Eligibility Traces, Impala

Milan Straka

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EUROPEAN UNION
European Structural and Investment Fund
Operational Programme Research,
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Charles University in Prague
Faculty of Mathematics and Physics
Institute of Formal and Applied Linguistics

Off-policy Correction Using Control Variates

Let $G_{t:t+n}$ be the estimated n-step return

$$G_{t:t+n} \stackrel{\text{def}}{=} \left(\sum_{k=t}^{t+n-1} \gamma^{k-t} R_{k+1}\right) + \Big[\text{episode still running in } t+n\Big] \gamma^n V(S_{t+n}),$$

which can be written recursively as

$$G_{t:t+n} \begin{cases} 0 & \text{if episode ended before } t, \\ V(S_t) & \text{if } n = 0, \\ R_{t+1} + \gamma G_{t+1:t+n} & \text{otherwise.} \end{cases}$$

For simplicity, we do not explicitly handle the first case (“the episode has already ended”) in the following.

Off-policy Correction Using Control Variates

Note that we can write

$$\begin{aligned}
G_{t:t+n} - V(S_t) &= R_{t+1} + \gamma G_{t+1:t+n} - V(S_t) \\
&= R_{t+1} + \gamma\big(G_{t+1:t+n} - V(S_{t+1})\big) + \gamma V(S_{t+1}) - V(S_t),
\end{aligned}$$

which yields

$$G_{t:t+n} - V(S_t) = R_{t+1} + \gamma V(S_{t+1}) - V(S_t) + \gamma\big(G_{t+1:t+n} - V(S_{t+1})\big).$$

Denoting the TD error as $\delta_t \stackrel{\text{def}}{=} R_{t+1} + \gamma V(S_{t+1}) - V(S_t)$, we can therefore write the n-step estimated return as a sum of TD errors:

$$G_{t:t+n} = V(S_t) + \sum_{i=0}^{n-1} \gamma^i \delta_{t+i}.$$

Incidentally, to correctly handle the "the episode has already ended" case, it would be enough to define $\delta_t \stackrel{\text{def}}{=} R_{t+1} + [\neg\text{done}] \cdot \gamma V(S_{t+1}) - V(S_t)$.

Return Formulations

Recursive definition	Formulation with TD errors
$G_{t:t+n} \stackrel{\text{def}}{=} R_{t+1} + \gamma G_{t+1:t+n}$	$V(S_t) + \sum_{i=0}^{n-1} \gamma^i \delta_{t+i}$

Off-policy Correction Using Control Variates

Now consider applying the IS off-policy correction to $G_{t:t+n}$ using the importance sampling ratio

$$\rho_t \stackrel{\text{def}}{=} \frac{\pi(A_t|S_t)}{b(A_t|S_t)}, \quad \rho_{t:t+n} \stackrel{\text{def}}{=} \prod_{i=0}^{n} \rho_{t+i}.$$

First note that

$$\mathbb{E}_{A_t\sim b}\big[\rho_t\big] = \sum_{A_t} b(A_t|S_t)\frac{\pi(A_t|S_t)}{b(A_t|S_t)} = 1,$$

which can be extended to

$$\mathbb{E}_b\big[\rho_{t:t+n}\big] = 1.$$

Off-policy Correction Using Control Variates

Until now, we used

$$G_{t:t+n}^{\mathrm{IS}} \stackrel{\text{def}}{=} \rho_{t:t+n-1} G_{t:t+n}.$$

However, such correction has unnecessary variance. Notably, when expanding $G_{t:t+n}$

$$G_{t:t+n}^{\mathrm{IS}} = \rho_{t:t+n-1}\big(R_{t+1} + \gamma G_{t+1:t+n}\big),$$

the R_{t+1} depends only on ρ_t, not on $\rho_{t+1:t+n-1}$, and given that the expectation of the importance sampling ratio is 1, we can simplify to

$$G_{t:t+n}^{\mathrm{IS}} = \rho_t R_{t+1} + \rho_{t:t+n-1}\gamma G_{t+1:t+n}.$$

Such an estimate can be written recursively as

$$G_{t:t+n}^{\mathrm{IS}} = \rho_t\big(R_{t+1} + \gamma G_{t+1:t+n}^{\mathrm{IS}}\big).$$

Recursive definition	Formulation with TD errors
$G_{t:t+n} \stackrel{\text{def}}{=} R_{t+1} + \gamma G_{t+1:t+n}$	$V(S_t) + \sum_{i=0}^{n-1} \gamma^i \delta_{t+i}$
$G_{t:t+n}^{\text{IS}} \stackrel{\text{def}}{=} \rho_t \big(R_{t+1} + \gamma G_{t+1:t+n}^{\text{IS}}\big)$	

Off-policy Correction Using Control Variates

We can reduce the variance even further – when $\rho_t = 0$, we might consider estimating the return using $V(S_t)$ instead of 0.

To utilize this idea, we turn to **control variates**, which is a general method of reducing variance of Monte Carlo estimators. Let μ be an unknown expectation, which we estimate using an unbiased estimator m. Assume we have another **correlated** statistic k with a known expectation κ.

We can then use an estimate $m^* \stackrel{\text{def}}{=} m - c(k - \kappa)$, which is also an unbiased estimator of μ, with variance

$$\operatorname{Var}(m^*) = \operatorname{Var}(m) + c^2 \operatorname{Var}(k) - 2c \operatorname{Cov}(m, k).$$

To arrive at the optimal value of c, we can set the derivative of $\operatorname{Var}(m^*)$ to 0, obtaining

$$c = \frac{\operatorname{Cov}(m, k)}{\operatorname{Var}(k)}.$$

Off-policy Correction Using Control Variates

In case of the value function estimate

$$G^{\text{IS}}_{t:t+n} = \rho_t\big(R_{t+1} + \gamma G^{\text{IS}}_{t+1:t+n}\big),$$

we might consider using ρ_t as the correlated statistic k, with known expectation $\kappa = 1$, because if $\rho_t \gg 1$, then our return estimate is probably an overestimate, and vice versa.

The optimal value of c should then be

$$c = \frac{\operatorname{Cov}(m, k)}{\operatorname{Var}(k)} = \frac{\mathbb{E}_b\Big[\big(G^{\text{IS}}_{t:t+n} - v_\pi(S_t)\big)(\rho_t - 1)\Big]}{\mathbb{E}_b\big[(\rho_t - 1)^2\big]},$$

which is however difficult to compute. Instead, considering the estimate when $\rho_t = 0$, we get

$$\rho_t\big(R_{t+1} + \gamma G^{\text{IS}}_{t+1:t+n}\big) + c(1 - \rho_t) \xlongequal{\rho_t=0} c.$$

Because a reasonable estimate in case of $\rho_t = 0$ is $V(S_t)$, we use $c = V(S_t)$.

The estimate with the **control variate** term is therefore

$$G_{t:t+n}^{\mathrm{CV}} \stackrel{\text{def}}{=} \rho_t\big(R_{t+1} + \gamma G_{t+1:t+n}^{\mathrm{CV}}\big) + (1-\rho_t)V(S_t),$$

which adds no bias, since the expected value of $1-\rho_t$ is zero and ρ_t and S_t are independent.

Similarly as before, rewriting to

$$\begin{aligned} G_{t:t+n}^{\mathrm{CV}} - V(S_t) &= \rho_t\big(R_{t+1} + \gamma G_{t+1:t+n}^{\mathrm{CV}}\big) - \rho_t V(S_t) \\ &= \rho_t\big(R_{t+1} + \gamma V(S_{t+1}) - V(S_t) + \gamma(G_{t+1:t+n}^{\mathrm{CV}} - V(S_{t+1}))\big) \end{aligned}$$

results in

$$G_{t:t+n}^{\mathrm{CV}} = V(S_t) + \sum\nolimits_{i=0}^{n-1} \gamma^i \rho_{t:t+i} \delta_{t+i}.$$

Return Formulations

Recursive definition	Formulation with TD errors
$G_{t:t+n} \stackrel{\text{def}}{=} R_{t+1} + \gamma G_{t+1:t+n}$	$V(S_t) + \sum_{i=0}^{n-1} \gamma^i \delta_{t+i}$
$G_{t:t+n}^{\text{IS}} \stackrel{\text{def}}{=} \rho_t \big(R_{t+1} + \gamma G_{t+1:t+n}^{\text{IS}}\big)$	
$G_{t:t+n}^{\text{CV}} \stackrel{\text{def}}{=} \rho_t \big(R_{t+1} + \gamma G_{t+1:t+n}^{\text{CV}}\big) + (1 - \rho_t)V(S_t)$	$V(S_t) + \sum_{i=0}^{n-1} \gamma^i \rho_{t:t+i} \delta_{t+i}$

Eligibility Traces

Eligibility traces are a mechanism of combining multiple n-step return estimates for various values of n.

First note instead of an n-step return, we can use any average of n-step returns for different values of n, for example $\frac{2}{3}G_{t:t+2} + \frac{1}{3}G_{t:t+4}$.

For a given $\lambda \in [0,1]$, we define **λ-return** as

$$G_t^\lambda \stackrel{\text{def}}{=} (1-\lambda)\sum_{i=1}^{\infty} \lambda^{i-1} G_{t:t+i}.$$

Alternatively, the λ-return can be written recursively as

$$G_t^\lambda = (1-\lambda)G_{t:t+1} + \lambda(R_{t+1} + \gamma G_{t+1}^\lambda).$$

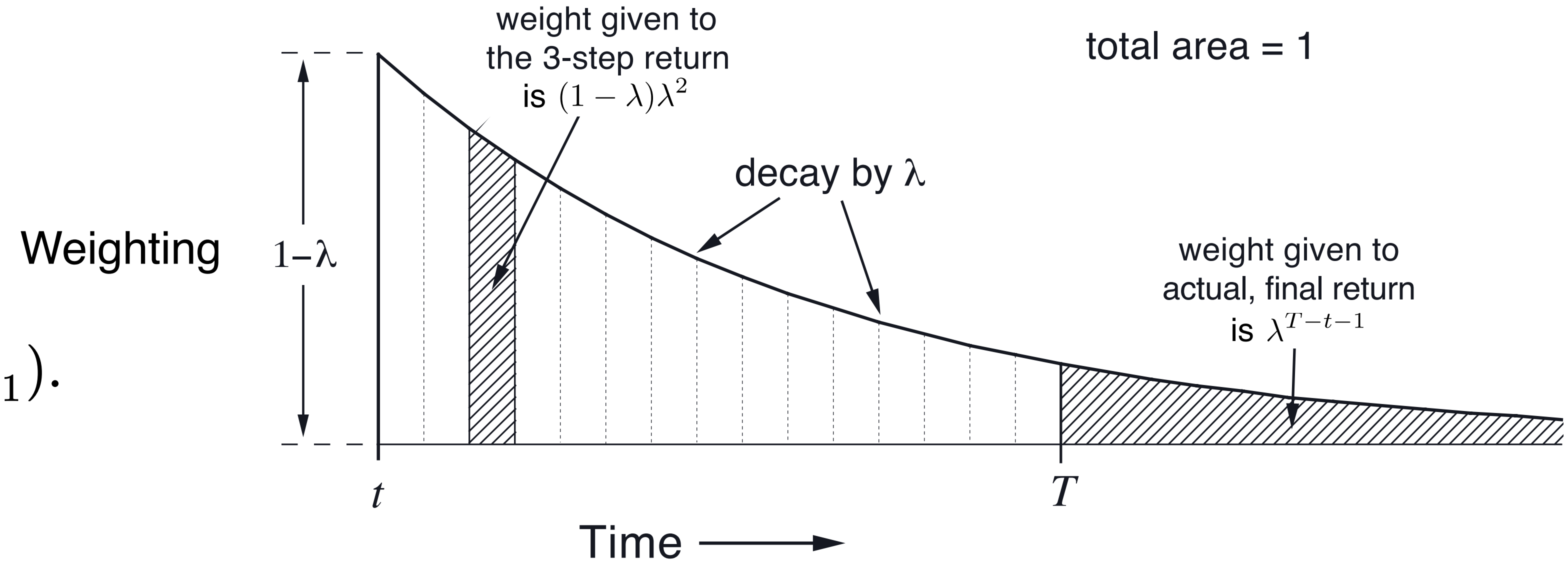


Figure 12.2: Weighting given in the λ-return to each of the n-step returns.

Figure 12.2 of "Reinforcement Learning: An Introduction, Second Edition".

λ-return

In an episodic task with time of termination T, we can rewrite the λ-return to

$$G_t^\lambda = (1-\lambda)\sum_{i=1}^{T-t-1}\lambda^{i-1}G_{t:t+i} + \lambda^{T-t-1}G_t.$$

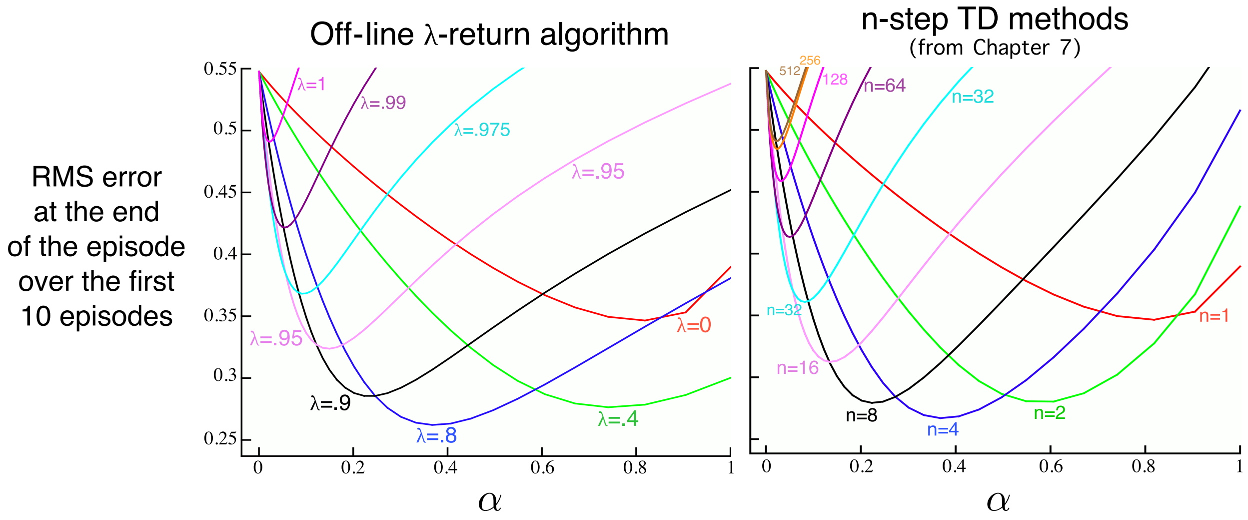


Figure 12.3: 19-state Random walk results (Example 7.1): Performance of the off-line λ-return algorithm alongside that of the n-step TD methods. In both case, intermediate values of the bootstrapping parameter (λ or n) performed best. The results with the off-line λ-return algorithm are slightly better at the best values of α and λ, and at high α.

Figure 12.3 of "Reinforcement Learning: An Introduction, Second Edition".

Truncated λ-return

We might also set a limit on the largest value of n, obtaining **truncated λ-return**

$$G^\lambda_{t:t+n} \stackrel{\text{def}}{=} (1-\lambda)\sum_{i=1}^{n-1} \lambda^{i-1} G_{t:t+i} + \lambda^{n-1} G_{t:t+n}.$$

The truncated λ return can be again written recursively as

$$G^\lambda_{t:t+n} = (1-\lambda) G_{t:t+1} + \lambda(R_{t+1} + \gamma G^\lambda_{t+1:t+n}),\ \ G^\lambda_{t:t+1} = G_{t:t+1}.$$

Similarly to before, we can express the truncated λ return as a sum of TD errors

$$\begin{aligned} G^\lambda_{t:t+n} - V(S_t) &= (1-\lambda)\big(R_{t+1} + \gamma V(S_{t+1})\big) + \lambda(R_{t+1} + \gamma G^\lambda_{t+1:t+n}) - V(S_t) \\ &= R_{t+1} + \gamma V(S_{t+1}) - V(S_t) + \lambda\gamma\big(G^\lambda_{t+1:t+n} - V(S_{t+1})\big), \end{aligned}$$

obtaining an analogous estimate $G^\lambda_{t:t+n} = V(S_t) + \sum_{i=0}^{n-1} \gamma^i \lambda^i \delta_{t+i}$.

Variable λs

The (truncated) λ-return can be generalized to utilize different λ_i at each step i. Notably, we can generalize the recursive definition

$$G_{t:t+n}^\lambda = (1-\lambda)G_{t:t+1} + \lambda(R_{t+1} + \gamma G_{t+1:t+n}^\lambda)$$

to

$$G_{t:t+n}^{\lambda_i} = (1-\lambda_{t+1})G_{t:t+1} + \lambda_{t+1}(R_{t+1} + \gamma G_{t+1:t+n}^{\lambda_i}),$$

and express this quantity again by a sum of TD errors:

$$G_{t:t+n}^{\lambda_i} = V(S_t) + \sum_{i=0}^{n-1} \gamma^i \left(\prod_{j=1}^{i} \lambda_{t+j}\right) \delta_{t+i}.$$

Off-policy Traces with Control Variates

Finally, we can combine the eligibility traces with off-policy estimation using control variates:

$$G_{t:t+n}^{\lambda,\mathrm{CV}} \stackrel{\text{def}}{=} (1-\lambda)\sum_{i=1}^{n-1}\lambda^{i-1}G_{t:t+i}^{\mathrm{CV}} + \lambda^{n-1}G_{t:t+n}^{\mathrm{CV}}.$$

Recalling that

$$G_{t:t+n}^{\mathrm{CV}} = \rho_t\big(R_{t+1} + \gamma G_{t+1:t+n}^{\mathrm{CV}}\big) + (1-\rho_t)V(S_t),$$

we can rewrite $G_{t:t+n}^{\lambda,\mathrm{CV}}$ recursively as

$$G_{t:t+n}^{\lambda,\mathrm{CV}} = (1-\lambda)G_{t:t+1}^{\mathrm{CV}} + \lambda\Big(\rho_t\big(R_{t+1} + \gamma G_{t+1:t+n}^{\lambda,\mathrm{CV}}\big) + (1-\rho_t)V(S_t)\Big),$$

which we can simplify by expanding $G_{t:t+1}^{\mathrm{CV}} = \rho_t(R_{t+1} + \gamma V(S_{t+1})) + (1-\rho_t)V(S_t)$ to

$$G_{t:t+n}^{\lambda,\mathrm{CV}} - V(S_t) = \rho_t\big(R_{t+1} + \gamma V(S_{t+1}) - V(S_t)\big) + \gamma\lambda\rho_t\big(G_{t+1:t+n}^{\lambda,\mathrm{CV}} - V(S_{t+1})\big).$$

Off-policy Traces with Control Variates

Consequently, analogously as before, we can write the off-policy traces estimate with control variates as

$$G_{t:t+n}^{\lambda,\mathrm{CV}} = V(S_t) + \sum\nolimits_{i=0}^{n-1} \gamma^i \lambda^i \rho_{t:t+i} \delta_{t+i},$$

and by repeating the above derivation we can extend the result also for time-variable λ_i, we obtain

$$G_{t:t+n}^{\lambda_\mathrm{i},\mathrm{CV}} = V(S_t) + \sum\nolimits_{i=0}^{n-1} \gamma^i \left(\prod_{j=1}^{i} \lambda_{t+j}\right) \rho_{t:t+i} \delta_{t+i}.$$

Return Recapitulation

Recursive definition	Formulation with TD errors
$G_{t:t+n} \stackrel{\text{def}}{=} R_{t+1} + \gamma G_{t+1:t+n}$	$V(S_t) + \sum_{i=0}^{n-1} \gamma^i \delta_{t+i}$
$G_{t:t+n}^{\text{IS}} \stackrel{\text{def}}{=} \rho_t \big(R_{t+1} + \gamma G_{t+1:t+n}^{\text{IS}}\big)$	
$G_{t:t+n}^{\text{CV}} \stackrel{\text{def}}{=} \rho_t \big(R_{t+1} + \gamma G_{t+1:t+n}^{\text{CV}}\big) + (1-\rho_t)V(S_t)$	$V(S_t) + \sum_{i=0}^{n-1} \gamma^i \rho_{t:t+i} \delta_{t+i}$
$G_{t:t+n}^{\lambda} \stackrel{\text{def}}{=} (1-\lambda)G_{t:t+1} + \lambda(R_{t+1} + \gamma G_{t+1:t+n}^{\lambda})$	$V(S_t) + \sum_{i=0}^{n-1} \gamma^i \lambda^i \delta_{t+i}$
$G_{t:t+n}^{\lambda_i} \stackrel{\text{def}}{=} (1-\lambda_{t+1})G_{t:t+1} + \lambda_{t+1}\big(R_{t+1} + \gamma G_{t+1:t+n}^{\lambda_i}\big)$	$V(S_t) + \sum_{i=0}^{n-1} \gamma^i \left(\prod_{j=1}^{i} \lambda_{t+j}\right) \delta_{t+i}$
$G_{t:t+n}^{\lambda,\text{CV}} \stackrel{\text{def}}{=} (1-\lambda)G_{t:t+1}^{\text{CV}}$ $+ \lambda\Big(\rho_t\big(R_{t+1} + \gamma G_{t+1:t+n}^{\lambda,\text{CV}}\big) + (1-\rho_t)V(S_t)\Big)$	$V(S_t) + \sum_{i=0}^{n-1} \gamma^i \lambda^i \rho_{t:t+i} \delta_{t+i}$
$G_{t:t+n}^{\lambda_i,\text{CV}} \stackrel{\text{def}}{=} (1-\lambda_{t+1})G_{t:t+1}^{\text{CV}}$ $+\lambda_{t+1}\Big(\rho_t\big(R_{t+1} + \gamma G_{t+1:t+n}^{\lambda_i,\text{CV}}\big) + (1-\rho_t)V(S_t)\Big)$	$V(S_t)$ $+ \sum_{i=0}^{n-1} \gamma^i \left(\prod_{j=1}^{i} \lambda_{t+j}\right) \rho_{t:t+i} \delta_{t+i}$

TD(λ)

We have defined the λ-return in the so-called **forward view**.

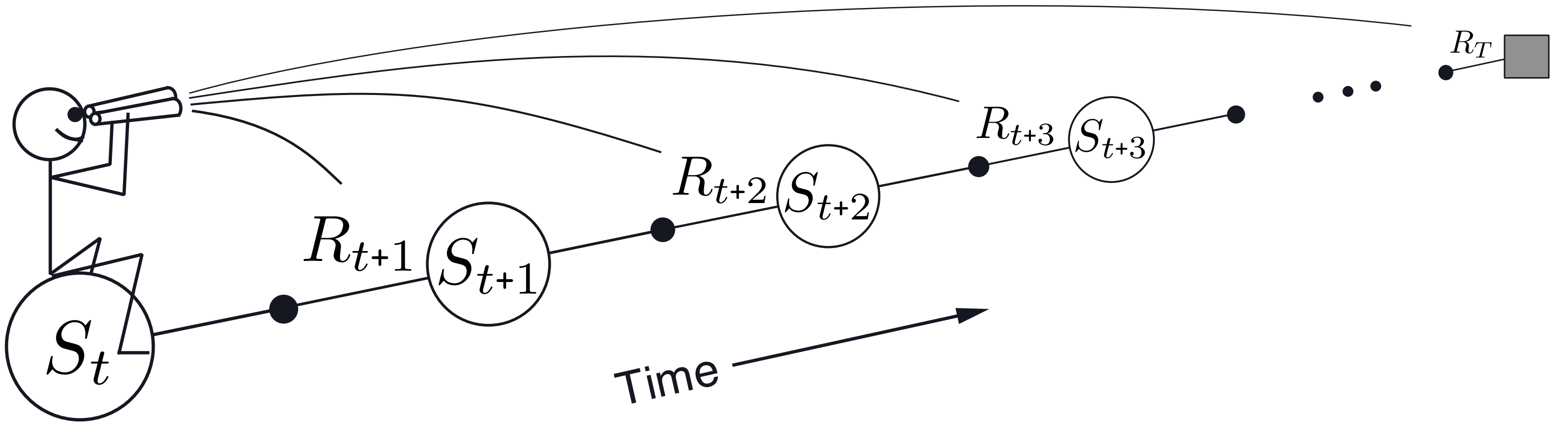


Figure 12.4: The forward view. We decide how to update each state by looking forward to future rewards and states.

Figure 12.4 of "Reinforcement Learning: An Introduction, Second Edition".

However, to allow on-line updates, we might consider also the **backward view**

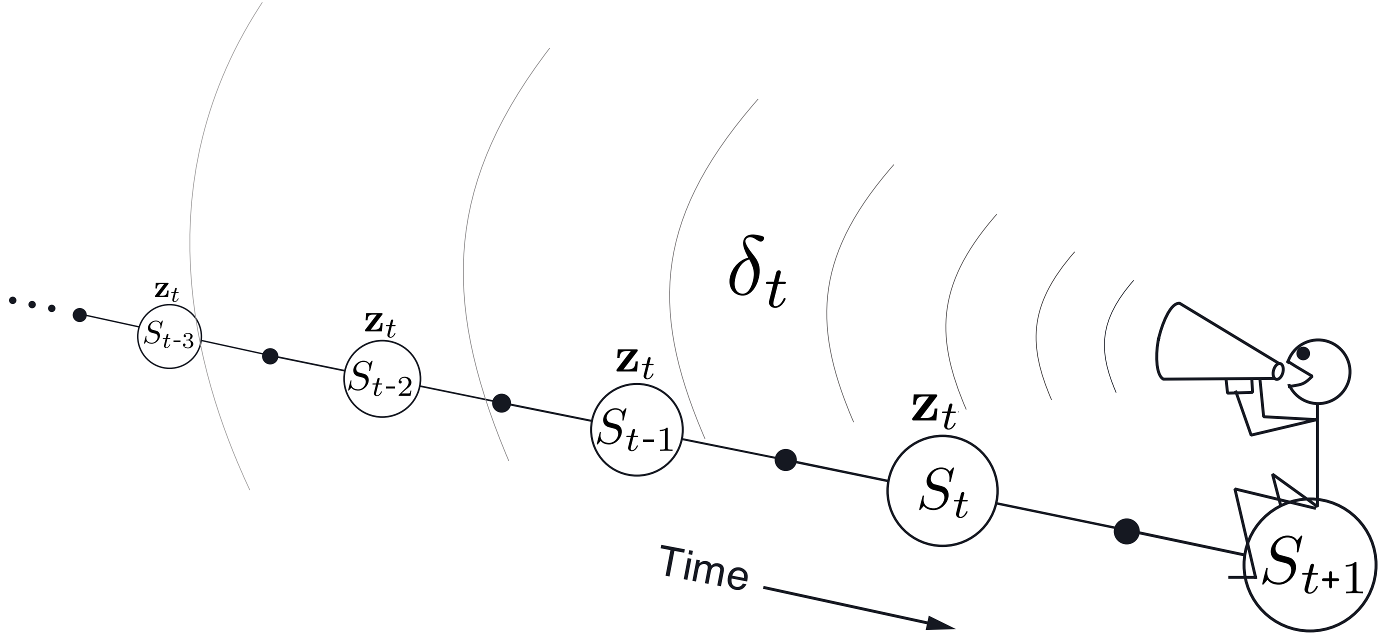


Figure 12.5: The backward or mechanistic view of TD(λ). Each update depends on the current TD error combined with the current eligibility traces of past events.

Figure 12.5 of "Reinforcement Learning: An Introduction, Second Edition".

TD(λ) is an algorithm implementing on-line policy evaluation utilizing the backward view.

Semi-gradient TD(λ) for estimating $\hat v \approx v_\pi$

Input: the policy π to be evaluated
Input: a differentiable function $\hat v : \mathcal{S}^+ \times \mathbb{R}^d \rightarrow \mathbb{R}$ such that $\hat v(\text{terminal},\cdot) = 0$
Algorithm parameters: step size $\alpha > 0$, trace decay rate $\lambda \in [0, 1]$
Initialize value-function weights $\mathbf{w}$ arbitrarily (e.g., $\mathbf{w} = \mathbf{0}$)

Loop for each episode:
  Initialize S
  $\mathbf{z} \leftarrow \mathbf{0}$ (a d-dimensional vector)
  Loop for each step of episode:
  | Choose $A \sim \pi(\cdot|S)$
  | Take action A, observe R, S'
  | $\mathbf{z} \leftarrow \gamma\lambda\mathbf{z} + \nabla\hat v(S,\mathbf{w})$
  | $\delta \leftarrow R + \gamma\hat v(S',\mathbf{w}) - \hat v(S,\mathbf{w})$
  | $\mathbf{w} \leftarrow \mathbf{w} + \alpha\delta\mathbf{z}$
  | $S \leftarrow S'$
  until S' is terminal

Algorithm 12.2 of "Reinforcement Learning: An Introduction, Second Edition".

V-trace

V-trace is a modified version of n-step return with off-policy correction, defined in the Feb 2018 IMPALA paper as (using the notation from the paper):

$$G_{t:t+n}^{\text{V-trace}} \stackrel{\text{def}}{=} V(S_t) + \sum_{i=0}^{n-1} \gamma^i \left(\prod\nolimits_{j=0}^{i-1} \bar{c}_{t+j} \right) \bar{\rho}_{t+i} \delta_{t+i},$$

where $\bar{\rho}_t$ and $\bar{c}_t$ are the truncated importance sampling ratios for $\bar{\rho} \geq \bar{c}$:

$$\bar{\rho}_t \stackrel{\text{def}}{=} \min\left(\bar{\rho}, \frac{\pi(A_t|S_t)}{b(A_t|S_t)}\right), \quad \bar{c}_t \stackrel{\text{def}}{=} \min\left(\bar{c}, \frac{\pi(A_t|S_t)}{b(A_t|S_t)}\right).$$

Note that if $b = \pi$ and assuming $\bar{c} \geq 1$, v_s reduces to n-step Bellman target.

V-trace

Note that the truncated IS weights $\bar{\rho}_t$ and $\bar{c}_t$ play different roles:

- The $\bar{\rho}_t$ appears defines the fixed point of the update rule. For $\bar{\rho} = \infty$, the target is the value function v_π, if $\bar{\rho} < \infty$, the fixed point is somewhere between v_π and v_b. Notice that we do not compute a product of these $\bar{\rho}_t$ coefficients.

  Concretely, the fixed point of an operator defined by $G_{t:t+n}^{\text{V-trace}}$ corresponds to a value function of the policy

$$\pi_{\bar{\rho}}(a|s) \propto \min\left(\bar{\rho}b(a|s), \pi(a|s)\right).$$

- The $\bar{c}_t$ impacts the speed of convergence (the contraction rate of the Bellman operator), not the sought policy. Because a product of the $\bar{c}_t$ ratios is computed, it plays an important role in variance reduction.

However, the paper utilizes $\bar{c} = 1$ and out of $\bar{\rho} \in \{1, 10, 100\}$, $\bar{\rho} = 1$ works empirically the best, so the distinction between $\bar{c}_t$ and $\bar{\rho}_t$ is not useful in practice.

V-trace Analysis

Let us define the (untruncated for simplicity; similar results can be proven for a truncated one) V-trace operator $\mathcal{R}$ as:

$$\mathcal{R}V(S_t) \stackrel{\text{def}}{=} V(S_t) + \mathbb{E}_b\left[\sum\nolimits_{i\geq 0} \gamma^i \left(\prod\nolimits_{j=0}^{i-1} \bar{c}_{t+j}\right) \bar{\rho}_{t+i}\delta_{t+i}\right],$$

where the expectation $\mathbb{E}_b$ is with respect to trajectories generated by behaviour policy b.

Assuming there exists $\beta \in (0, 1]$ such that $\mathbb{E}_b\bar{\rho}_0 \geq \beta$, it can be proven (see Theorem 1 in Appendix A.1 in the Impala paper if interested) that such an operator is a contraction with a contraction constant

$$\gamma^{-1} - (\gamma^{-1} - 1)\underbrace{\sum\nolimits_{i\geq 0} \gamma^i \mathbb{E}_b\left[\left(\prod\nolimits_{j=0}^{i-1} \bar{c}_j\right) \bar{\rho}_i\right]}_{\geq 1 + \gamma\mathbb{E}_b\bar{\rho}_0} \leq 1 - (1-\gamma)\beta < 1,$$

therefore, $\mathcal{R}$ has a unique fixed point.

V-trace Analysis

We now prove that the fixed point of $\mathcal{R}$ is $V^{\pi_{\bar\rho}}$. We have:

$$\begin{aligned}
\mathbb{E}_b\left[\bar\rho_t\delta_t\right] &= \mathbb{E}_b\Big[\bar\rho_t\big(R_{t+1} + \gamma V^{\pi_{\bar\rho}}(S_{t+1}) - V^{\pi_{\bar\rho}}(S_t)\big)\Big|S_t\Big] \\
&= \sum\nolimits_a b(a|S_t)\min\left(\bar\rho, \frac{\pi(a|S_t)}{b(a|S_t)}\right)\Big[R_{t+1} + \gamma\mathbb{E}_{s'\sim p(S_t,a)}V^{\pi_{\bar\rho}}(s') - V^{\pi_{\bar\rho}}(S_t)\Big] \\
&= \underbrace{\sum\nolimits_a \pi_{\bar\rho}(a|S_t)\Big[R_{t+1} + \gamma\mathbb{E}_{s'\sim p(S_t,a)}V^{\pi_{\bar\rho}}(s') - V^{\pi_{\bar\rho}}(S_t)\Big]}_{=0}\sum_{a'}\min\big(\bar\rho b(a'|S_t), \pi(a'|S_t)\big) \\
&= 0,
\end{aligned}$$

where the tagged part is zero, since it is the Bellman equation for $V^{\pi_{\bar\rho}}$. This shows that $\mathcal{R}V^{\pi_{\bar\rho}}(s) = V^{\pi_{\bar\rho}}(s) + \mathbb{E}_b\Big[\sum_{i\geq 0}\gamma^i\Big(\prod_{j=0}^{i-1}\bar c_{t+j}\Big)\bar\rho_{t+i}\delta_{t+i}\Big] = V^{\pi_{\bar\rho}}$, and therefore $V^{\pi_{\bar\rho}}$ is the unique fixed point of $\mathcal{R}$.

Consequently, in $G_{t:t+n}^{\lambda_\mathrm{i},\mathrm{CV}} = V(S_t) + \sum_{i=0}^{n-1}\gamma^i\big(\prod_{j=1}^{i}\lambda_{t+j}\big)\rho_{t:t+i}\delta_{t+i}$, only the last ρ_{t+i} from every $\rho_{t:t+i}$ is actually needed for off-policy correction; $\rho_{t:t+i-1}$ can be considered as traces.

Impala (**Imp**ortance Weighted **A**ctor-**L**earner **A**rchitecture) was suggested in Feb 2018 paper and allows massively distributed implementation of an actor-critic-like learning algorithm.

Compared to A3C-based agents, which communicate gradients with respect to the parameters of the policy, IMPALA actors communicate trajectories to the centralized learner.

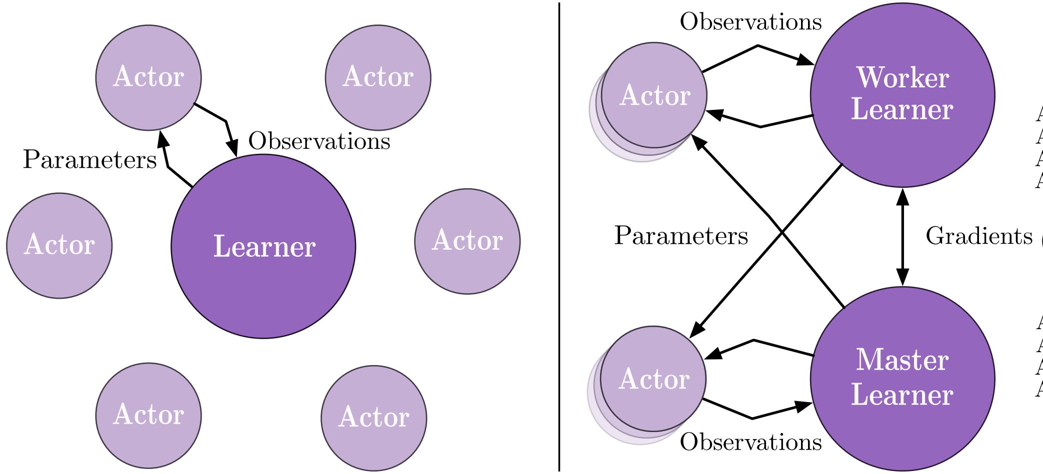


Figure 1 of "IMPALA: Scalable Distributed Deep-RL with Importance Weighted Actor-Learner Architectures" by Lasse Espeholt et al.

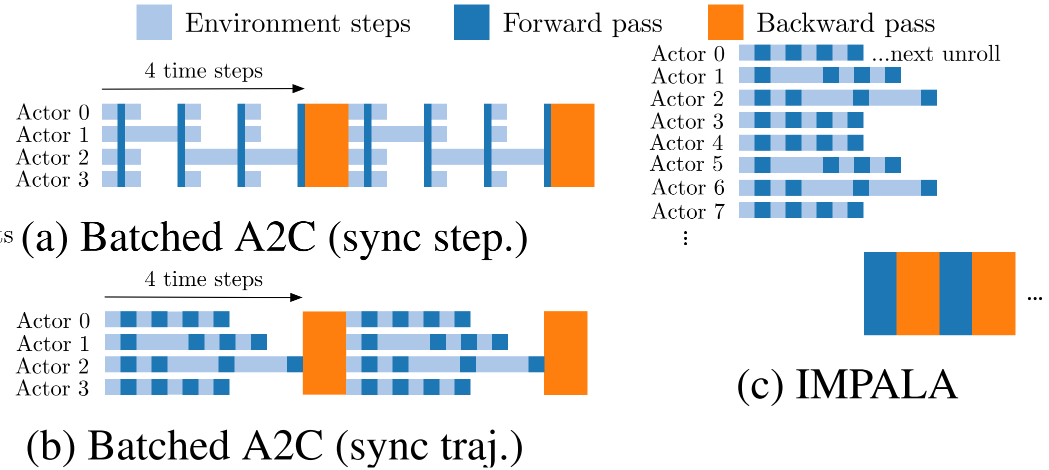


Figure 2 of "IMPALA: Scalable Distributed Deep-RL with Importance Weighted Actor-Learner Architectures" by Lasse Espeholt et al.

If many actors are used, the policy used to generate a trajectory can lag behind the latest policy. Therefore, the V-trace off-policy actor-critic algorithm is employed.

Consider a parametrized functions computing $v(s; \boldsymbol\theta)$ and $\pi(a|s; \boldsymbol\omega)$, we update the critic in the direction of

$$\Big(G_{t:t+n}^\text{V-trace} - v(S_t; \boldsymbol\theta)\Big) \nabla_{\boldsymbol\theta} v(S_t; \boldsymbol\theta),$$

and the actor in the direction of the policy gradient

$$\bar\rho_t \nabla_{\boldsymbol\omega} \log \pi(A_t|S_t; \boldsymbol\omega) \big(R_{t+1} + \gamma G_{t+1:t+n}^\text{V-trace} - v(S_t; \boldsymbol\theta)\big).$$

Finally, we again add the entropy regularization term $\beta H\big(\pi(\cdot|S_t; \boldsymbol\omega)\big)$ to the loss function.

Architecture	CPUs	GPUs[1]	FPS[2]	
Single-Machine			Task 1	Task 2
A3C 32 workers	64	0	6.5K	9K
Batched A2C (sync step)	48	0	9K	5K
Batched A2C (sync step)	48	1	13K	5.5K
Batched A2C (sync traj.)	48	0	16K	17.5K
Batched A2C (dyn. batch)	48	1	16K	13K
IMPALA 48 actors	48	0	17K	20.5K
IMPALA (dyn. batch) 48 actors[3]	48	1	21K	24K
Distributed				
A3C	200	0	46K	50K
IMPALA	150	1	80K	
IMPALA (optimised)	375	1	200K	
IMPALA (optimised) batch 128	500	1	250K	

[1] Nvidia P100 [2] In frames/sec (4 times the agent steps due to action repeat). [3] Limited by amount of rendering possible on a single machine.

Table 1 of "IMPALA: Scalable Distributed Deep-RL with Importance Weighted Actor-Learner Architectures" by Lasse Espeholt et al.

For Atari experiments, population based training with a population of 24 agents is used to adapt entropy regularization, learning rate, RMSProp ε and the global gradient norm clipping threshold.

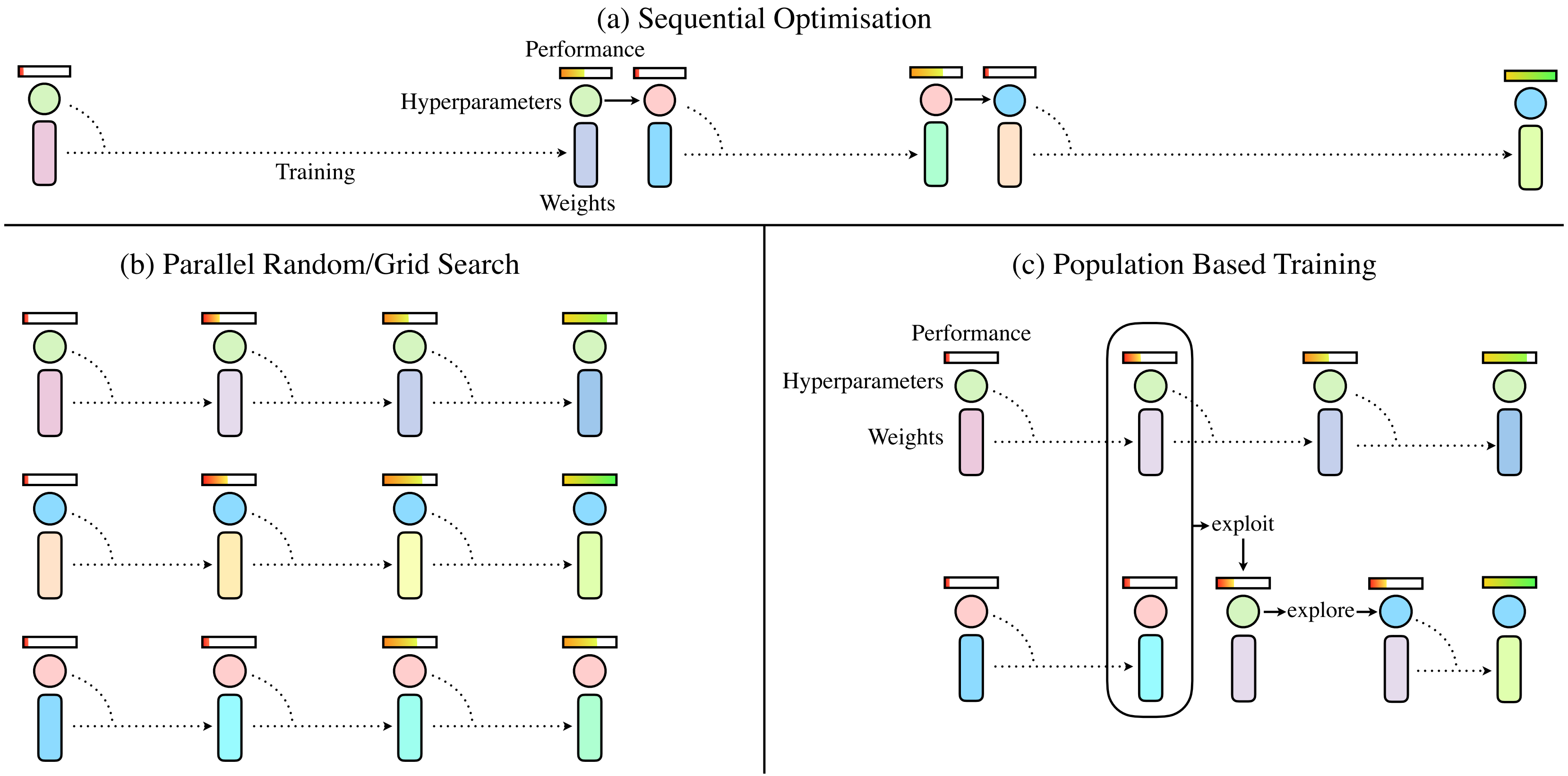


Figure 1 of "Population Based Training of Neural Networks" by Max Jaderberg et al.

IMPALA – Population Based Training

For Atari experiments, population based training with a population of 24 agents is used to adapt entropy regularization, learning rate, RMSProp ε and the global gradient norm clipping threshold.

In population based training, several agents are trained in parallel. When an agent is *ready* (after 5000 episodes), then:

- it may be overwritten by parameters and hyperparameters of another randomly chosen agent, if it is sufficiently better (5000 episode mean capped human normalized score returns are 5% better);
- and independently, the hyperparameters may undergo a change (multiplied by either 1.2 or 1/1.2 with 33% chance).

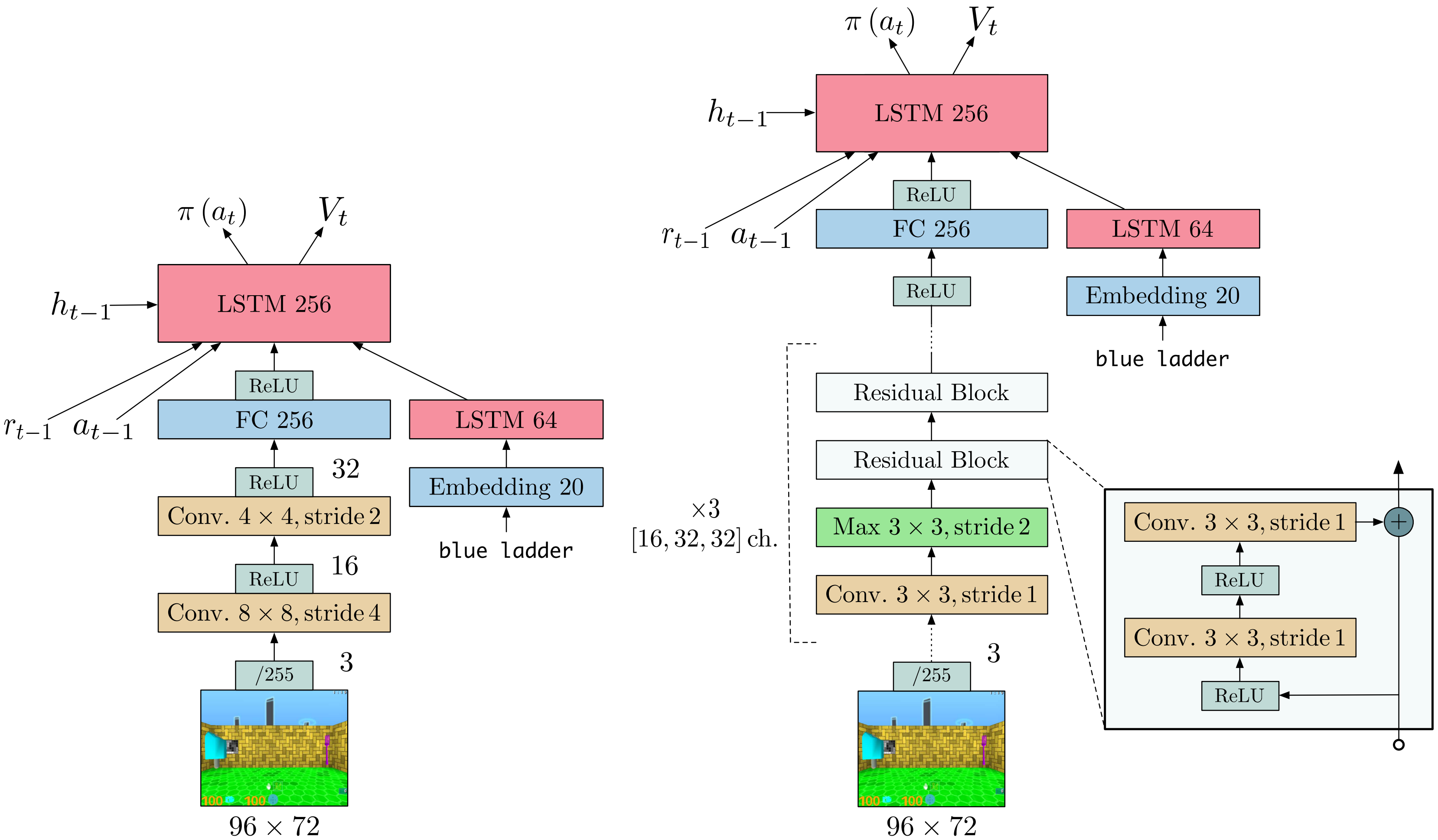


Figure 3 of "IMPALA: Scalable Distributed Deep-RL with Importance Weighted Actor-Learner Architectures" by Lasse Espeholt et al.

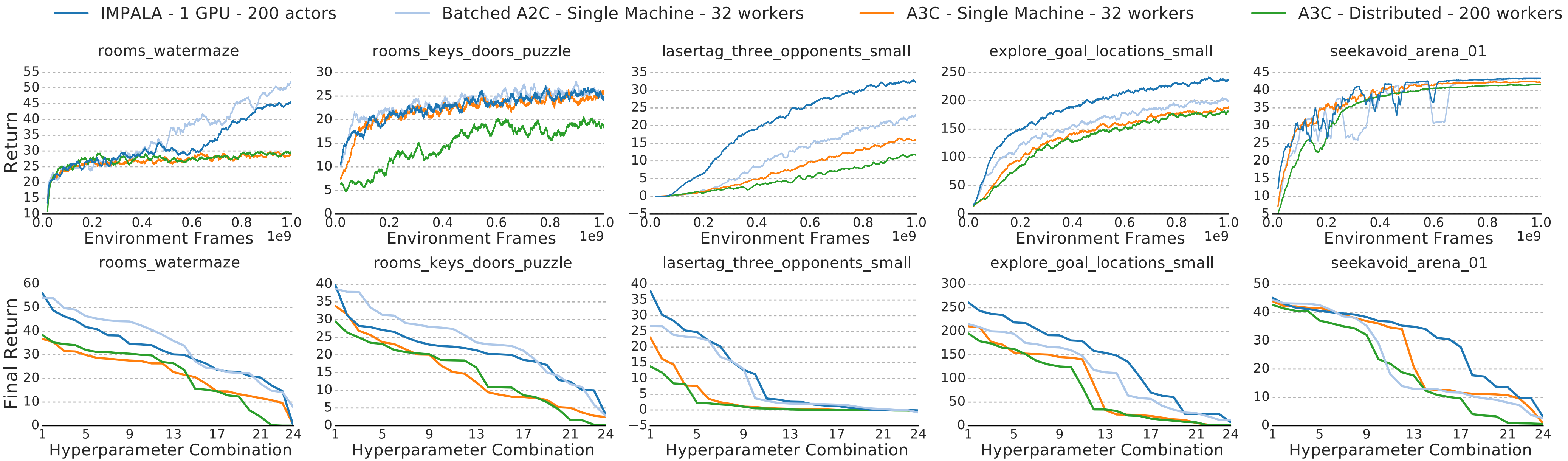
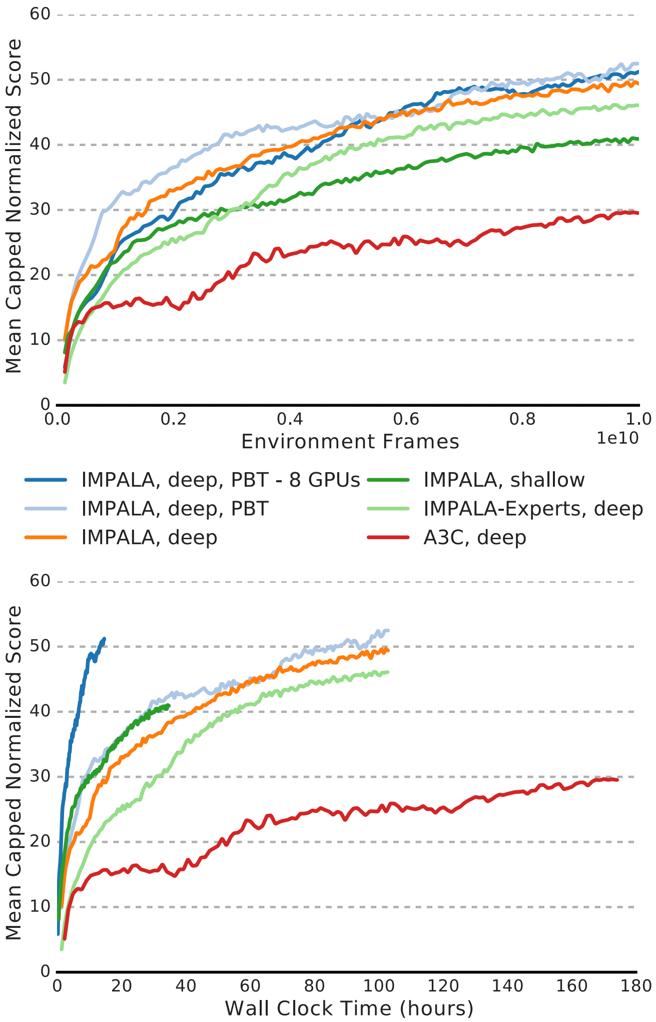


Figure 4 of "IMPALA: Scalable Distributed Deep-RL with Importance Weighted Actor-Learner Architectures" by Lasse Espeholt et al.

IMPALA – Learning Curves



Figures 5, 6 of "IMPALA: Scalable Distributed Deep-RL with Importance Weighted Actor-Learner Architectures" by Lasse Espeholt et al.

IMPALA – Atari Games

Human Normalised Return	Median	Mean
A3C, shallow, experts	54.9%	285.9%
A3C, deep, experts	117.9%	503.6%
Reactor, experts	187%	N/A
IMPALA, shallow, experts	93.2%	466.4%
IMPALA, deep, experts	191.8%	957.6%
IMPALA, deep, multi-task	59.7%	176.9%

Table 4 of "IMPALA: Scalable Distributed Deep-RL with Importance Weighted Actor-Learner Architectures" by Lasse Espeholt et al.

IMPALA – Atari Hyperparameters

Parameter	Value
Image Width	84
Image Height	84
Grayscaling	Yes
Action Repetitions	4
Max-pool over last N action repeat frames	2
Frame Stacking	4
End of episode when life lost	Yes
Reward Clipping	[-1, 1]
Unroll Length (n)	20
Batch size	32
Discount (γ)	0.99
Baseline loss scaling	0.5
Entropy Regularizer	0.01
RMSProp momentum	0.0
RMSProp ε	0.01
Learning rate	0.0006
Clip global gradient norm	40.0
Learning rate schedule	Anneal linearly to 0
	From beginning to end of training.
Population based training (only multi-task agent)	
- Population size	24
- Start parameters	Same as DMLab-30 sweep
- Fitness	Mean capped human normalised scores
	$\left(\sum_l \min\left[1, (s_t - r_t)/(h_t - r_t)\right]\right)/N$
- Adapted parameters	Gradient clipping threshold
	Entropy regularisation
	Learning rate
	RMSProp ε

Table G1 of "IMPALA: Scalable Distributed Deep-RL with Importance Weighted Actor-Learner Architectures" by Lasse Espeholt et al.

IMPALA – Ablations

- **No-correction**: no off-policy correction;
- **ε-correction**: add a small value $\varepsilon = 10^{-6}$ during gradient calculation to prevent π to be very small and lead to unstabilities during $\log \pi$ computation;
- **1-step**: no off-policy correction in the update of the value function, TD errors in the policy gradient are multiplied by the corresponding ρ but no cs; it can be considered V-trace "without traces".

	Task 1	Task 2	Task 3	Task 4	Task 5
Without Replay					
V-trace	46.8	32.9	**31.3**	**229.2**	**43.8**
1-Step	**51.8**	**35.9**	25.4	215.8	43.7
ε-correction	44.2	27.3	4.3	107.7	41.5
No-correction	40.3	29.1	5.0	94.9	16.1
With Replay					
V-trace	47.1	**35.8**	**34.5**	**250.8**	**46.9**
1-Step	**54.7**	34.4	26.4	204.8	41.6
ε-correction	30.4	30.2	3.9	101.5	37.6
No-correction	35.0	21.1	2.8	85.0	11.2

Tasks: `rooms_watermaze`, `rooms_keys_doors_puzzle`, `lasertag_three_opponents_small`, `explore_goal_locations_small`, `seekavoid_arena_01`

Table 2 of "IMPALA: Scalable Distributed Deep-RL with Importance Weighted Actor-Learner Architectures" by Lasse Espeholt et al.

The effect of the policy lag (the number of updates the actor is behind the learned policy) on the performance.

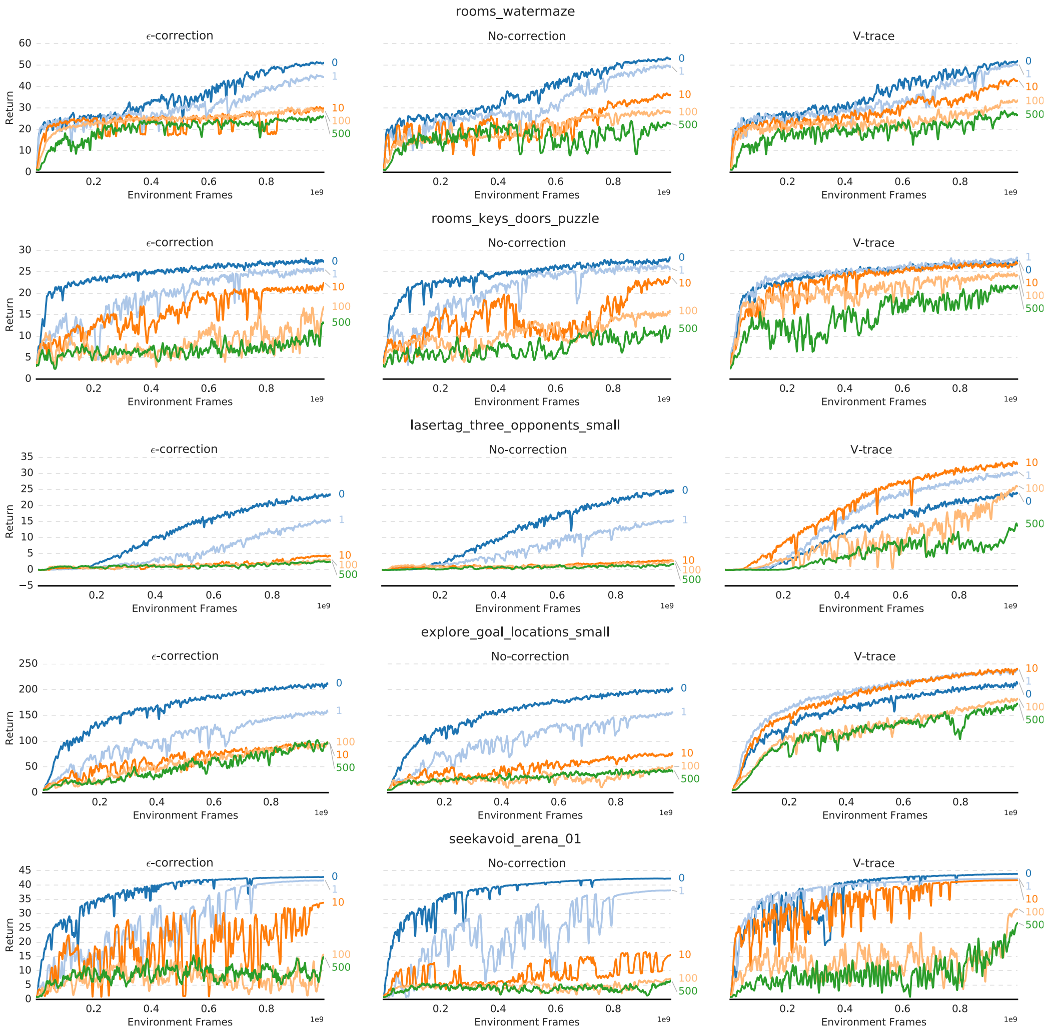


Figure E.1 of "IMPALA: Scalable Distributed Deep-RL with Importance Weighted Actor-Learner Architectures" by Lasse Espeholt et al.

PopArt Normalization

An improvement of IMPALA from Sep 2018, which performs normalization of task rewards instead of just reward clipping. PopArt stands for *Preserving Outputs Precisely, while Adaptively Rescaling Targets*.

Assume the value estimate $v(s; \boldsymbol\theta, \sigma, \mu)$ is computed using a normalized value predictor $n(s; \boldsymbol\theta)$

$$v(s; \boldsymbol\theta, \sigma, \mu) \stackrel{\text{def}}{=} \sigma n(s; \boldsymbol\theta) + \mu,$$

and further assume that $n(s; \boldsymbol\theta)$ is an output of a linear function

$$n(s; \boldsymbol\theta) \stackrel{\text{def}}{=} \boldsymbol\omega^T f(s; \boldsymbol\theta - \{\boldsymbol\omega, b\}) + b.$$

We can update the σ and μ using exponentially moving average with decay rate β (in the paper, first moment μ and second moment υ is tracked, and the standard deviation is computed as $\sigma = \sqrt{\upsilon - \mu^2}$; decay rate $\beta = 3 \cdot 10^{-4}$ is employed).

PopArt Normalization

Utilizing the parameters μ and σ, we can normalize the observed (unnormalized) returns as $(G-\mu)/\sigma$, and use an actor-critic algorithm with advantage $(G-\mu)/\sigma - n(S;\boldsymbol\theta)$.

However, in order to make sure the value function estimate does not change when the normalization parameters change, the parameters $\boldsymbol\omega, b$ used to compute the value estimate

$$v(s;\boldsymbol\theta,\sigma,\mu) \stackrel{\text{def}}{=} \sigma \cdot \Big(\boldsymbol\omega^T f(s;\boldsymbol\theta - \{\boldsymbol\omega, b\}) + b\Big) + \mu$$

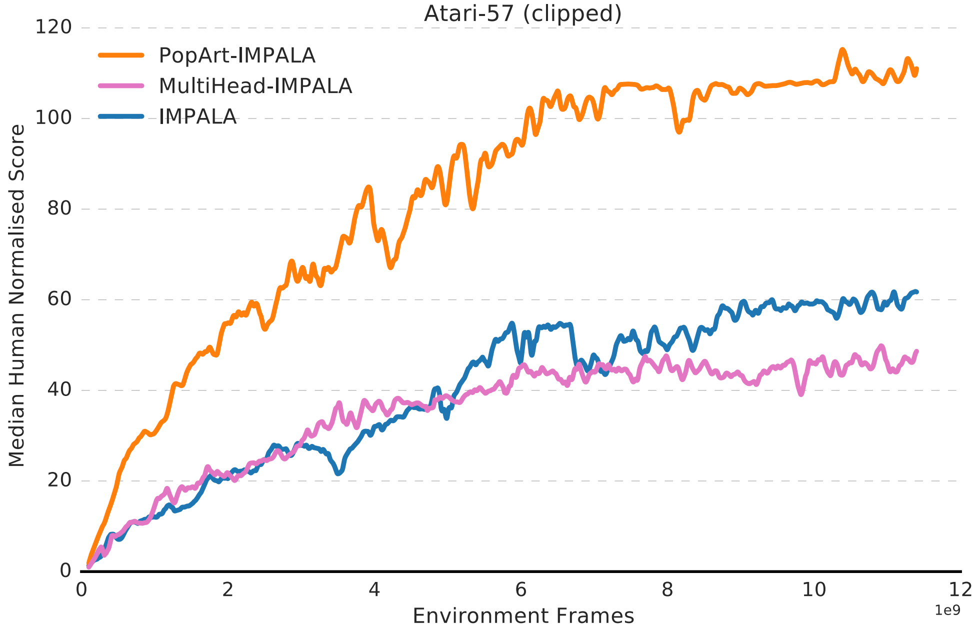
are updated under any change $\mu \rightarrow \mu'$ and $\sigma \rightarrow \sigma'$ as

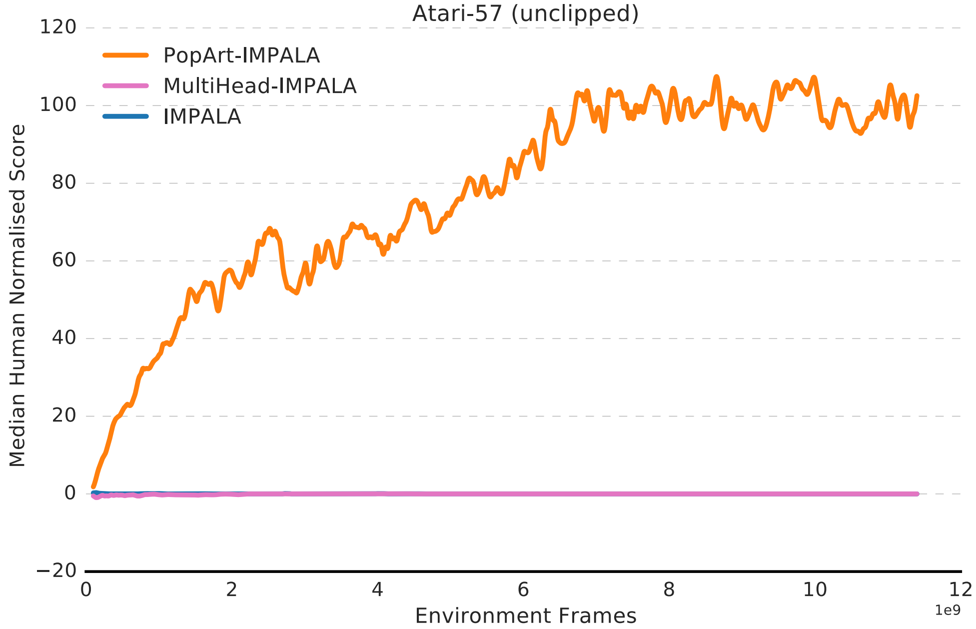
$$\begin{aligned} \boldsymbol\omega' &\leftarrow \frac{\sigma}{\sigma'}\boldsymbol\omega, \\ b' &\leftarrow \frac{\sigma b + \mu - \mu'}{\sigma'}. \end{aligned}$$

In multi-task settings, we train a task-agnostic policy and task-specific value functions (therefore, $\boldsymbol\mu$, $\boldsymbol\sigma$, and $\boldsymbol n(s;\boldsymbol\theta)$ are vectors).

	Atari-57		Atari-57 (unclipped)		DmLab-30	
Agent	Random	Human	Random	Human	Train	Test
IMPALA	59.7%	28.5%	0.3%	1.0%	60.6%	58.4%
PopArt-IMPALA	110.7%	101.5%	107.0%	93.7%	73.5%	72.8%

Table 1 of "Multi-task Deep Reinforcement Learning with PopArt" by Matteo Hessel et al.





Figures 1, 2 of "Multi-task Deep Reinforcement Learning with PopArt" by Matteo Hessel et al.

PopArt Results

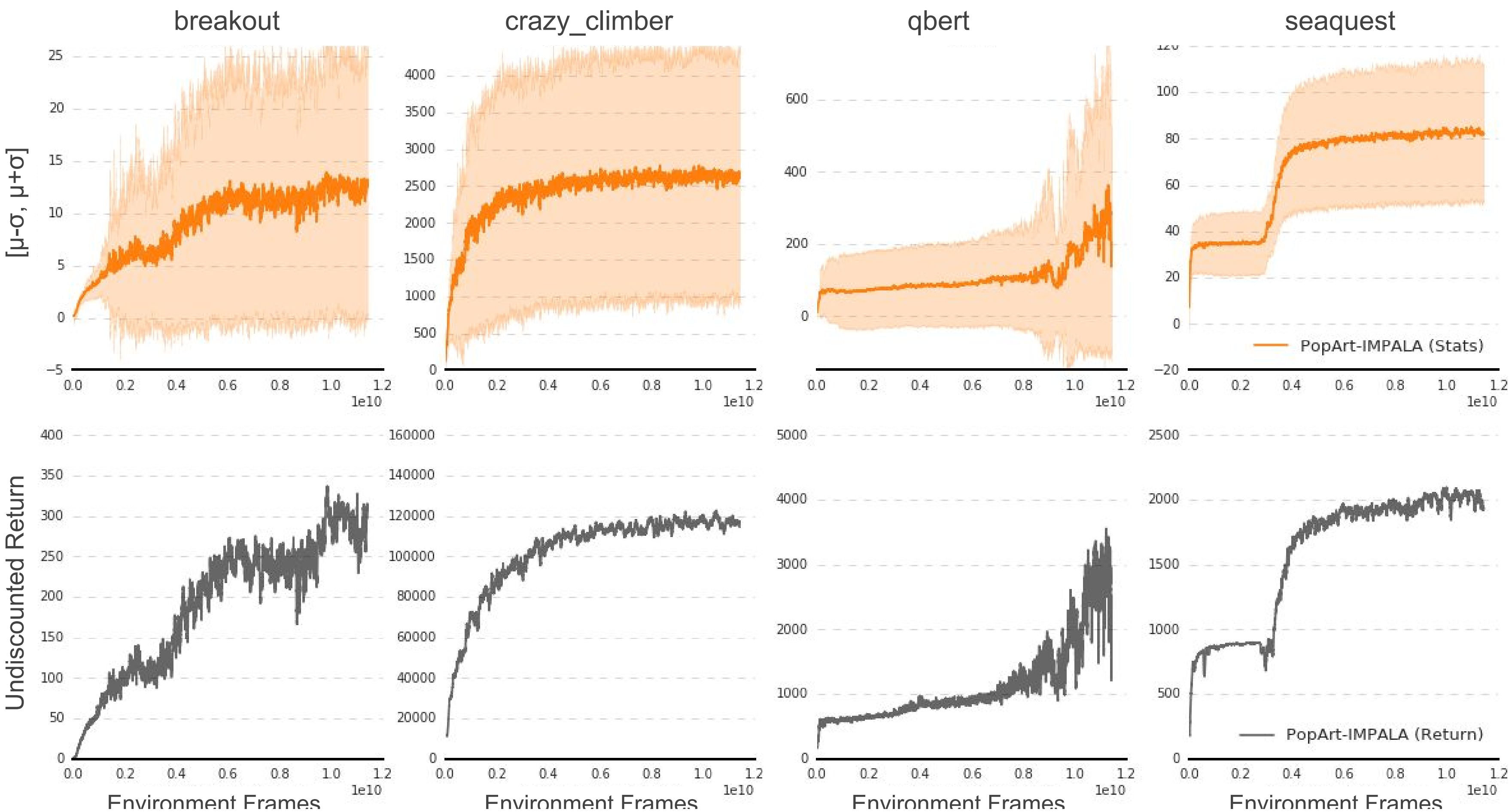


Figure 3 of "Multi-task Deep Reinforcement Learning with PopArt" by Matteo Hessel et al.

Normalization statistics on chosen environments.