N-step Temporal Difference Methods

Self-Study

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n-step Methods

Full return is

$$G_t = \sum_{k=t}^{\infty} R_{k+1},$$

one-step return is

$$G_{t:t+1} = R_{t+1} + \gamma V(S_{t+1}).$$

We can generalize both into n-step returns:

$$G_{t:t+n} \stackrel{\text{def}}{=} \left(\sum_{k=t}^{t+n-1} \gamma^{k-t} R_{k+1}\right) + \gamma^n V(S_{t+n}).$$

with $G_{t:t+n} \stackrel{\text{def}}{=} G_t$ if $t+n \geq T$ (episode length).

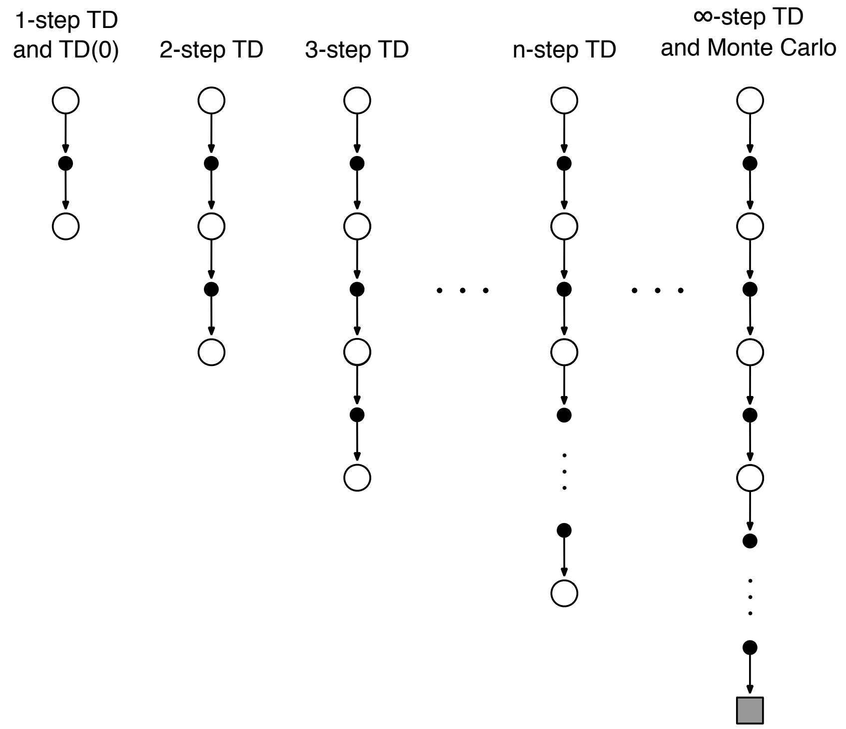


Figure 7.1 of "Reinforcement Learning: An Introduction, Second Edition".

n-step Methods

A natural update rule is

$$V(S_t) \leftarrow V(S_t) + \alpha \left[G_{t:t+n} - V(S_t)\right].$$

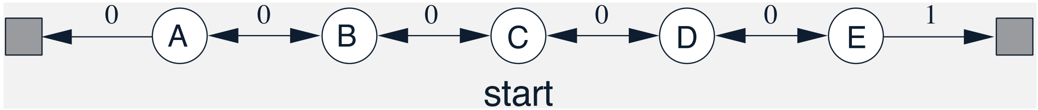
n-step TD for estimating $V \approx v_\pi$

Input: a policy π
Algorithm parameters: step size $\alpha \in (0, 1]$, a positive integer n
Initialize $V(s)$ arbitrarily, for all $s \in \mathcal{S}$
All store and access operations (for S_t and R_t) can take their index mod $n+1$

Loop for each episode:
  Initialize and store $S_0 \neq$ terminal
  $T \leftarrow \infty$
  Loop for $t = 0, 1, 2, \ldots$:
  | If $t < T$, then:
  | Take an action according to $\pi(\cdot|S_t)$
  | Observe and store the next reward as R_{t+1} and the next state as S_{t+1}
  | If S_{t+1} is terminal, then $T \leftarrow t+1$
  | $\tau \leftarrow t - n + 1$ (τ is the time whose state's estimate is being updated)
  | If $\tau \geq 0$:
  | $G \leftarrow \sum_{i=\tau+1}^{\min(\tau+n, T)} \gamma^{i-\tau-1} R_i$
  | If $\tau + n < T$, then: $G \leftarrow G + \gamma^n V(S_{\tau+n})$ $(G_{\tau:\tau+n})$
  | $V(S_\tau) \leftarrow V(S_\tau) + \alpha \left[G - V(S_\tau)\right]$
  Until $\tau = T - 1$

Algorithm 7.1 of "Reinforcement Learning: An Introduction, Second Edition".

Using the random walk example, but with 19 states instead of 5,



Example 6.2 of "Reinforcement Learning: An Introduction, Second Edition".

we obtain the following comparison of different values of n:

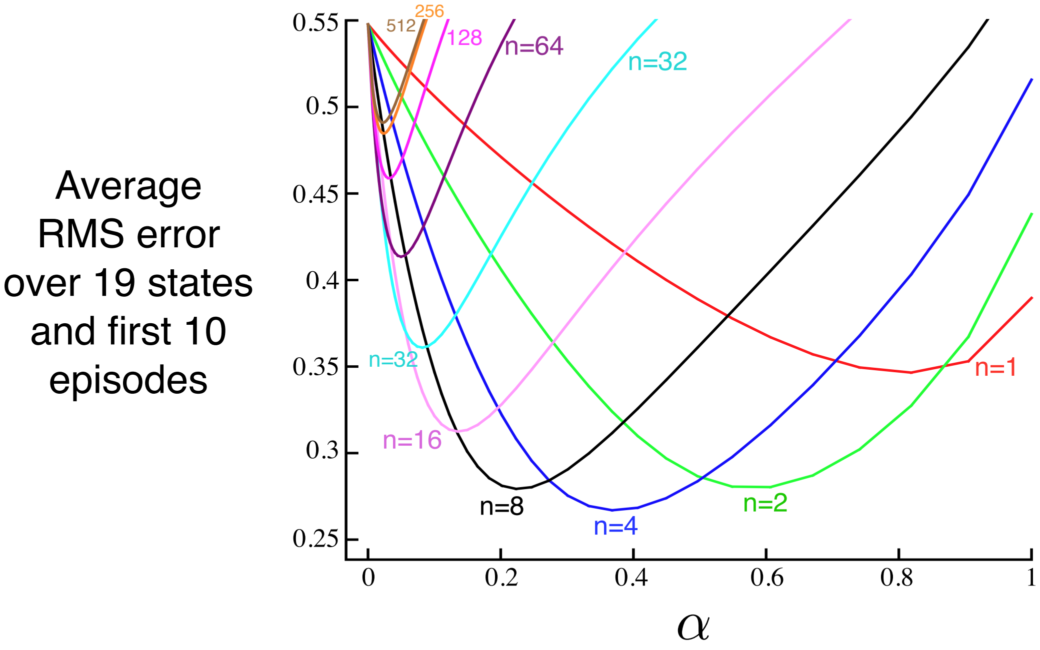


Figure 7.2 of "Reinforcement Learning: An Introduction, Second Edition".

n-step Sarsa

Defining the n-step return to utilize action-value function as

$$G_{t:t+n} \stackrel{\text{def}}{=} \left(\sum_{k=t}^{t+n-1} \gamma^{k-t} R_{k+1}\right) + \gamma^n Q(S_{t+n}, A_{t+n})$$

with $G_{t:t+n} \stackrel{\text{def}}{=} G_t$ if $t+n \geq T$, we get the following straightforward algorithm:

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \left[G_{t:t+n} - Q(S_t, A_t)\right].$$

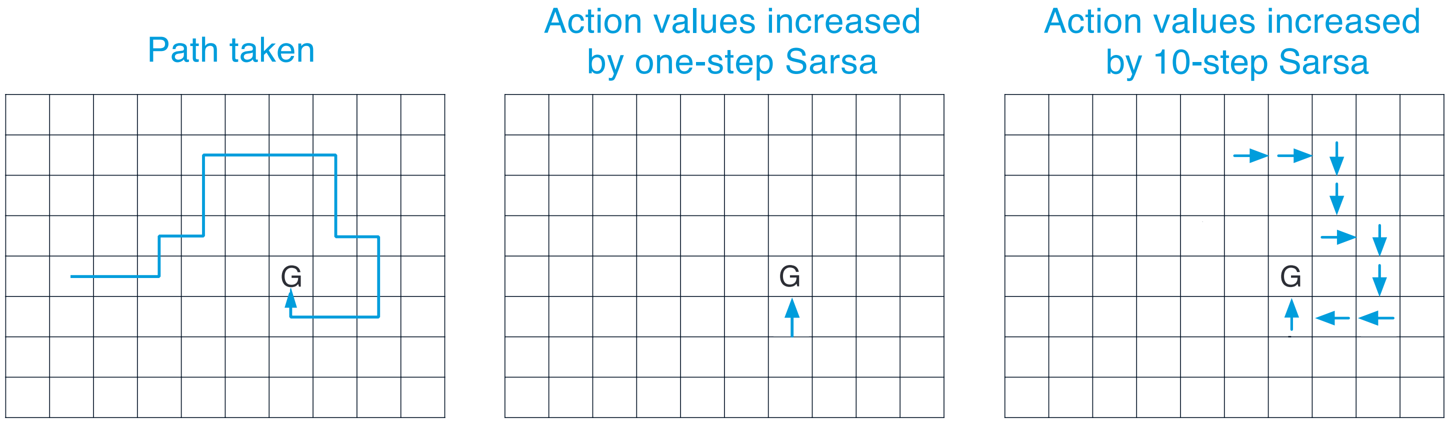


Figure 7.4 of "Reinforcement Learning: An Introduction, Second Edition".

n-step Sarsa Algorithm

n-step Sarsa for estimating $Q \approx q_*$ or q_π

Initialize $Q(s,a)$ arbitrarily, for all $s \in \mathcal{S}, a \in \mathcal{A}$
Initialize π to be ε-greedy with respect to Q, or to a fixed given policy
Algorithm parameters: step size $\alpha \in (0,1]$, small $\varepsilon > 0$, a positive integer n
All store and access operations (for S_t, A_t, and R_t) can take their index mod $n+1$

Loop for each episode:
    Initialize and store $S_0 \neq$ terminal
    Select and store an action $A_0 \sim \pi(\cdot|S_0)$
    $T \leftarrow \infty$
    Loop for $t = 0, 1, 2, \ldots$:
    | If $t < T$, then:
    | Take action A_t
    | Observe and store the next reward as R_{t+1} and the next state as S_{t+1}
    | If S_{t+1} is terminal, then:
    | $T \leftarrow t+1$
    | else:
    | Select and store an action $A_{t+1} \sim \pi(\cdot|S_{t+1})$
    | $\tau \leftarrow t - n + 1$ (τ is the time whose estimate is being updated)
    | If $\tau \geq 0$:
    | $G \leftarrow \sum_{i=\tau+1}^{\min(\tau+n,T)} \gamma^{i-\tau-1} R_i$
    | If $\tau + n < T$, then $G \leftarrow G + \gamma^n Q(S_{\tau+n}, A_{\tau+n})$ $(G_{\tau:\tau+n})$
    | $Q(S_\tau, A_\tau) \leftarrow Q(S_\tau, A_\tau) + \alpha \left[G - Q(S_\tau, A_\tau)\right]$
    | If π is being learned, then ensure that $\pi(\cdot|S_\tau)$ is ε-greedy wrt Q
    Until $\tau = T - 1$

Algorithm 7.2 of "Reinforcement Learning: An Introduction, Second Edition".

Off-policy n-step Sarsa

Recall the relative probability of a trajectory under the target and behaviour policies, which we now generalize as

$$\rho_{t:t+n} \stackrel{\text{def}}{=} \prod_{k=t}^{\min(t+n,T-1)} \frac{\pi(A_k|S_k)}{b(A_k|S_k)}.$$

Then a simple off-policy n-step TD can be computed as

$$V(S_t) \leftarrow V(S_t) + \alpha\rho_{t:t+n-1}\left[G_{t:t+n} - V(S_t)\right].$$

Similarly, n-step Sarsa becomes

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha\rho_{\boldsymbol{t+1:t+n}}\left[G_{t:t+n} - Q(S_t, A_t)\right].$$

Off-policy n-step Sarsa

Off-policy n-step Sarsa for estimating $Q \approx q_*$ or q_π

Input: an arbitrary behavior policy b such that $b(a|s) > 0$, for all $s \in \mathcal{S}, a \in \mathcal{A}$
Initialize $Q(s, a)$ arbitrarily, for all $s \in \mathcal{S}, a \in \mathcal{A}$
Initialize π to be greedy with respect to Q, or as a fixed given policy
Algorithm parameters: step size $\alpha \in (0, 1]$, a positive integer n
All store and access operations (for S_t, A_t, and R_t) can take their index mod $n + 1$

Loop for each episode:
    Initialize and store $S_0 \neq$ terminal
    Select and store an action $A_0 \sim b(\cdot|S_0)$
    $T \leftarrow \infty$
    Loop for $t = 0, 1, 2, \ldots$:
    | If $t < T$, then:
    | Take action A_t
    | Observe and store the next reward as R_{t+1} and the next state as S_{t+1}
    | If S_{t+1} is terminal, then:
    | $T \leftarrow t + 1$
    | else:
    | Select and store an action $A_{t+1} \sim b(\cdot|S_{t+1})$
    | $\tau \leftarrow t - n + 1$ (τ is the time whose estimate is being updated)
    | If $\tau \geq 0$:
    | $\rho \leftarrow \prod_{i=\tau+1}^{\min(\tau+n\ \ \ \ ,T-1)} \frac{\pi(A_i|S_i)}{b(A_i|S_i)}$ ($\rho_{\tau+1:t+n}$)
    | $G \leftarrow \sum_{i=\tau+1}^{\min(\tau+n,T)} \gamma^{i-\tau-1} R_i$
    | If $\tau + n < T$, then: $G \leftarrow G + \gamma^n Q(S_{\tau+n}, A_{\tau+n})$ ($G_{\tau:\tau+n}$)
    | $Q(S_\tau, A_\tau) \leftarrow Q(S_\tau, A_\tau) + \alpha\rho\left[G - Q(S_\tau, A_\tau)\right]$
    | If π is being learned, then ensure that $\pi(\cdot|S_\tau)$ is greedy wrt Q
    Until $\tau = T - 1$

Modified from Algorithm 7.3 of "Reinforcement Learning: An Introduction, Second Edition" by changing ρ_{τ+1:τ+n-1} to ρ_{τ+1:τ+n}.

Off-policy n-step Without Importance Sampling

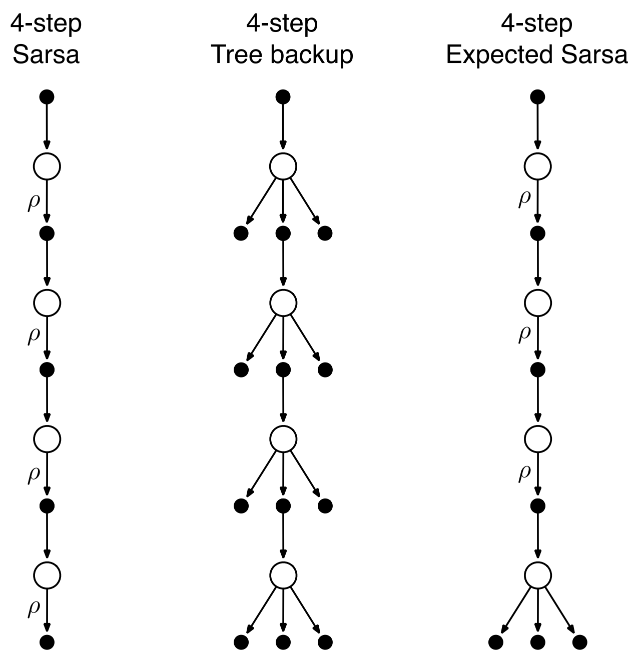


Figure 7.5 of "Reinforcement Learning: An Introduction, Second Edition".

Q-learning and Expected Sarsa can learn off-policy without importance sampling.

To generalize to n-step off-policy method, we must compute expectations over actions in each step of n-step update. However, we have not obtained a return for the non-sampled actions.

Luckily, we can estimate their values by using the current action-value function.

We now derive the n-step reward, starting from one-step:

$$G_{t:t+1} \stackrel{\text{def}}{=} R_{t+1} + \sum\nolimits_a \pi(a|S_{t+1})Q(S_{t+1}, a).$$

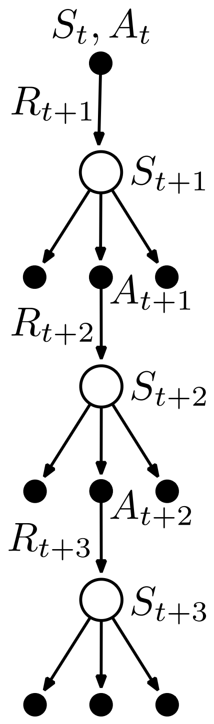
For two-step, we get:

$$G_{t:t+2} \stackrel{\text{def}}{=} R_{t+1} + \gamma \sum\nolimits_{a \neq A_{t+1}} \pi(a|S_{t+1})Q(S_{t+1}, a) + \gamma\pi(A_{t+1}|S_{t+1})G_{t+1:t+2}.$$

Therefore, we can generalize to:

$$G_{t:t+n} \stackrel{\text{def}}{=} R_{t+1} + \gamma \sum\nolimits_{a \neq A_{t+1}} \pi(a|S_{t+1})Q(S_{t+1}, a) + \gamma\pi(A_{t+1}|S_{t+1})G_{t+1:t+n}.$$

The resulting algorithm is n-step **Tree backup** and it is an off-policy n-step temporal difference method not requiring importance sampling.



the 3-step tree-backup update

Example in Section 7.5 of "Reinforcement Learning: An Introduction, Second Edition".

Off-policy n-step Without Importance Sampling

n-step Tree Backup for estimating $Q \approx q_*$ or q_π

Initialize $Q(s,a)$ arbitrarily, for all $s \in \mathcal{S}, a \in \mathcal{A}$
Initialize π to be greedy with respect to Q, or as a fixed given policy
Algorithm parameters: step size $\alpha \in (0,1]$, a positive integer n
All store and access operations can take their index mod $n+1$

```
Loop for each episode:
   Initialize and store S_0 ≠ terminal
   Choose an action A_0 arbitrarily as a function of S_0; Store A_0
   T ← ∞
   Loop for t = 0, 1, 2, ... :
   |  If t < T:
   |     Take action A_t; observe and store the next reward and state as R_{t+1}, S_{t+1}
   |     If S_{t+1} is terminal:
   |        T ← t + 1
   |     else:
   |        Choose an action A_{t+1} arbitrarily as a function of S_{t+1}; Store A_{t+1}
   |  τ ← t + 1 − n    (τ is the time whose estimate is being updated)
   |  If τ ≥ 0:
   |     If t + 1 ≥ T:
   |        G ← R_T
   |     else
   |        G ← R_{t+1} + γ Σ_a π(a|S_{t+1}) Q(S_{t+1}, a)
   |     Loop for k = min(t, T − 1) down through τ + 1:
   |        G ← R_k + γ Σ_{a≠A_k} π(a|S_k) Q(S_k, a) + γ π(A_k|S_k) G
   |     Q(S_τ, A_τ) ← Q(S_τ, A_τ) + α [G − Q(S_τ, A_τ)]
   |     If π is being learned, then ensure that π(·|S_τ) is greedy wrt Q
   Until τ = T − 1
```

Algorithm 7.5 of "Reinforcement Learning: An Introduction, Second Edition".