i=1..n} \lambda_i$) (n=3)

- Estimation using MLE:
  - <u>fix</u> the p_3, p_2, p_1 and $|V|$ parameters as estimated from the training data
  - then find such $\{\lambda_i\}$ which minimizes the cross entropy (maximizes probability of data): $-(1/|D|)\Sigma_{i=1..|D|}\log_2(p'_\lambda(w_i|h_i))$

Held-out (Cross-validation) Data

- What data to use?
  - try the training data T: but we will always get $\lambda_3 = 1$
    - **why? (let p_{iT} be an i-gram distribution estimated using r.f. from T)**
    - **minimizing $H_T(p'_\lambda)$ over a vector λ, $p'_\lambda = \lambda_3 p_{3T} + \lambda_2 p_{2T} + \lambda_1 p_{1T} + \lambda_0/|V|$**
      - remember: $H_T(p'_\lambda) = H(p_{3T}) + D(p_{3T} \| p'_\lambda)$;
        - (p_{3T} fixed $\rightarrow$ $H(p_{3T})$ fixed, best)
      - which p'_λ minimizes $H_T(p'_\lambda)$? ... a p'_λ for which $D(p_{3T} \| p'_\lambda)=0$
      - ...and that's p_{3T} (because $D(p\|p) = 0$, as we know).
      - ...and certainly $p'_\lambda = p_{3T}$ if $\lambda_3 = 1$ (maybe in some other cases, too).
      - ($p'_\lambda = 1 \times p_{3T} + 0 \times p_{2T} + 0 \times p_{1T} + 0/|V|$)
  - thus: do <u>not use the training data for estimation of λ!</u>
    - **must hold out part of the training data (*heldout* data, <u>H</u>):**
    - **...call the remaining data the (true/raw) *training* data, <u>T</u>**
    - **the *test* data <u>S</u> (e.g., for comparison purposes): still different data!**

The Formulas

- Repeat: minimizing $-(1/|H|)\Sigma_{i=1..|H|}\log_2(p'_\lambda(w_i|h_i))$ over λ

$$\boxed{\begin{aligned} p'_\lambda(w_i|\ h_i) = p'_\lambda(w_i|\ w_{i-2}, w_{i-1}) = \lambda_3\ p_3(w_i|\ w_{i-2}, w_{i-1}) + \\ \lambda_2\ p_2(w_i|\ w_{i-1}) + \lambda_1\ p_1(w_i) + \lambda_0\ /|V| \end{aligned}}\quad \textbf{!}$$

- "Expected Counts (of lambdas)": $j = 0..3$

E-step

$$\boxed{c(\lambda_j) = \Sigma_{i=1..|H|}\ (\lambda_j p_j(w_i|h_i)\ /\ p'_\lambda(w_i|h_i)) \quad \textbf{!}}$$

- "Next λ": $j = 0..3$

M-step

$$\boxed{\lambda_{j,next} = c(\lambda_j)\ /\ \Sigma_{k=0..3}\ (c(\lambda_k)) \quad \textbf{!}}$$

The (Smoothing) EM Algorithm

1. Start with some λ, such that $\lambda_j > 0$ for all $j \in 0..3$.
2. Compute "Expected Counts" for each λ_j.
3. Compute new set of λ_j, using the "Next λ" formula.
4. Start over at step 2, unless a termination condition is met.

- Termination condition: convergence of λ.
  - Simply set an ε, and finish if $|\lambda_j - \lambda_{j,next}| < \varepsilon$ for each j (step 3).
- Guaranteed to converge:

  follows from Jensen's inequality, plus a technical proof.

Remark on Linear Interpolation Smoothing

- "Bucketed" smoothing:
  - use several vectors of λ instead of one, based on (the frequency of) history: $\lambda(h)$
    - **e.g. for h = (micrograms,per) we will have**

      $\lambda(h)$ = (.999,.0009,.00009,.00001)

      (because "cubic" is the only word to follow...)
  - actually: not a separate set for each history, but rather a set for "similar" histories ("bucket"):

    $\lambda(b(h))$, where b: $V^2 \rightarrow N$ (in the case of trigrams)

    <u>b</u> classifies histories according to their reliability (~ frequency)

Bucketed Smoothing: The Algorithm

- First, determine the bucketing function b (use heldout!):
  - decide in advance you want e.g. 1000 buckets
  - compute the total frequency of histories in 1 bucket ($f_{max}(b)$)
  - gradually fill your buckets from the most frequent bigrams so that the sum of frequencies does not exceed $f_{max}(b)$ (you might end up with slightly more than 1000 buckets)
- Divide your heldout data according to buckets
- Apply the previous algorithm to each bucket and its data

Simple Example

- Raw distribution (unigram only; smooth with uniform):

  $p(a) = .25$, $p(b) = .5$, $p(\alpha) = 1/64$ for $\alpha \in \{c..r\}$, $= 0$ for the rest: s,t,u,v,w,x,y,z

- Heldout data: <u>baby</u>; use one set of λ (λ_1: unigram, λ_0: uniform)
- Start with $\lambda_1 = .5$; $p'_\lambda(b) = .5 \times .5 + .5 / 26 = .27$

  $p'_\lambda(a) = .5 \times .25 + .5 / 26 = .14$

  $p'_\lambda(y) = .5 \times 0 + .5 / 26 = .02$

$c(\lambda_1) = .5x.5/.27 + .5x.25/.14 + .5x.5/.27 + .5x0/.02 = 2.72$

$c(\lambda_0) = .5x.04/.27 + .5x.04/.14 + .5x.04/.27 + .5x.04/.02 = 1.28$

Normalize: $\lambda_{1,next} = .68$, $\lambda_{0,next} = .32$.

Repeat from step 2 (recompute p'_λ first for efficient computation, then $c(\lambda_i)$, ...)

Finish when new lambdas almost equal to the old ones (say, < 0.01 difference).