PopArt Normalization, R2D2, MuZero

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December 14, 2020

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Operational Programme Research, Development and Education

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PopArt Normalization

An improvement of IMPALA from Sep 2018, which performs normalization of task rewards instead of just reward clipping. PopArt stands for *Preserving Outputs Precisely, while Adaptively Rescaling Targets*.

Assume the value estimate $v(s; \boldsymbol\theta, \sigma, \mu)$ is computed using a normalized value predictor $n(s; \boldsymbol\theta)$

$$v(s; \boldsymbol\theta, \sigma, \mu) \stackrel{\text{def}}{=} \sigma n(s; \boldsymbol\theta) + \mu$$

and further assume that $n(s; \boldsymbol\theta)$ is an output of a linear function

$$n(s; \boldsymbol\theta) \stackrel{\text{def}}{=} \boldsymbol\omega^T f(s; \boldsymbol\theta - \{\boldsymbol\omega, b\}) + b.$$

We can update the σ and μ using exponentially moving average with decay rate β (in the paper, first moment μ and second moment υ is tracked, and the standard deviation is computed as $\sigma = \sqrt{\upsilon - \mu^2}$; decay rate $\beta = 3 \cdot 10^{-4}$ is employed).

PopArt Normalization

Utilizing the parameters μ and σ, we can normalize the observed (unnormalized) returns as $(G - \mu)/\sigma$ and use an actor-critic algorithm with advantage $(G - \mu)/\sigma - n(S; \boldsymbol\theta)$.

However, in order to make sure the value function estimate does not change when the normalization parameters change, the parameters $\boldsymbol\omega, b$ used to compute the value estimate

$$v(s; \boldsymbol\theta, \sigma, \mu) \stackrel{\text{def}}{=} \sigma\Big(\boldsymbol\omega^T f(s; \boldsymbol\theta - \{\boldsymbol\omega, b\}) + b\Big) + \mu$$

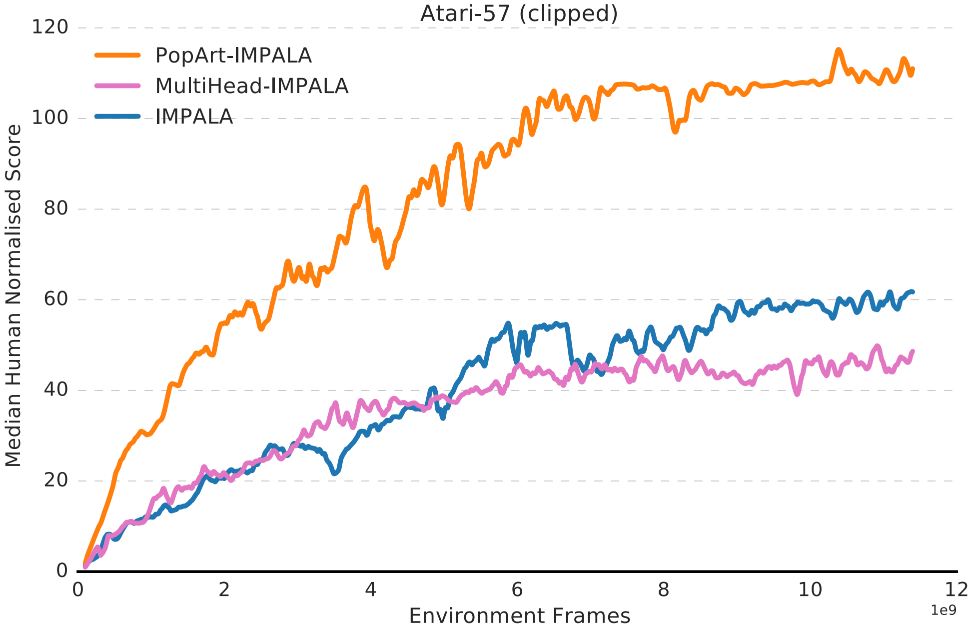
are updated under any change $\mu \rightarrow \mu'$ and $\sigma \rightarrow \sigma'$ as

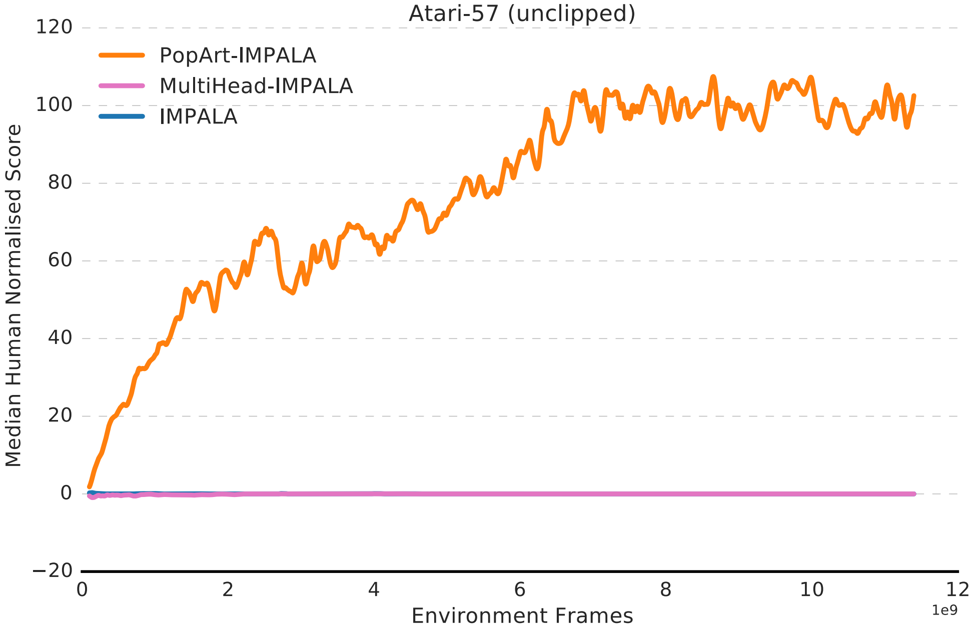
$$\begin{aligned} \boldsymbol\omega' &\leftarrow \frac{\sigma}{\sigma'}\boldsymbol\omega, \\ b' &\leftarrow \frac{\sigma b + \mu - \mu'}{\sigma'}. \end{aligned}$$

In multi-task settings, we train a task-agnostic policy and task-specific value functions (therefore, $\boldsymbol\mu$, $\boldsymbol\sigma$ and $\boldsymbol n(s; \boldsymbol\theta)$ are vectors).

Agent	Atari-57 Random	Atari-57 Human	Atari-57 (unclipped) Random	Atari-57 (unclipped) Human	DmLab-30 Train	DmLab-30 Test
IMPALA	59.7%	28.5%	0.3%	1.0%	60.6%	58.4%
PopArt-IMPALA	110.7%	101.5%	107.0%	93.7%	73.5%	72.8%

Table 1 of paper "Multi-task Deep Reinforcement Learning with PopArt" by Matteo Hessel et al.





Figures 1, 2 of paper "Multi-task Deep Reinforcement Learning with PopArt" by Matteo Hessel et al.

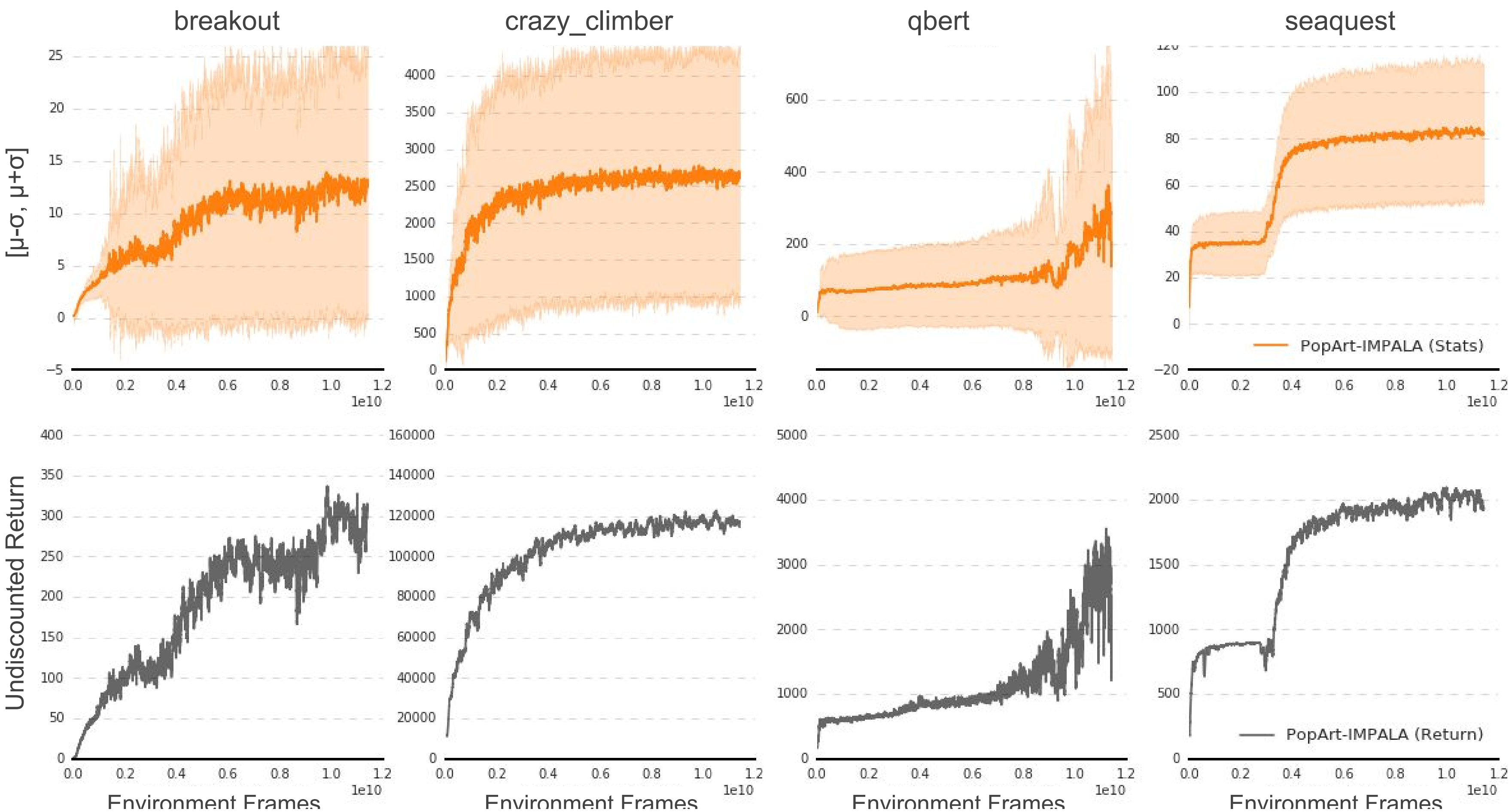
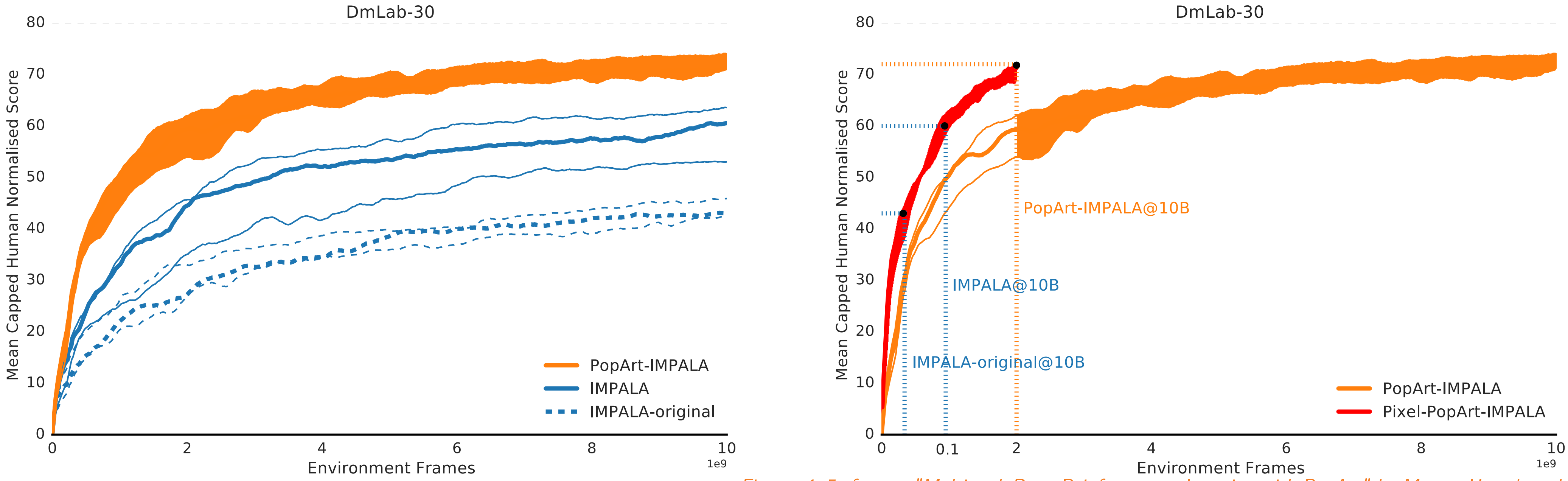


Figure 3 of paper "Multi-task Deep Reinforcement Learning with PopArt" by Matteo Hessel et al.

Normalization statistics on chosen environments.



Figures 4, 5 of paper "Multi-task Deep Reinforcement Learning with PopArt" by Matteo Hessel et al.

Transformed Rewards

So far, we have clipped the rewards in DQN on Atari environments.

Consider a Bellman operator $\mathcal{T}$

$$(\mathcal{T}q)(s,a) \stackrel{\text{def}}{=} \mathbb{E}_{s',r\sim p}\Big[r + \gamma \max_{a'} q(s',a')\Big].$$

Instead of reducing the magnitude of rewards, we might use a function $h: \mathbb{R} \rightarrow \mathbb{R}$ to reduce their scale. We define a transformed Bellman operator $\mathcal{T}_h$ as

$$(\mathcal{T}_h q)(s,a) \stackrel{\text{def}}{=} \mathbb{E}_{s',r\sim p}\Big[h\Big(r + \gamma \max_{a'} h^{-1}\big(q(s',a')\big)\Big)\Big].$$

Transformed Rewards

It is easy to prove the following two propositions from a 2018 paper *Observe and Look Further: Achieving Consistent Performance on Atari* by Tobias Pohlen et al.

1. If $h(z) = \alpha z$ for $\alpha > 0$, then $\mathcal{T}_h^k q \xrightarrow{k\rightarrow\infty} h \circ q_* = \alpha q_*$.

   The statement follows from the fact that it is equivalent to scaling the rewards by a constant α.

2. When h is strictly monotonically increasing and the MDP is deterministic, then $\mathcal{T}_h^k q \xrightarrow{k\rightarrow\infty} h \circ q_*$.

   This second proposition follows from

   $$h \circ q_* = h \circ \mathcal{T} q_* = h \circ \mathcal{T}(h^{-1} \circ h \circ q_*) = \mathcal{T}_h(h \circ q_*),$$

   where the last equality only holds if the MDP is deterministic.

Transformed Rewards

The authors use the following transformation for the Atari environments

$$h(x) \stackrel{\text{def}}{=} \operatorname{sign}(x)\left(\sqrt{|x|+1}-1\right)+\varepsilon x$$

with $\varepsilon = 10^{-2}$. The additive regularization term ensures that h^{-1} is Lipschitz continuous.

It is straightforward to verify that

$$h^{-1}(x) = \operatorname{sign}(x)\left(\left(\frac{\sqrt{1+4\varepsilon(|x|+1+\varepsilon)}-1}{2\varepsilon}\right)^2-1\right).$$

Recurrent Replay Distributed DQN (R2D2)

Proposed in 2019, to study the effects of recurrent state, experience replay and distributed training.

R2D2 utilizes prioritized replay, n-step double Q-learning with $n=5$, convolutional layers followed by a 512-dimensional LSTM passed to duelling architecture, generating experience by a large number of actors (256; each performing approximately 260 steps per second) and learning from batches by a single learner (achieving 5 updates per second using mini-batches of 64 sequences of length 80).

Instead of individual transitions, the replay buffer consists of fixed-length $(m=80)$ sequences of (s, a, r), with adjacent sequences overlapping by 40 time steps.

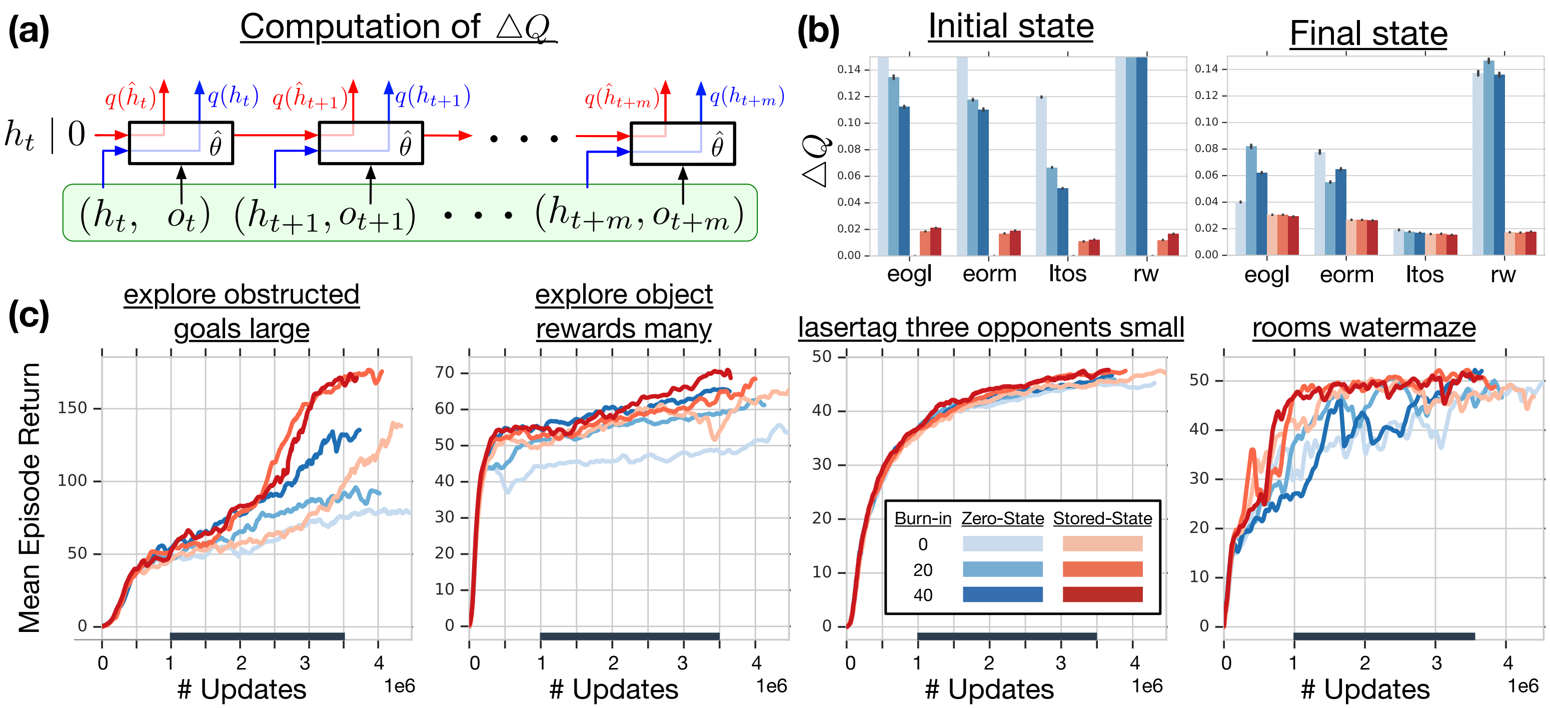


Figure 1: Top row shows Q-value discrepancy ΔQ as a measure for recurrent state staleness. **(a)** Diagram of how ΔQ is computed, with green box indicating a whole sequence sampled from replay. For simplicity, $l = 0$ (no burn-in). **(b)** ΔQ measured at first state and last state of replay sequences, for agents training on a selection of DMLab levels (indicated by initials) with different training strategies. Bars are averages over seeds and through time indicated by bold line on x-axis in bottom row. **(c)** Learning curves on the same levels, varying the training strategy, and averaged over 3 seeds.

Figure 1 of the paper "Recurrent Experience Replay in Distributed Reinforcement Learning" by Steven Kapturowski et al.

Recurrent Replay Distributed DQN (R2D2)

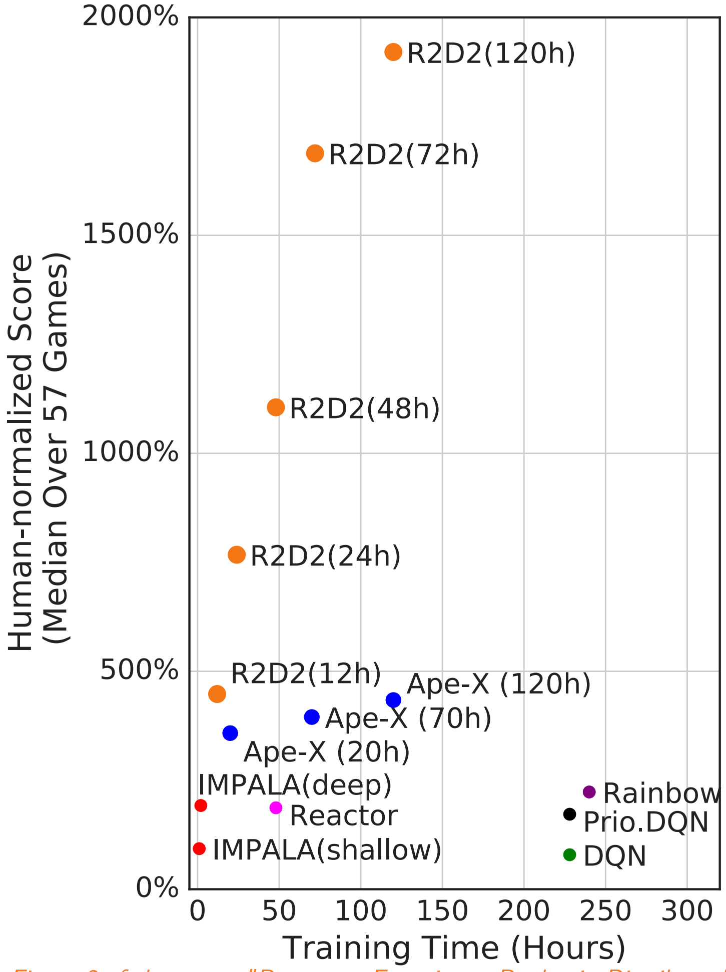


Figure 2 of the paper "Recurrent Experience Replay in Distributed Reinforcement Learning" by Steven Kapturowski et al.

	Atari-57		DMLab-30	
Human-Normalized Score	**Median**	**Mean**	**Median**	**Mean-Capped**
Ape-X (Horgan et al., 2018)	434.1%	1695.6%	–	–
Reactor (Gruslys et al., 2018)	187.0%	–	–	–
IMPALA, deep (Espeholt et al., 2018)	191.8%	957.6%	49.0%	45.8%
IMPALA, shallow (re-run)	–	–	89.7%	73.6%
IMPALA, deep (re-run)	–	–	**107.5%**	85.1%
R2D2+	–	–	99.5%	**85.7%**
R2D2, feed-forward	589.2%	1974.4%	–	–
R2D2	**1920.6%**	**4024.9%**	96.9%	78.3%

Table 1 of the paper "Recurrent Experience Replay in Distributed Reinforcement Learning" by Steven Kapturowski et al.

Number of actors	256
Actor parameter update interval	400 environment steps
Sequence length m	80 (+ prefix of $l = 40$ in burn-in experiments)
Replay buffer size	4×10^6 observations (10^5 part-overlapping sequences)
Priority exponent	0.9
Importance sampling exponent	0.6
Discount γ	0.997
Minibatch size	64 (32 for R2D2+ on DMLab)
Optimizer	Adam (Kingma & Ba, 2014)
Optimizer settings	learning rate $= 10^{-4}$, $\varepsilon = 10^{-3}$
Target network update interval	2500 updates
Value function rescaling	$h(x) = \operatorname{sign}(x)(\sqrt{\lvert x\rvert + 1} - 1) + \epsilon x,\ \epsilon = 10^{-3}$

Table 2: Hyper-parameters values used in R2D2. All missing parameters follow the ones in Ape-X (Horgan et al., 2018).

Table 2 of the paper "Recurrent Experience Replay in Distributed Reinforcement Learning" by Steven Kapturowski et al.

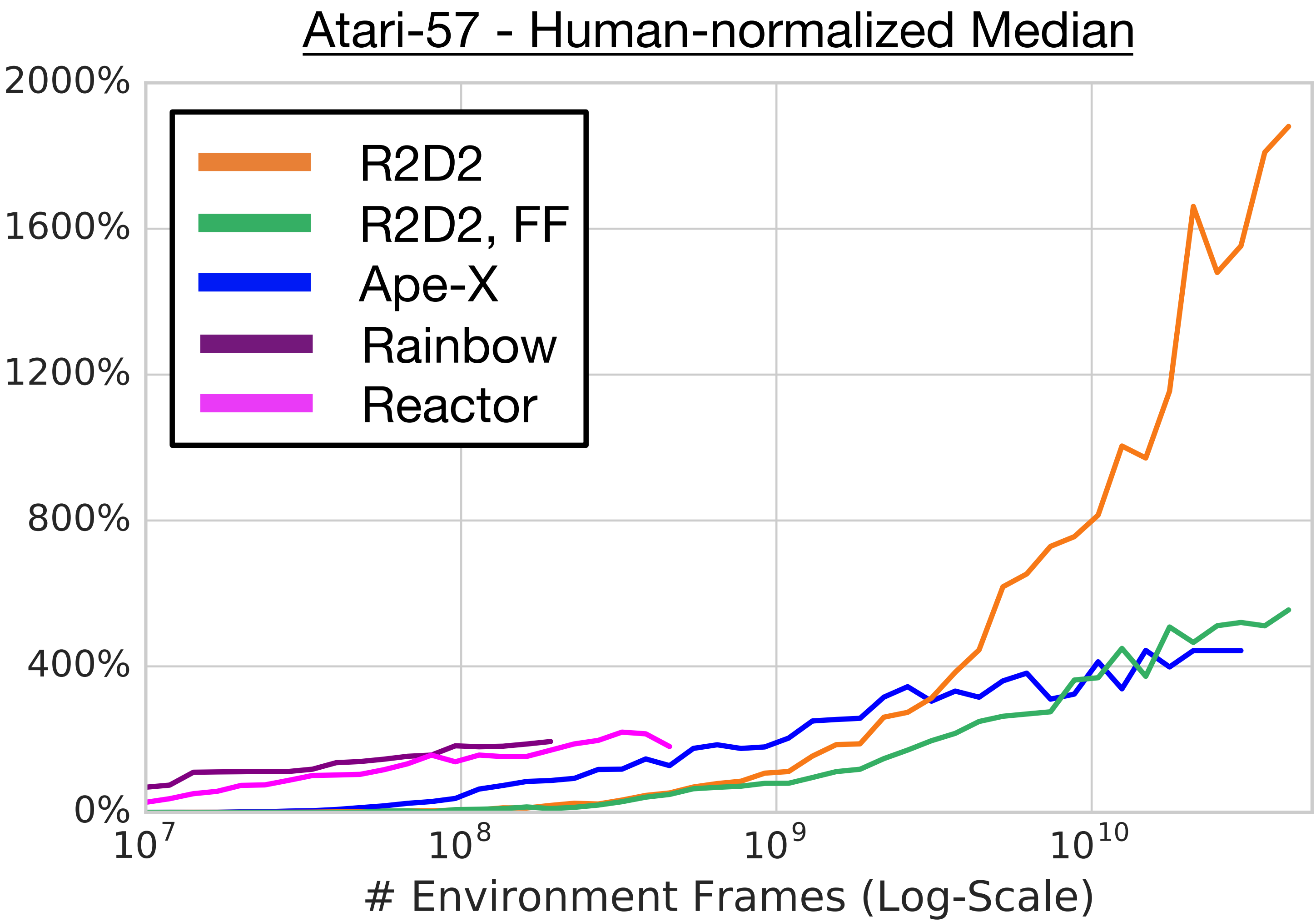


Figure 9 of the paper "Recurrent Experience Replay in Distributed Reinforcement Learning" by Steven Kapturowski et al.

Recurrent Replay Distributed DQN (R2D2)

Ablations comparing the reward clipping instead of value rescaling (**Clipped**), smaller discount factor $\gamma = 0.99$ (**Discount**) and **Feed-Forward** variant of R2D2. Furthermore, life-loss **reset** evaluates resetting an episode on life loss, with **roll** preventing value bootstrapping (but not LSTM unrolling).

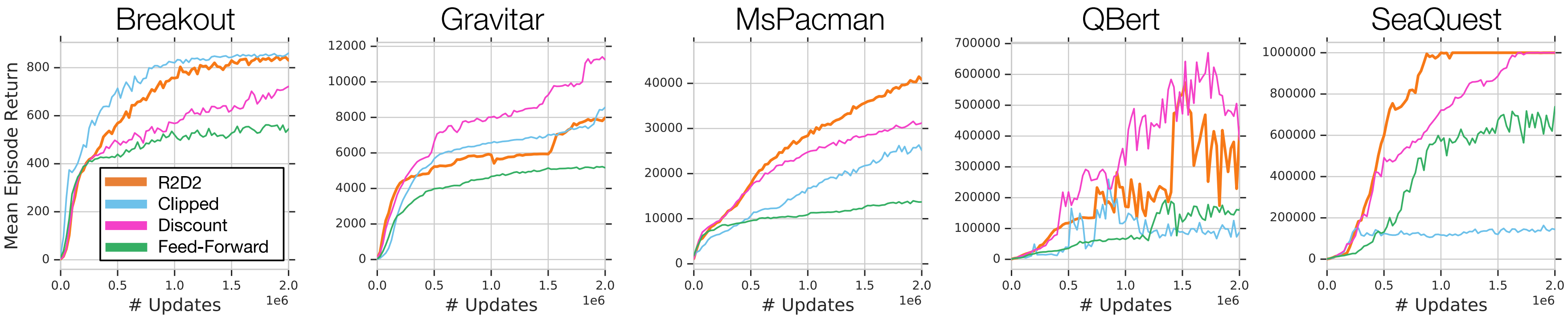


Figure 4 of the paper "Recurrent Experience Replay in Distributed Reinforcement Learning" by Steven Kapturowski et al.

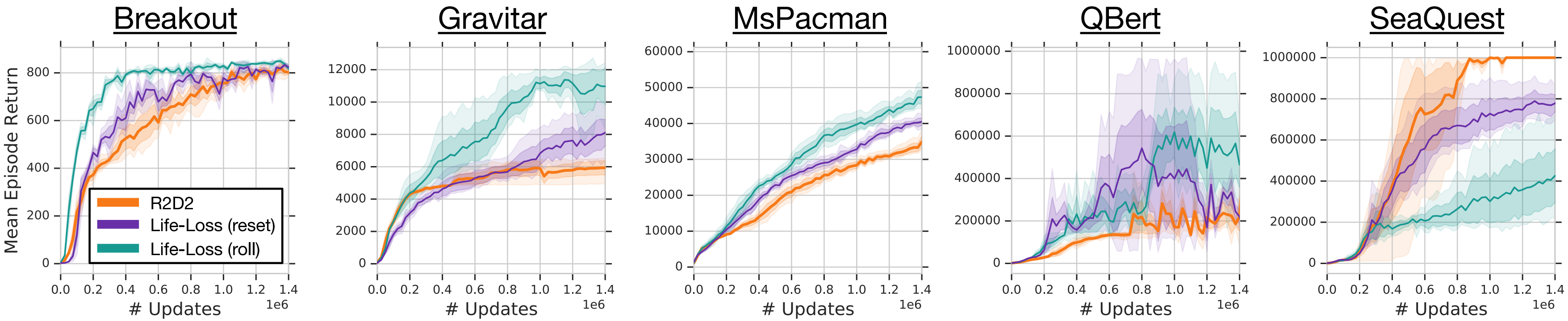


Figure 7 of the paper "Recurrent Experience Replay in Distributed Reinforcement Learning" by Steven Kapturowski et al.

Utilization of LSTM Memory During Inference

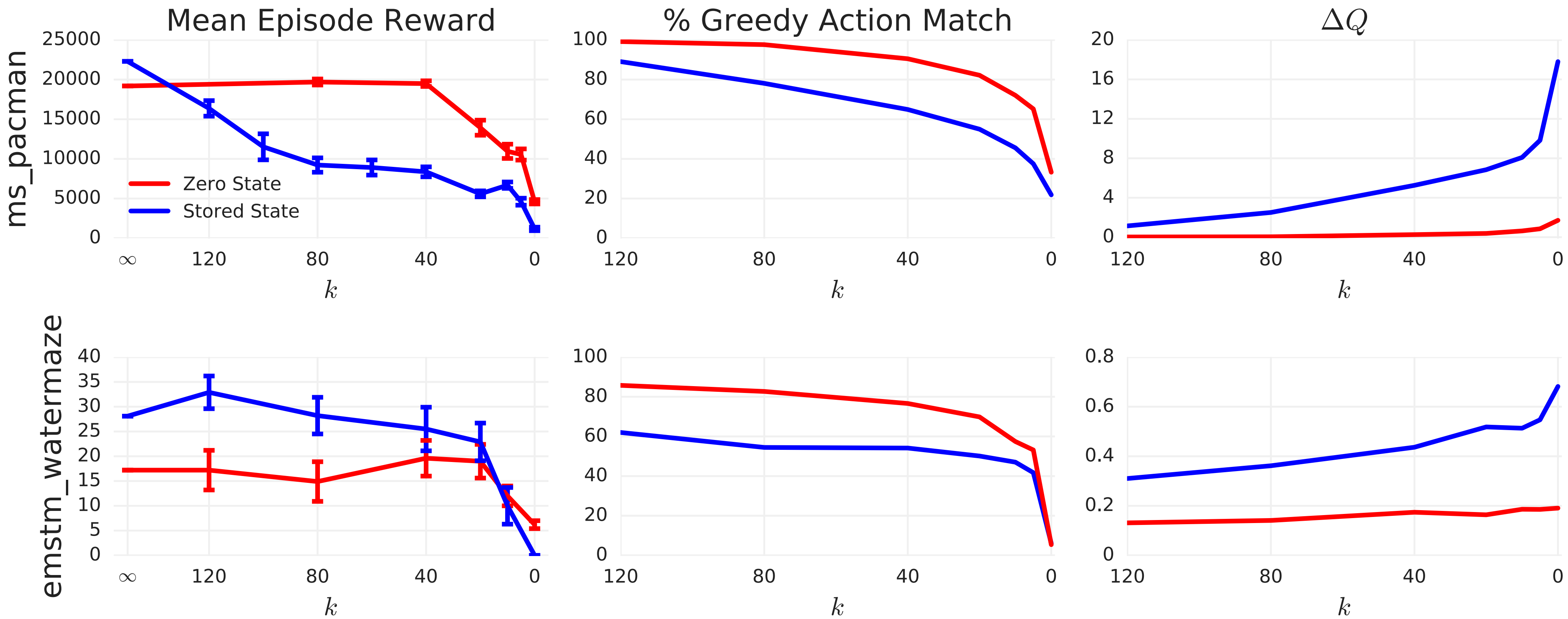


Figure 5 of the paper "Recurrent Experience Replay in Distributed Reinforcement Learning" by Steven Kapturowski et al.

The MuZero algorithm extends the AlphaZero by a **trained model**, alleviating the requirement for a known MDP dynamics.

At each time-step t, for each of $1 \le k \le K$ steps, a model μ_θ, with parameters θ, conditioned on past observations $o_1, \ldots, o_t$ and future actions $a_{t+1}, \ldots, a_{t+k}$, predicts three future quantities:

- the policy $\boldsymbol{p}_t^k \approx \pi(a_{t+k+1} | o_1, \ldots, o_t, a_{t+1}, \ldots, a_{t+k})$,
- the value function $v_t^k \approx \mathbb{E}\big[u_{t+k+1} + \gamma u_{t+k+2} + \ldots \big| o_1, \ldots, o_t, a_{t+1}, \ldots, a_{t+k}\big]$,
- the immediate reward $r_t^k \approx u_{t+k}$,

where u_i are the observed rewards and π is the behaviour policy.

At each time-step t (omitted from now on for simplicity), the model is composed of three components, a *representation* function, a *dynamics* function and a *prediction* function.

- The dynamics function, $\boldsymbol{r}^k, \boldsymbol{s}^k = g_\theta(\boldsymbol{s}^{k-1}, \boldsymbol{a}^k)$, simulates the MDP dynamics and predicts an immediate reward $\boldsymbol{r}^k$ and an internal state $\boldsymbol{s}^k$. The internal state has no explicit semantics, its only goal is to accurately predict rewards, values and policies.
- The prediction function $\boldsymbol{p}^k, v^k = f_\theta(\boldsymbol{s}^k)$, computes the policy and value function, similarly as in AlphaZero.
- The representation function, $\boldsymbol{s}^0 = h_\theta(o_1, \ldots, o_t)$, generates an internal state encoding the past observations.

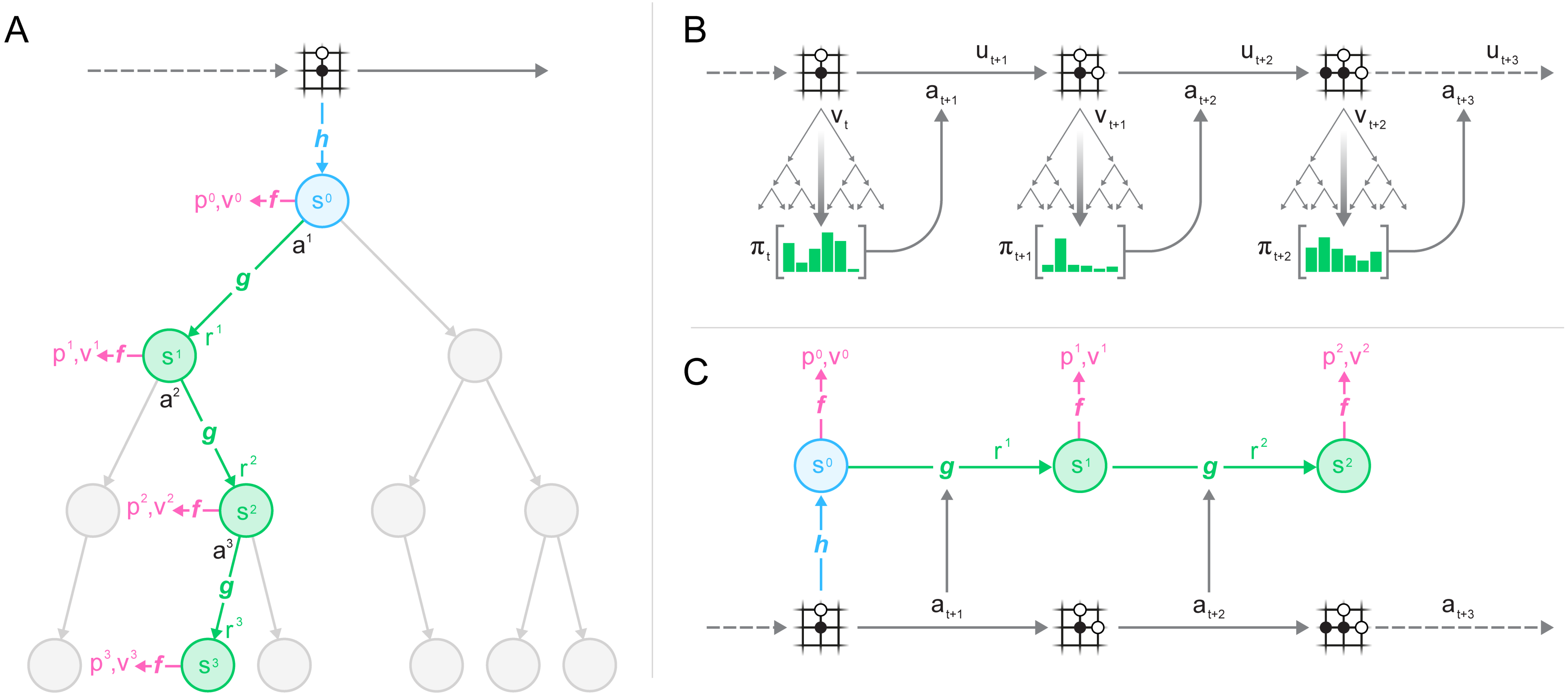


Figure 1 of the paper "Mastering Atari, Go, Chess and Shogi by Planning with a Learned Model" by Julian Schrittwieser et al.

To select a move, we employ a MCTS algorithm similar to the AlphaZero. It produces a policy π_t and value estimate ν_t, and an action is then sampled from the policy $a_{t+1} \sim \pi_t$.

During training, we utilize a sequence of k moves. We estimate the return using bootstrapping $z_t = u_{t+1} + \gamma u_{t+2} + \ldots + \gamma^{n-1} u_{t+n} + \gamma^n \nu_{t+n}$. The values $k = 5$ and $n = 10$ are used in the paper.

The loss is then composed of the following components:

$$\mathcal{L}_t(\theta) = \sum_{k=0}^{K} \mathcal{L}^r(u_{t+k}, r_t^k) + \mathcal{L}^v(z_{t+k}, v_t^k) + \mathcal{L}^p(\pi_{t+k}, \boldsymbol{p}_t^k) + c\|\theta\|^2.$$

Note that in Atari, rewards are scaled by $\operatorname{sign}(x)\big(\sqrt{|x| + 1} - 1\big) + \varepsilon x$ for $\varepsilon = 10^{-3}$, and authors utilize a cross-entropy loss with 601 categories for values $-300, \ldots, 300$, which they claim to be more stable.

Model

$$\left.\begin{aligned} s^0 &= h_\theta(o_1, \ldots, o_t) \\ r^k, s^k &= g_\theta(s^{k-1}, a^k) \\ \mathbf{p}^k, v^k &= f_\theta(s^k) \end{aligned}\right\} \quad \mathbf{p}^k, v^k, r^k = \mu_\theta(o_1, \ldots, o_t, a^1, \ldots, a^k)$$

Search

$$\nu_t, \pi_t = MCTS(s_t^0, \mu_\theta)$$

$$a_t \sim \pi_t$$

Learning Rule

$$\mathbf{p}_t^k, v_t^k, r_t^k = \mu_\theta(o_1, \ldots, o_t, a_{t+1}, ..., a_{t+k})$$

$$z_t = \begin{cases} u_T & \text{for games} \\ u_{t+1} + \gamma u_{t+2} + ... + \gamma^{n-1} u_{t+n} + \gamma^n \nu_{t+n} & \text{for general MDPs} \end{cases}$$

$$\mathcal{L}_t(\theta) = \sum_{k=0}^{K} \mathcal{L}^r(u_{t+k}, r_t^k) + \mathcal{L}^v(z_{t+k}, v_t^k) + \mathcal{L}^p(\pi_{t+k}, \boldsymbol{p}_t^k) + c\|\theta\|^2$$

Losses

$$\mathcal{L}^r(u, r) = \begin{cases} 0 & \text{for games} \\ \boldsymbol{\phi}(u)^T \log \mathbf{r} & \text{for general MDPs} \end{cases}$$

$$\mathcal{L}^v(z, q) = \begin{cases} (z-q)^2 & \text{for games} \\ \boldsymbol{\phi}(z)^T \log \mathbf{q} & \text{for general MDPs} \end{cases}$$

$$\mathcal{L}^p(\pi, p) = \boldsymbol{\pi}^T \log \mathbf{p}$$

MuZero – Evaluation

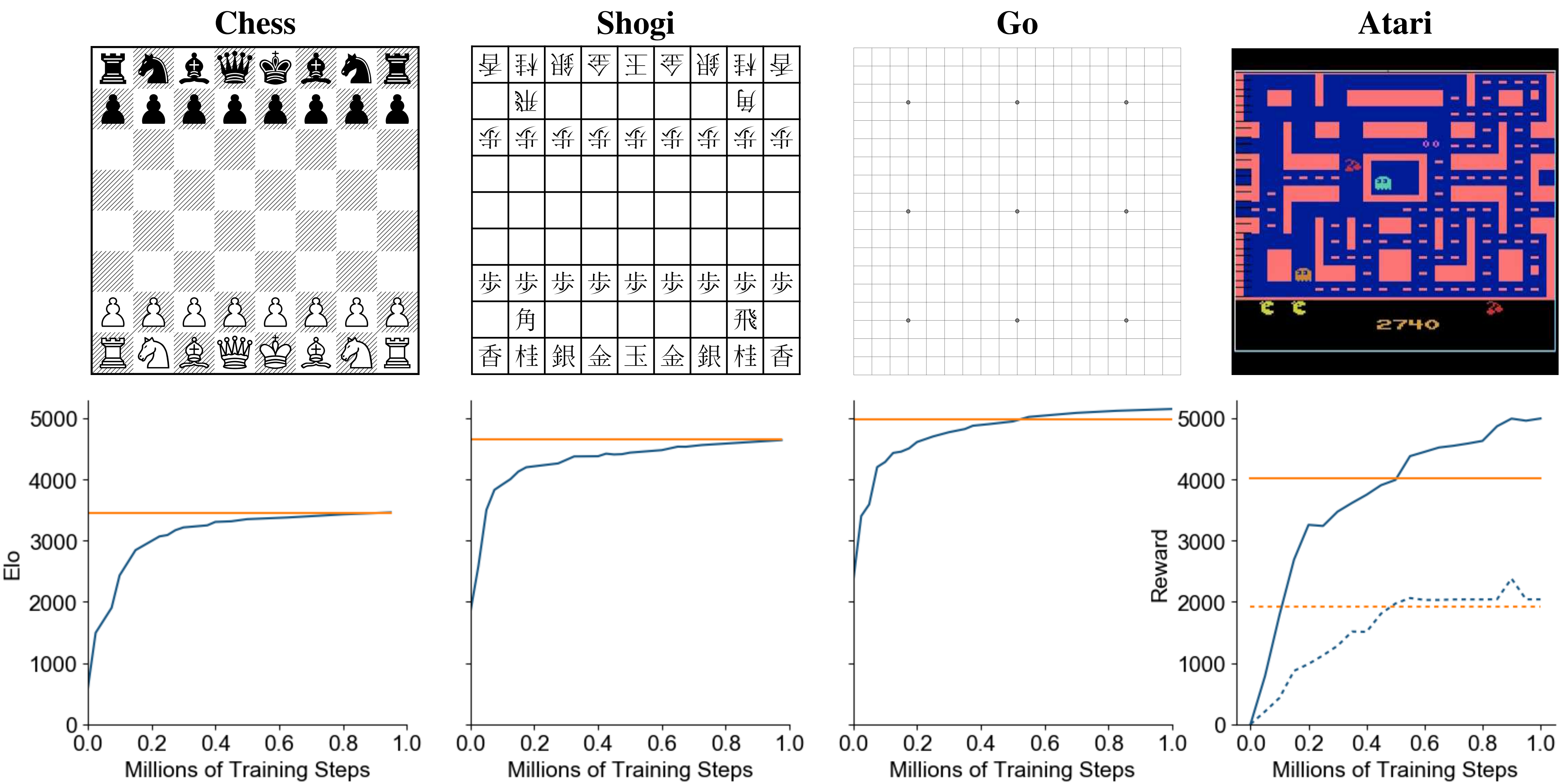


Figure 2 of the paper "Mastering Atari, Go, Chess and Shogi by Planning with a Learned Model" by Julian Schrittwieser et al.

MuZero – Atari Results

Agent	Median	Mean	Env. Frames	Training Time	Training Steps
Ape-X [18]	434.1%	1695.6%	22.8B	5 days	8.64M
R2D2 [21]	1920.6%	4024.9%	37.5B	5 days	2.16M
MuZero	**2041.1%**	**4999.2%**	20.0B	12 hours	1M
IMPALA [9]	191.8%	957.6%	200M	–	–
Rainbow [17]	231.1%	–	200M	10 days	–
UNREAL[a] [19]	250%[a]	880%[a]	250M	–	–
LASER [36]	431%	–	200M	–	–
MuZero Reanalyze	**731.1%**	**2168.9%**	200M	12 hours	1M

Table 1 of the paper "Mastering Atari, Go, Chess and Shogi by Planning with a Learned Model" by Julian Schrittwieser et al.

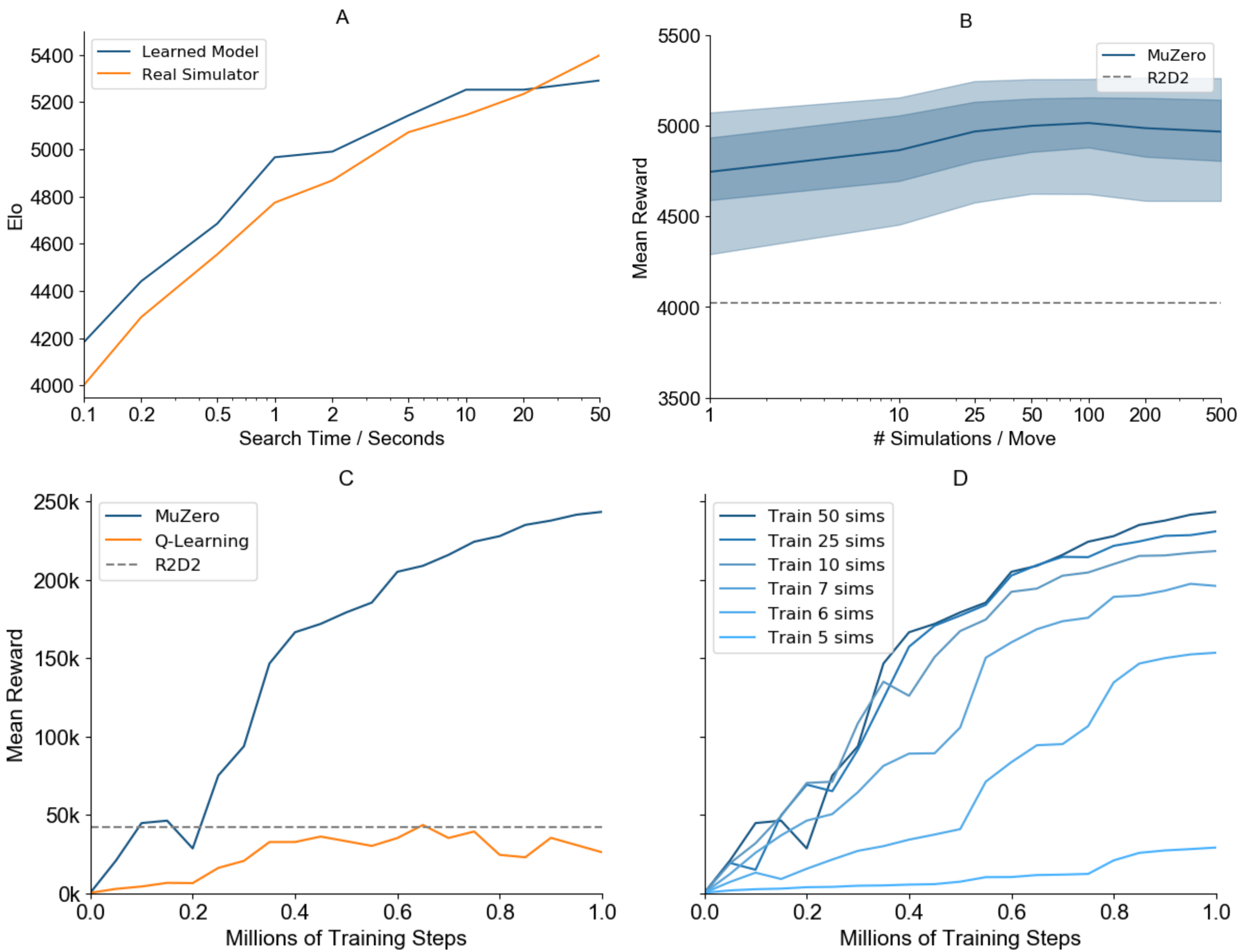


Figure 3 of the paper "Mastering Atari, Go, Chess and Shogi by Planning with a Learned Model" by Julian Schrittwieser et al.

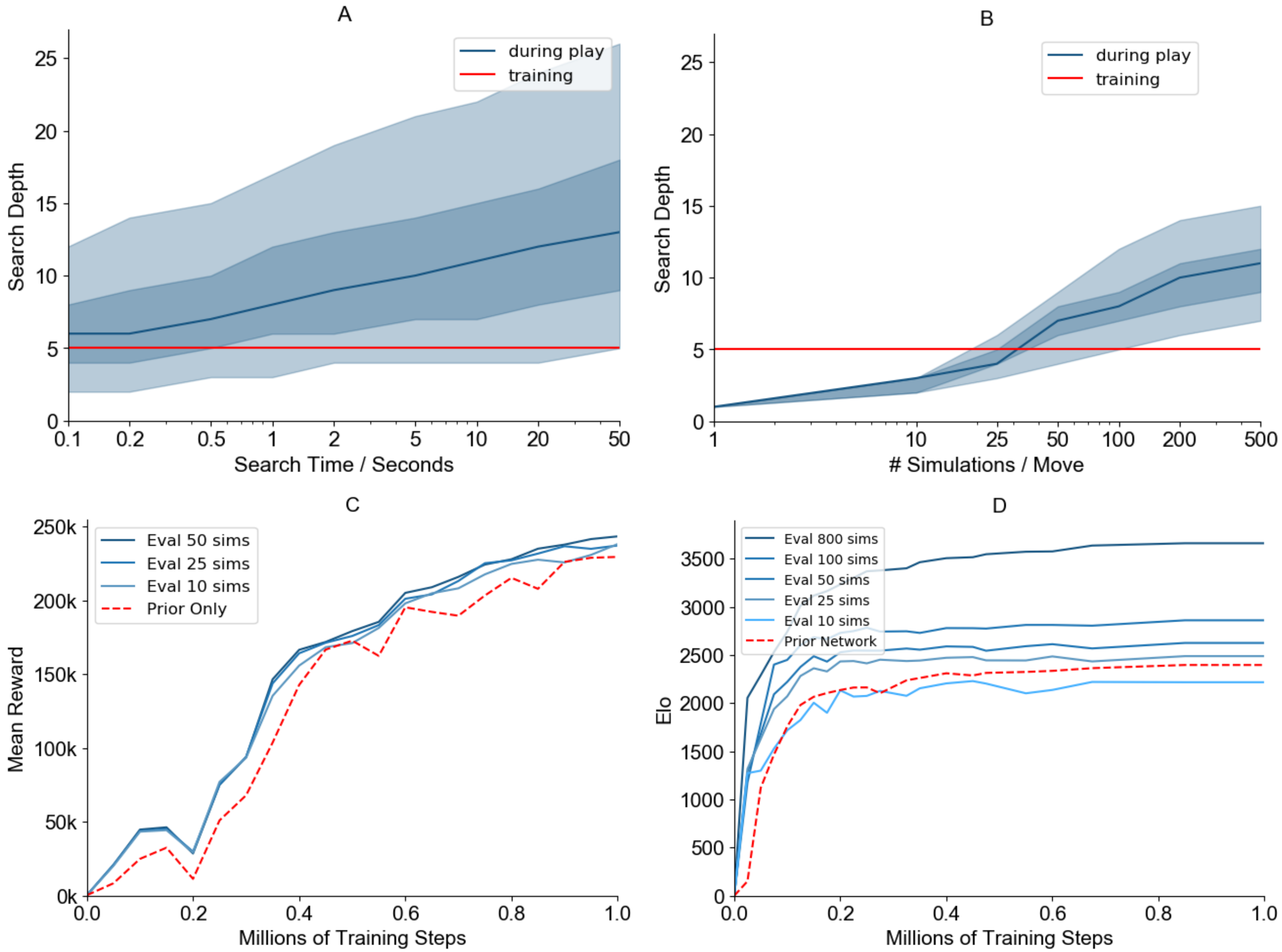


Figure S3 of the paper "Mastering Atari, Go, Chess and Shogi by Planning with a Learned Model" by Julian Schrittwieser et al.

MuZero – Detailed Atari Results

Game	Random	Human	SimPLe [20]	Ape-X [18]	R2D2 [21]	*MuZero*	*MuZero* normalized
alien	227.75	7,127.80	616.90	40,805.00	229,496.90	**741,812.63**	10,747.5 %
amidar	5.77	1,719.53	74.30	8,659.00	**29,321.40**	28,634.39	1,670.5 %
assault	222.39	742.00	527.20	24,559.00	108,197.00	**143,972.03**	27,664.9 %
asterix	210.00	8,503.33	1,128.30	313,305.00	**999,153.30**	998,425.00	12,036.4 %
asteroids	719.10	47,388.67	793.60	155,495.00	357,867.70	**678,558.64**	1,452.4 %
atlantis	12,850.00	29,028.13	20,992.50	944,498.00	1,620,764.00	**1,674,767.20**	10,272.6 %
bank heist	14.20	753.13	34.20	1,716.00	**24,235.90**	1,278.98	171.2 %
battle zone	2,360.00	37,187.50	4,031.20	98,895.00	751,880.00	**848,623.00**	2,429.9 %
beam rider	363.88	16,926.53	621.60	63,305.00	188,257.40	**454,993.53**	2,744.9 %
berzerk	123.65	2,630.42	-	57,197.00	53,318.70	**85,932.60**	3,423.1 %
bowling	23.11	160.73	30.00	18.00	219.50	**260.13**	172.2 %
boxing	0.05	12.06	7.80	**100.00**	98.50	**100.00**	832.2 %
breakout	1.72	30.47	16.40	801.00	837.70	**864.00**	2,999.2 %
centipede	2,090.87	12,017.04	-	12,974.00	599,140.30	**1,159,049.27**	11,655.6 %
chopper command	811.00	7,387.80	979.40	721,851.00	986,652.00	**991,039.70**	15,056.4 %
crazy climber	10,780.50	35,829.41	62,583.60	320,426.00	366,690.70	**458,315.40**	1,786.6 %
defender	2,874.50	18,688.89	-	411,944.00	665,792.00	**839,642.95**	5,291.2 %
demon attack	152.07	1,971.00	208.10	133,086.00	140,002.30	**143,964.26**	7,906.4 %
double dunk	-18.55	-16.40	-	**24.00**	23.70	23.94	1,976.3 %
enduro	0.00	860.53	-	2,177.00	2,372.70	**2,382.44**	276.9 %
fishing derby	-91.71	-38.80	-90.70	44.00	85.80	**91.16**	345.6 %
freeway	0.01	29.60	16.70	**34.00**	32.50	33.03	111.6 %
frostbite	65.20	4,334.67	236.90	9,329.00	315,456.40	**631,378.53**	14,786.7 %
gopher	257.60	2,412.50	596.80	120,501.00	124,776.30	**130,345.58**	6,036.8 %
gravitar	173.00	3,351.43	173.40	1,599.00	**15,680.70**	6,682.70	204.8 %
hero	1,026.97	30,826.38	2,656.60	31,656.00	39,537.10	**49,244.11**	161.8 %
ice hockey	-11.15	0.88	-11.60	33.00	**79.30**	67.04	650.0 %
jamesbond	29.00	302.80	100.50	21,323.00	25,354.00	**41,063.25**	14,986.9 %
kangaroo	52.00	3,035.00	51.20	1,416.00	14,130.70	**16,763.60**	560.2 %
# best	0	5	0	5	13	37	

Table S1 of the paper "Mastering Atari, Go, Chess and Shogi by Planning with a Learned Model" by Julian Schrittwieser et al.

MuZero – Detailed Atari Results

Game	Random	Human	SimPLe [20]	Ape-X [18]	R2D2 [21]	*MuZero*	*MuZero* normalized
krull	1,598.05	2,665.53	2,204.80	11,741.00	218,448.10	**269,358.27**	25,083.4 %
kung fu master	258.50	22,736.25	14,862.50	97,830.00	**233,413.30**	204,824.00	910.1 %
montezuma revenge	0.00	**4,753.33**	-	2,500.00	2,061.30	0.00	0.0 %
ms pacman	307.30	6,951.60	1,480.00	11,255.00	42,281.70	**243,401.10**	3,658.7 %
name this game	2,292.35	8,049.00	2,420.70	25,783.00	58,182.70	**157,177.85**	2,690.5 %
phoenix	761.40	7,242.60	-	224,491.00	864,020.00	**955,137.84**	14,725.3 %
pitfall	-229.44	**6,463.69**	-	-1.00	0.00	0.00	3.4 %
pong	-20.71	14.59	12.80	**21.00**	**21.00**	**21.00**	118.2 %
private eye	24.94	**69,571.27**	35.00	50.00	5,322.70	15,299.98	22.0 %
qbert	163.88	13,455.00	1,288.80	302,391.00	**408,850.00**	72,276.00	542.6 %
riverraid	1,338.50	17,118.00	1,957.80	63,864.00	45,632.10	**323,417.18**	2,041.1 %
road runner	11.50	7,845.00	5,640.60	222,235.00	599,246.70	**613,411.80**	7,830.5 %
robotank	2.16	11.94	-	74.00	100.40	**131.13**	1,318.7 %
seaquest	68.40	42,054.71	683.30	392,952.00	**999,996.70**	999,976.52	2,381.5 %
skiing	-17,098.09	**-4,336.93**	-	-10,790.00	-30,021.70	-29,968.36	-100.9 %
solaris	1,236.30	**12,326.67**	-	2,893.00	3,787.20	56.62	-10.6 %
space invaders	148.03	1,668.67	-	54,681.00	43,223.40	**74,335.30**	4,878.7 %
star gunner	664.00	10,250.00	-	434,343.00	**717,344.00**	549,271.70	5,723.0 %
surround	-9.99	6.53	-	7.00	9.90	**9.99**	120.9 %
tennis	-23.84	-8.27	-	**24.00**	-0.10	0.00	153.1 %
time pilot	3,568.00	5,229.10	-	87,085.00	445,377.30	**476,763.90**	28,486.9 %
tutankham	11.43	167.59	-	273.00	395.30	**491.48**	307.4 %
up n down	533.40	11,693.23	3,350.30	401,884.00	589,226.90	**715,545.61**	6,407.0 %
venture	0.00	1,187.50	-	1,813.00	**1,970.70**	0.40	0.0 %
video pinball	0.00	17,667.90	-	565,163.00	**999,383.20**	981,791.88	5,556.9 %
wizard of wor	563.50	4,756.52	-	46,204.00	144,362.70	**197,126.00**	4,687.9 %
yars revenge	3,092.91	54,576.93	5,664.30	148,595.00	**995,048.40**	553,311.46	1,068.7 %
zaxxon	32.50	9,173.30	-	42,286.00	224,910.70	**725,853.90**	7,940.5 %
# best	0	5	0	5	13	37	

Table S1 of the paper "Mastering Atari, Go, Chess and Shogi by Planning with a Learned Model" by Julian Schrittwieser et al.