V-trace, PopArt Normalization, Partially Observable MDPs

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Impala (Importance Weighted Actor-Learner Architecture) was suggested in Feb 2018 paper and allows massively distributed implementation of an actor-critic-like learning algorithm.

Compared to A3C-based agents, which communicates gradients with respect to the parameters of the policy, IMPALA actors communicates trajectories to the centralized learner.

If many actors are used, the policy used to generate a trajectory can lag behind the latest policy. Therefore, a new V-trace off-policy actor-critic algorithm is proposed.
Consider a trajectory \((S_t, A_t, R_{t+1})_{t=s}^{s+n}\) generated by a behaviour policy \(b\).

The \(n\)-step V-trace target for \(S_s\) is defined as

\[
v_s \overset{\text{def}}{=} V(S_s) + \sum_{t=s}^{s+n-1} \gamma^{t-s} \left( \prod_{i=s}^{t-1} c_i \right) \delta_t V,
\]

where \(\delta_t V\) is the temporal difference for \(V\)

\[
\delta_t V \overset{\text{def}}{=} \rho_t \left( R_{t+1} + \gamma V(s_{t+1}) - V(s_t) \right),
\]

and \(\rho_t\) and \(c_i\) are truncated importance sampling ratios with \(\bar{\rho} \geq \bar{c}\):

\[
\rho_t \overset{\text{def}}{=} \min \left( \bar{\rho}, \frac{\pi(A_t | S_t)}{b(A_t | S_t)} \right), \quad c_i \overset{\text{def}}{=} \min \left( \bar{c}, \frac{\pi(A_i | S_i)}{b(A_i | S_i)} \right).
\]

Note that if \(b = \pi\) and assuming \(\bar{c} \geq 1\), \(v_s\) reduces to \(n\)-step Bellman target.
Note that the truncated IS weights $\rho_t$ and $c_i$ play different roles:

- The $\rho_t$ appears in the definition of $\delta_t V$ and defines the fixed point of the update rule. For $\bar{\rho} = \infty$, the target is the value function $v_\pi$, if $\bar{\rho} < \infty$, the fixed point is somewhere between $v_\pi$ and $v_b$. Notice that we do not compute a product of these $\rho_t$ coefficients.
- The $c_i$ impacts the speed of convergence (the contraction rate of the Bellman operator), not the sought policy. Because a product of the $c_i$ ratios is computed, it plays an important role in variance reduction.

The paper utilizes $c = 1$ and out of $\bar{\rho} \in \{1, 10, 100\}$, $\bar{\rho} = 1$ works empirically the best.
Consider a parametrized functions computing $v(s; \theta)$ and $\pi(a|s; \omega)$. Assuming the defined $n$-step V-trace target

$$v_s \overset{\text{def}}{=} V(S_s) + \sum_{t=s}^{s+n-1} \gamma^{t-s} \left( \prod_{i=s}^{t-1} c_i \right) \delta_t V,$$

we update the critic in the direction of

$$(v_s - v(S_s; \theta)) \nabla_\theta v(S_s; \theta)$$

and the actor in the direction of the policy gradient

$$\rho_s \nabla_\omega \log \pi(A_s|S_s; \omega) \left( R_{s+1} + \gamma v_{s+1} - v(S_s; \theta) \right).$$

Finally, we again add the entropy regularization term $H(\pi(\cdot|S_s; \theta))$ to the loss function.
<table>
<thead>
<tr>
<th>Architecture</th>
<th>CPUs</th>
<th>GPUs$^1$</th>
<th>FPS$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Task 1</td>
</tr>
<tr>
<td><strong>Single-Machine</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A3C 32 workers</td>
<td>64</td>
<td>0</td>
<td>6.5K</td>
</tr>
<tr>
<td>Batched A2C (sync step)</td>
<td>48</td>
<td>0</td>
<td>9K</td>
</tr>
<tr>
<td>Batched A2C (sync step)</td>
<td>48</td>
<td>1</td>
<td>13K</td>
</tr>
<tr>
<td>Batched A2C (sync traj.)</td>
<td>48</td>
<td>0</td>
<td>16K</td>
</tr>
<tr>
<td>Batched A2C (dyn. batch)</td>
<td>48</td>
<td>1</td>
<td>16K</td>
</tr>
<tr>
<td>IMPALA 48 actors</td>
<td>48</td>
<td>0</td>
<td>17K</td>
</tr>
<tr>
<td>IMPALA (dyn. batch) 48 actors$^3$</td>
<td>48</td>
<td>1</td>
<td>21K</td>
</tr>
<tr>
<td><strong>Distributed</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A3C</td>
<td>200</td>
<td>0</td>
<td>46K</td>
</tr>
<tr>
<td>IMPALA</td>
<td>150</td>
<td>1</td>
<td>80K</td>
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<tr>
<td>IMPALA (optimised)</td>
<td>375</td>
<td>1</td>
<td>200K</td>
</tr>
<tr>
<td>IMPALA (optimised) batch 128</td>
<td>500</td>
<td>1</td>
<td>250K</td>
</tr>
</tbody>
</table>

$^1$ Nvidia P100  $^2$ In frames/sec (4 times the agent steps due to action repeat).  $^3$ Limited by amount of rendering possible on a single machine.

Table 1 of the paper "IMPALA: Scalable Distributed Deep-RL with Importance Weighted Actor-Learner Architectures" by Lasse Espeholt et al.
For Atari experiments, population based training with a population of 24 agents is used to adapt entropy regularization, learning rate, RMSProp $\varepsilon$ and the global gradient norm clipping threshold.

Figure 1 of paper “Population Based Training of Neural Networks” by Max Jaderberg et al.
For Atari experiments, population based training with a population of 24 agents is used to adapt entropy regularization, learning rate, RMSProp $\varepsilon$ and the global gradient norm clipping threshold.

In population based training, several agents are trained in parallel. When an agent is ready (after 5000 episodes), then:

- it may be overwritten by parameters and hyperparameters of another agent, if it is sufficiently better (5000 episode mean capped human normalized score returns are 5% better);
- and independently, the hyperparameters may undergo a change (multiplied by either 1.2 or $1/1.2$ with 33% chance).
Figure 4 of the paper "IMPALA: Scalable Distributed Deep-RL with Importance Weighted Actor-Learner Architectures" by Lasse Espeholt et al.
Figures 5, 6 of the paper "IMPALA: Scalable Distributed Deep-RL with Importance Weighted Actor-Learner Architectures" by Lasse Espeholt et al.
### Human Normalised Return

<table>
<thead>
<tr>
<th></th>
<th>Median</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>A3C, shallow, experts</td>
<td>54.9%</td>
<td>285.9%</td>
</tr>
<tr>
<td>A3C, deep, experts</td>
<td>117.9%</td>
<td>503.6%</td>
</tr>
<tr>
<td>Reactor, experts</td>
<td>187%</td>
<td>N/A</td>
</tr>
<tr>
<td>IMPALA, shallow, experts</td>
<td>93.2%</td>
<td>466.4%</td>
</tr>
<tr>
<td>IMPALA, deep, experts</td>
<td>191.8%</td>
<td>957.6%</td>
</tr>
<tr>
<td>IMPALA, deep, multi-task</td>
<td>59.7%</td>
<td>176.9%</td>
</tr>
</tbody>
</table>

*Table 4 of the paper “IMPALA: Scalable Distributed Deep-RL with Importance Weighted Actor-Learner Architectures” by Lasse Espeholt et al.*
## IMPALA – Ablations

<table>
<thead>
<tr>
<th>Task</th>
<th>Task 1</th>
<th>Task 2</th>
<th>Task 3</th>
<th>Task 4</th>
<th>Task 5</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Without Replay</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V-trace</td>
<td>46.8</td>
<td>32.9</td>
<td>31.3</td>
<td>229.2</td>
<td>43.8</td>
</tr>
<tr>
<td>1-Step</td>
<td><strong>51.8</strong></td>
<td><strong>35.9</strong></td>
<td>25.4</td>
<td>215.8</td>
<td>43.7</td>
</tr>
<tr>
<td>$\varepsilon$-correction</td>
<td>44.2</td>
<td>27.3</td>
<td>4.3</td>
<td>107.7</td>
<td>41.5</td>
</tr>
<tr>
<td>No-correction</td>
<td>40.3</td>
<td>29.1</td>
<td>5.0</td>
<td>94.9</td>
<td>16.1</td>
</tr>
<tr>
<td><strong>With Replay</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V-trace</td>
<td>47.1</td>
<td><strong>35.8</strong></td>
<td><strong>34.5</strong></td>
<td><strong>250.8</strong></td>
<td><strong>46.9</strong></td>
</tr>
<tr>
<td>1-Step</td>
<td><strong>54.7</strong></td>
<td>34.4</td>
<td>26.4</td>
<td>204.8</td>
<td>41.6</td>
</tr>
<tr>
<td>$\varepsilon$-correction</td>
<td>30.4</td>
<td>30.2</td>
<td>3.9</td>
<td>101.5</td>
<td>37.6</td>
</tr>
<tr>
<td>No-correction</td>
<td>35.0</td>
<td>21.1</td>
<td>2.8</td>
<td>85.0</td>
<td>11.2</td>
</tr>
</tbody>
</table>

Tasks: rooms_watermaze, rooms_keys_doors_puzzle, lasertag_three_opponents_small, explore_goal_locations_small, seekavoid_arena_01

Table 2 of the paper “IMPALA: Scalable Distributed Deep-RL with Importance Weighted Actor-Learner Architectures” by Lasse Espeholt et al.
Figure E.1 of the paper "IMPALA: Scalable Distributed Deep-RL with Importance Weighted Actor-Learner Architectures" by Lasse Espeholt et al.
PopArt Normalization

An improvement of IMPALA from Sep 2018, which performs normalization of task rewards instead of just reward clipping.

Assume the value estimate \( v(s; \theta, \sigma, \mu) \) is normalized with respect to an unnormalized value predictor \( n(s; \theta) \) as

\[
v(s; \theta, \sigma, \mu) \overset{\text{def}}{=} \sigma n(s; \theta) + \mu
\]

and further assume that \( n(s; \theta) \) is an output of a linear function

\[
n(s; \theta) \overset{\text{def}}{=} \omega^T f(s; \theta - \{\omega, b\}) + b.
\]

We can update the \( \sigma \) and \( \mu \) using exponentially moving average with decay rate \( \beta \) (in the paper, first moment \( \mu \) and second moment \( \nu \) is tracked, and standard deviation is computed as \( \sigma = \sqrt{\nu - \mu^2}; \beta = 3 \cdot 10^{-4} \) is used).
PopArt Normalization

Using the normalization parameters $\mu$ and $\sigma$, we can normalize returns as $(G - \mu)/\sigma$ and utilize actor-critic algorithm with advantage $(G - \mu)/\sigma - n(S; \theta)$.

However, in order to make sure the value function estimate does not change when the normalization parameters change, the parameters $\omega, b$ computing the unnormalized value estimate are updated under any change $\mu \rightarrow \mu'$ and $\sigma \rightarrow \sigma'$ as:

$$\omega' \overset{\text{def}}{=} \frac{\sigma}{\sigma'} \omega, \quad b' \overset{\text{def}}{=} \frac{\sigma b + \mu - \mu'}{\sigma'}.$$

In multi-task settings, we train a task-agnostic policy and task-specific value functions (therefore, $\mu$, $\sigma$ and $n(s; \theta)$ are vectors).
PopArt Results

<table>
<thead>
<tr>
<th>Agent</th>
<th>Atari-57</th>
<th>Atari-57 (unclipped)</th>
<th>DmLab-30</th>
</tr>
</thead>
<tbody>
<tr>
<td>IMPALA</td>
<td>Random</td>
<td>Human</td>
<td>Train</td>
</tr>
<tr>
<td>IMPALA</td>
<td>59.7%</td>
<td>0.3%</td>
<td>60.6%</td>
</tr>
<tr>
<td>PopArt-IMPALA</td>
<td>110.7%</td>
<td>107.0%</td>
<td>73.5%</td>
</tr>
<tr>
<td>PopArt-IMPALA</td>
<td>28.5%</td>
<td>1.0%</td>
<td>58.4%</td>
</tr>
<tr>
<td>PopArt-IMPALA</td>
<td>101.5%</td>
<td>93.7%</td>
<td>72.8%</td>
</tr>
</tbody>
</table>

Table 1 of paper “Multi-task Deep Reinforcement Learning with PopArt” by Matteo Hessel et al.

Figures 1, 2 of paper “Multi-task Deep Reinforcement Learning with PopArt” by Matteo Hessel et al.
Figure 3 of paper “Multi-task Deep Reinforcement Learning with PopArt” by Matteo Hessel et al.

- Breakout
- Crazy Climber
- Qbert
- Seaquest
PopArt Results

Figures 4, 5 of paper “Multi-task Deep Reinforcement Learning with PopArt” by Matteo Hessel et al.
Partially Observable MDPs

Recall that a Markov decision process (MDP) is a quadruple \((S, A, p, \gamma)\), where:

1. \(S\) is a set of states,
2. \(A\) is a set of actions,
3. \(p(S_{t+1} = s', R_{t+1} = r | S_t = s, A_t = a)\) is a probability that action \(a \in A\) will lead from state \(s \in S\) to \(s' \in S\), producing a reward \(r \in \mathbb{R}\),
4. \(\gamma \in [0, 1]\) is a discount factor.

Partially observable Markov decision process extends the Markov decision process to a sextuple \((S, A, p, \gamma, O, o)\), where in addition to an MDP

1. \(O\) is a set of observations,
2. \(o(O_t | S_t, A_{t-1})\) is an observation model.

In robotics (out of the domain of this course), several approaches are used to handle POMDPs, to model uncertainty, imprecise mechanisms and inaccurate sensors.
Partially Observable MDPs

In Deep RL, partially observable MDPs are usually handled using recurrent networks. After suitable encoding of input observation $O_t$ and previous action $A_{t-1}$, a RNN (usually LSTM) unit is used to model the current $S_t$ (or its suitable latent representation), which is in turn utilized to produce $A_t$.

**a. RL-LSTM**

![Diagram](image)

Figure 1a of paper "Unsupervised Predictive Memory in a Goal-Directed Agent" by Greg Wayne et al.
However, keeping all information in the RNN state is substantially limiting. Therefore, memory-augmented networks can be used to store suitable information in external memory (in the lines of NTM, DNC or MANN models).

We now describe an approach used by Merlin architecture (*Unsupervised Predictive Memory in a Goal-Directed Agent* DeepMind Mar 2018 paper).

**b. RL-MEM**

![Diagram](image)

*Figure 1b of paper "Unsupervised Predictive Memory in a Goal-Directed Agent" by Greg Wayne et al.*
Let $M$ be a memory matrix of size $N_{mem} \times 2|z|$.

Assume we have already encoded observations as $e_t$ and previous action $a_{t-1}$. We concatenate them with $K$ previously read vectors and process by a deep LSTM (two layers are used in the paper) to compute $h_t$.

Then, we apply a linear layer to $h_t$, computing $K$ key vectors $k_1, \ldots k_K$ of length $2|z|$ and $K$ positive scalars $\beta_1, \ldots, \beta_K$.

**Reading:** For each $i$, we compute cosine similarity of $k_i$ and all memory rows $M_j$, multiply the similarities by $\beta_i$ and pass them through a $\text{softmax}$ to obtain weights $\omega_i$. The read vector is then computed as $M\omega_i$.

**Writing:** We find one-hot write index $v_{wr}$ to be the least used memory row (we keep usage indicators and add read weights to them). We then compute $v_{ret} \leftarrow \gamma v_{ret} + (1 - \gamma) v_{wr}$, and update the memory matrix using $M \leftarrow M + v_{wr}[e_t, 0] + v_{ret}[0, e_t]$. 

![Figure 1b of paper "Unsupervised Predictive Memory in a Goal-Directed Agent" by Greg Wayne et al.](image)
However, updating the encoder and memory content purely using RL is inefficient. Therefore, MERLIN includes a *memory-based predictor (MBP)* in addition to policy. The goal of MBP is to compress observations into low-dimensional state representations $z$ and storing them in memory.

According to the paper, the idea of unsupervised and predictive modeling has been entertained for decades, and recent discussions have proposed such modeling to be connected to hippocampal memory.

We want the state variables not only to faithfully represent the data, but also emphasise rewarding elements of the environment above irrelevant ones. To accomplish this, the authors follow the hippocampal representation theory of Gluck and Myers, who proposed that hippocampal representations pass through a compressive bottleneck and then reconstruct input stimuli together with task reward.

In MERLIN, a *prior* distribution over $z_t$ predicts next state variable conditioned on history of state variables and actions $p(z_t|z_{t-1}, a_{t-1}, \ldots, z_1, a_1)$, and *posterior* corrects the prior using the new observation $o_t$, forming a better estimate $q(z_t|o_t, z_{t-1}, a_{t-1}, \ldots, z_1, a_1)$. 
To achieve the mentioned goals, we add two terms to the loss.

- We try reconstructing input stimuli, action, reward and return using a sample from the state variable posterior, and add the difference of the reconstruction and ground truth to the loss.
- We also add KL divergence of the prior and posterior to the loss, to ensure consistency between the prior and posterior.

**c. MERLIN**

![MERLIN Diagram](image)

*Figure 1c of paper "Unsupervised Predictive Memory in a Goal-Directed Agent" by Greg Wayne et al.*
Algorithm 1 MERLIN Worker Pseudocode

// Assume global shared parameter vectors $\theta$ for the policy network and $\chi$ for the memory-based predictor; global shared counter $T$ := 0
// Assume thread-specific parameter vectors $\theta^t, \chi^t$
// Assume discount factor $\gamma \in [0, 1]$ and bootstrapping parameter $\lambda \in [0, 1]$
Initialize thread step counter $t$ := 1
repeat
  Synchronize thread-specific parameters $\theta^t := \theta^t, \chi^t := \chi^t$
  Zero model's memory & recurrent state if new episode begins
  $t_{end} := t$
  repeat
    Prior $N(\mu^t, \log \Sigma^t) = p(h_{t-1}, m_{t-1})$
    $\sigma_t = \exp(\alpha_t)$
    Posterior $N(\mu^t, \log \Sigma^t) = q(\tau_t, h_{t-1}, m_{t-1}, \mu^t, \log \Sigma^t)$
    Sample $\tau_t \sim N(\mu^t, \log \Sigma^t)$
    Policy network update $h_t := \text{rec}((h_{t-1}, m_{t-1}, \text{StopGradient}(\tau_t)))$
    Policy distribution $m_t := \pi(h_t, \text{StopGradient}(\tau_t))$
    Sample $a_t \sim \pi_t$
    $h_t := \text{rec}(h_{t-1}, m_{t}, a_t)$
    Update memory with $\tau_t$ by Methods Eq. 2
    $R_t, o'_t := \text{dec}(\tau_t, m_{t}, a_t)$
    Apply $a_t$ to environment and receive reward $r_t$ and observation $o_{t+1}$
    $t := t + 1; T := T + 1$
  until environment termination or $T - t_{end} = \tau_{window}$
  If not terminated, run additional step to compute $V^\gamma(z_{t+1}, \log \pi_{t+1})$ and set $R_{t+1} := V^\gamma(z_{t+1}, \log \pi_{t+1})$ if (but don’t increment counters)
  Reset performance accumulators $A := 0; L := 0; H := 0$
  for $k$ from $1$ down to $t_{end}$ do
    $\gamma_k := \{\gamma, \text{otherwise}\}$
    $R_k := r_k + \gamma R_{k+1}$
    $\delta_k := r_k + \gamma V(z_{k+1}, \log \pi_{k+1}) - V(z_k, \log \pi_k)$
    $A_k := \delta_k + (\gamma \lambda) A_{k+1}$
    $L := L + L_k$ (Eq. 7)
    $A := A + A_k \log \pi_k$ (Empirical loss)
    $H := H - \gamma \text{entropy} \sum \pi_k (\log \pi_k)$ (Entropy loss)
  end for
  $dx' := \nabla_{\psi} L$
  $d\theta' := \nabla_{\theta} (A + H)$
  Asynchronously update via gradient ascent $\theta$ using $d\theta'$ and $\chi$ using $dx'$
  until $T > T_{max}$

Algorithm 1 of paper "Unsupervised Predictive Memory in a Goal-Directed Agent" by Greg Wayne et al.
Figure 2 of paper “Unsupervised Predictive Memory in a Goal-Directed Agent” by Greg Wayne et al.
Figure 3 of paper "Unsupervised Predictive Memory in a Goal-Directed Agent" by Greg Wayne et al.
Extended Figure 3 of paper "Unsupervised Predictive Memory in a Goal-Directed Agent" by Greg Wayne et al.
For the Win agent for Capture The Flag

(a) FTW Agent Architecture

(b) Progression During Training

Figure 2 of paper "Human-level performance in first-person multiplayer games with population-based deep reinforcement learning" by Max Jaderber et al.
For the Win agent for Capture The Flag

- Extension of the MERLIN architecture.
- Hierarchical RNN with two timescales.
- Population based training controlling KL divergence penalty weights, slow ticking RNN speed and gradient flow factor from fast to slow RNN.
For the Win agent for Capture The Flag

Figure S10 of paper "Human-level performance in first-person multiplayer games with population-based deep reinforcement learning" by Max Jaderber et al.
For the Win agent for Capture The Flag

Figure 4 of paper "Human-level performance in first-person multiplayer games with population-based deep reinforcement learning" by Max Jaderber et al.