

# Convolutional Neural Networks

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unless otherwise stated

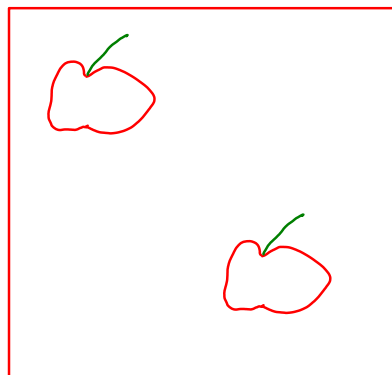
## Going Deeper

# Convolutional Networks

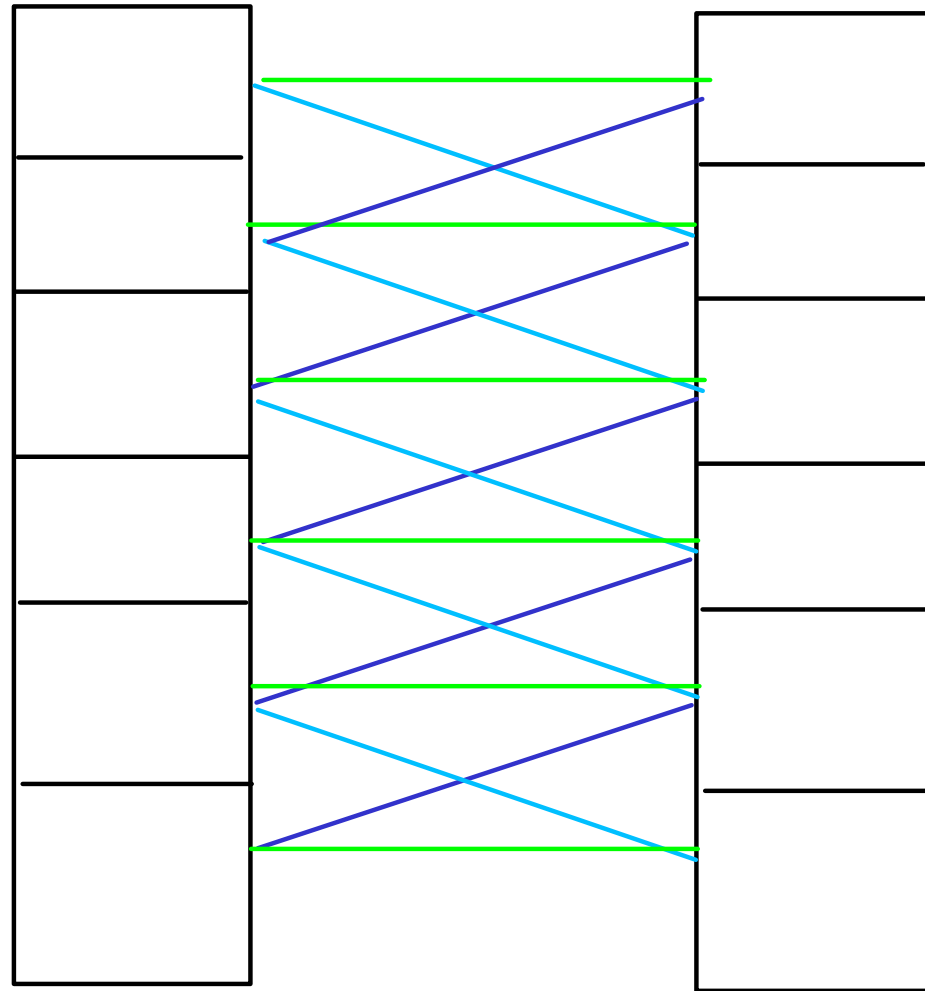
Consider data with some structure (temporal data, speech, images, ...).

Unlike densely connected layers, we might want:

- local interactions only;
- shift invariance (equal response everywhere);



- parameter sharing.



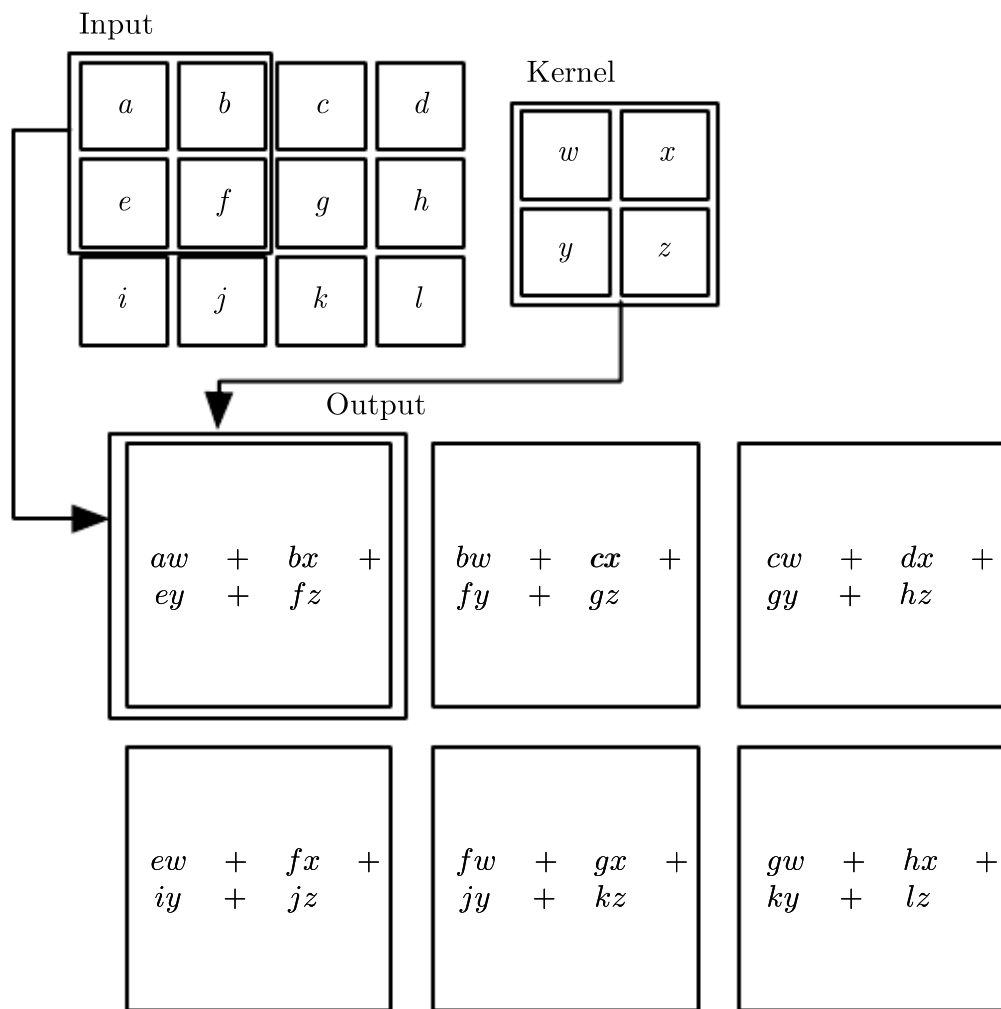
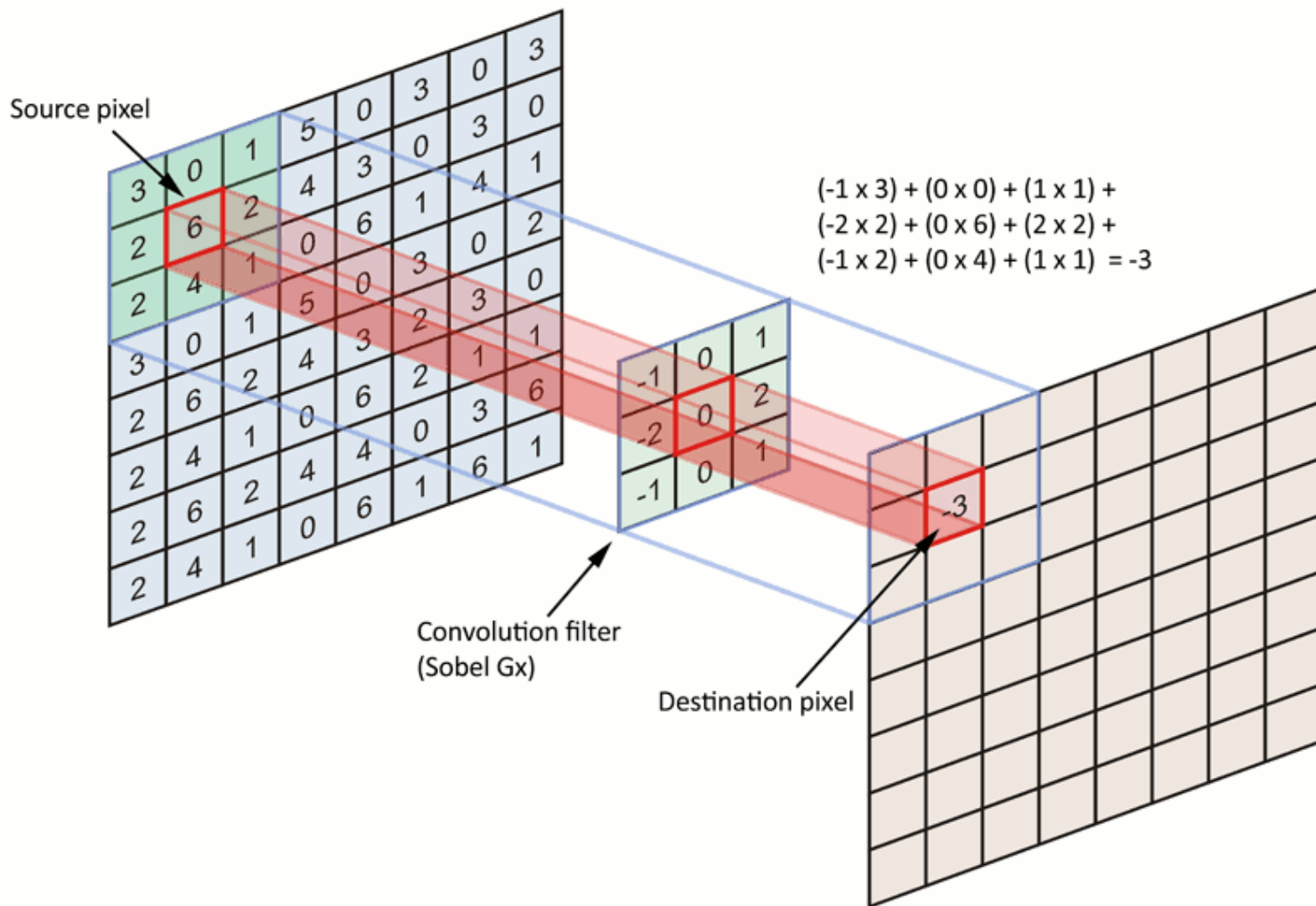


Figure 9.1 of "Deep Learning" book, <https://www.deeplearningbook.org>

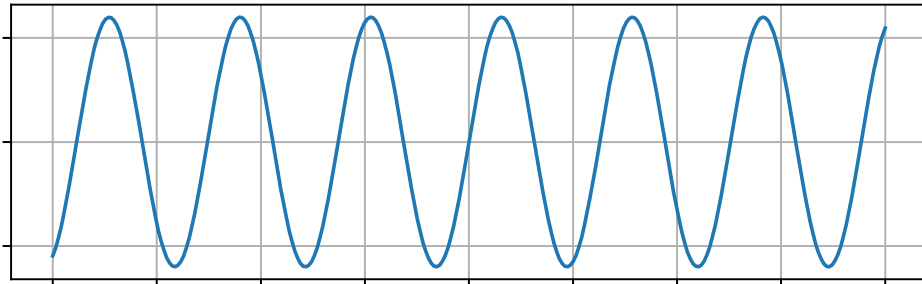


<https://i.stack.imgur.com/YDusp.png>

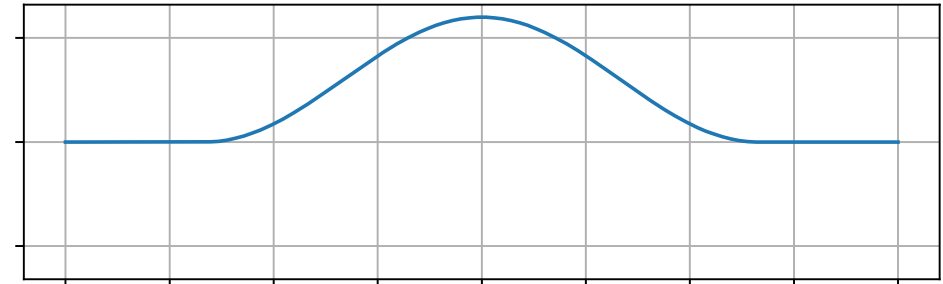
For a functions  $x$  and  $w$ , *convolution*  $w * x$  is defined as

$$(w * x)(t) = \int x(t - a)w(a) da.$$

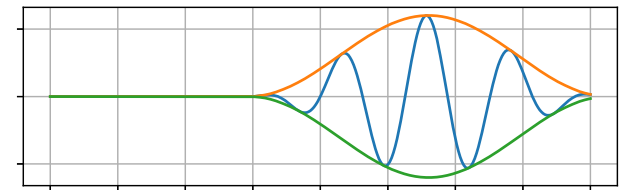
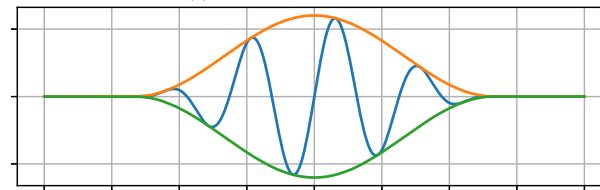
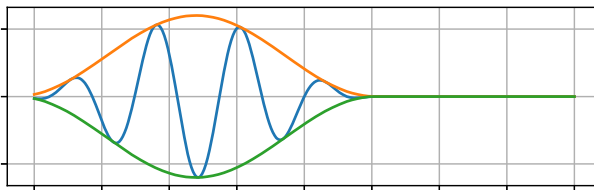
Time Signal  $x$



Window Function  $w$



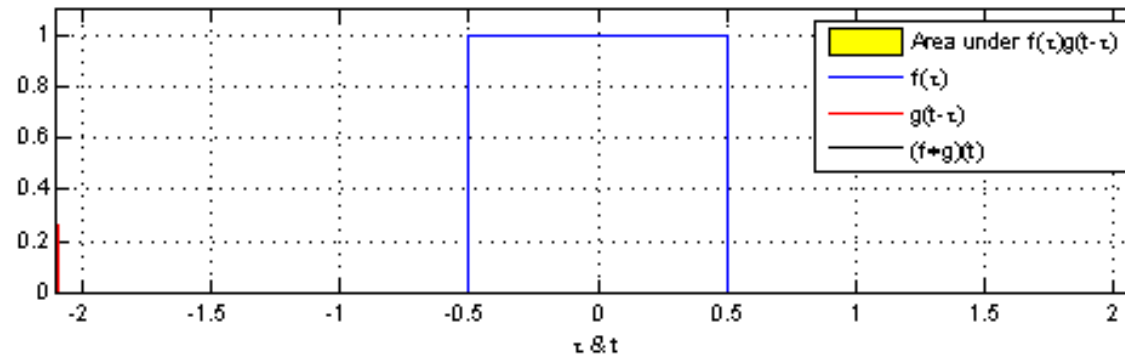
Applied Window Function



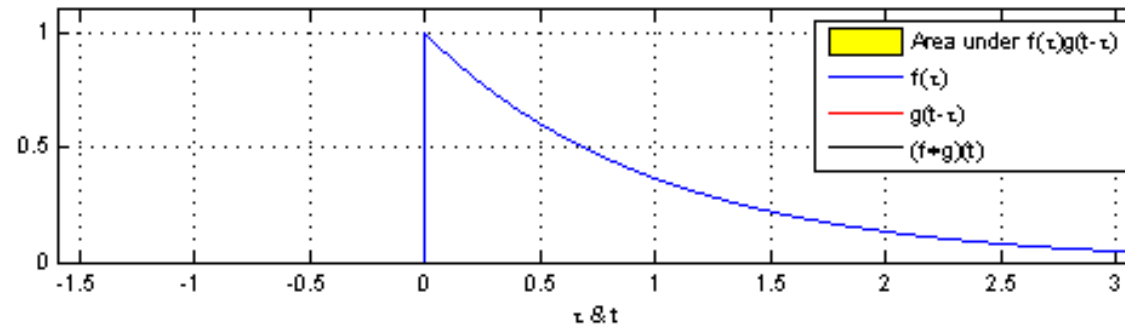
# Convolution Operation

For a functions  $x$  and  $w$ , *convolution*  $w * x$  is defined as

$$(w * x)(t) = \int x(t - a)w(a) da.$$



[https://commons.wikimedia.org/wiki/File:Convolution\\_of\\_box\\_signal\\_with\\_itself2.gif](https://commons.wikimedia.org/wiki/File:Convolution_of_box_signal_with_itself2.gif)



[https://commons.wikimedia.org/wiki/File:Convolution\\_of\\_spiky\\_function\\_with\\_box2.gif](https://commons.wikimedia.org/wiki/File:Convolution_of_spiky_function_with_box2.gif)



# Convolution Operation

For a functions  $x$  and  $w$ , *convolution*  $w * x$  is defined as

$$(w * x)(t) = \int x(t - a)w(a) da.$$

For vectors, we have

$$(w * x)_t = \sum_i x_{t-i}w_i.$$

Convolution operation can be generalized to two dimensions by

$$(\mathbf{K} * \mathbf{I})_{i,j} = \sum_{m,n} \mathbf{I}_{i-m,j-n} \mathbf{K}_{m,n}.$$

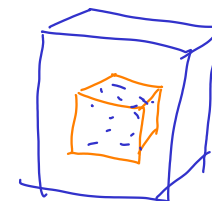
Closely related is *cross-correlation*, where  $K$  is flipped:

$$(\mathbf{K} \star \mathbf{I})_{i,j} = \sum_{m,n} \mathbf{I}_{i+m,j+n} \mathbf{K}_{m,n}.$$

The  $K$  is usually called a **kernel** or a **filter**.

Note that usually we have a whole vector of values for a single pixel, the so-called **channels**. These single pixel channel values have no longer any spacial structure, so the kernel contains a different set of weights for every input dimension, obtaining

$$(K \star I)_{i,j} = \sum_{m,n,c} I_{i+m,j+n,c} K_{m,n,c}.$$



Furthermore, we usually want to be able to specify the output dimensionality similarly to for example a fully connected layer – the number of **output channels** for every pixel. Each output channel is then the output of an independent convolution operation, so we can consider  $K$  to be a four-dimensional tensor and the convolution if computed as

$$(K \star I)_{i,j,o} = \sum_{m,n,c} I_{i+m,j+n,c} K_{m,n,c,o}.$$

# Convolution Layer

To arrive at the complete convolution layer, we need to specify:

- the width  $W$  and height  $H$  of the kernel;
- the number of output channels  $F$ ;
- the **stride** denoting that every output pixel is computed for every **stride**-th input pixel (e.g., the output is half the size if stride is 2).

Considering an input image with  $C$  channels, the convolution layer is then parametrized by a kernel  $\mathbf{K}$  of total size  $W \times H \times C \times F$  and is computed as

$$(\mathbf{K} \star \mathbf{I})_{i,j,o} = \sum_{m,n,c} I_{i \cdot S+m, j \cdot S+n, c} \mathbf{K}_{m,n,c,o}.$$

Note that while only local interactions are performed in the image spacial dimensions (width and height), we combine input channels in a fully connected manner.

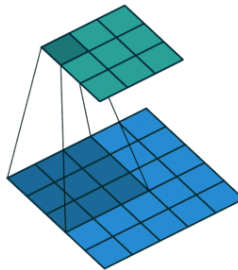
# Convolution Layer

There are multiple padding schemes, most common are:

- `valid`: Only use valid pixels, which causes the result to be smaller than the input.
- `same`: Pad original image with zero pixels so that the result is exactly the size of the input.

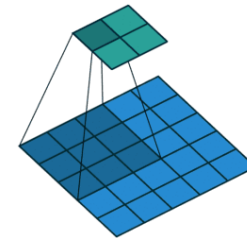
Illustration of the padding schemes and different strides for a  $3 \times 3$  kernel:

- **valid** padding, stride=1:



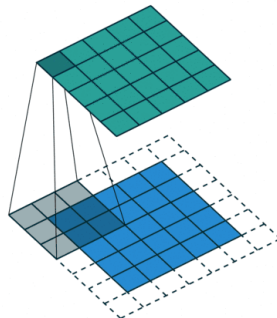
[https://github.com/vdumoulin/conv\\_arithmetic](https://github.com/vdumoulin/conv_arithmetic)

- stride=2:



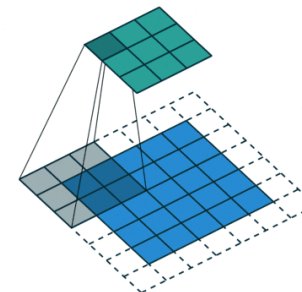
[https://github.com/vdumoulin/conv\\_arithmetic](https://github.com/vdumoulin/conv_arithmetic)

- **same** padding, stride=1:



[https://github.com/vdumoulin/conv\\_arithmetic](https://github.com/vdumoulin/conv_arithmetic)

- stride=2:



[https://github.com/vdumoulin/conv\\_arithmetic](https://github.com/vdumoulin/conv_arithmetic)

There are two prevalent image formats (called `data_format` in TensorFlow):

- `channels_last`: The dimensions of the 4-dimensional image tensor are batch, height, width, and channels.

The original TensorFlow format, faster on CPU.

- `channels_first`: The dimensions of the 4-dimensional image tensor are batch, channel, height, and width.

Usual GPU format (used by CUDA and nearly all frameworks); on TensorFlow, not all CPU kernels are available with this layout.

In TensorFlow, data is represented using the `channels_last` approach and the runtime will automatically convert it to `channels_first` if it is more suitable for available hardware (especially for a GPU).

Pooling is an operation similar to convolution, but we perform a fixed operation instead of multiplying by a kernel.

- Max pooling (minor translation invariance)
- Average pooling

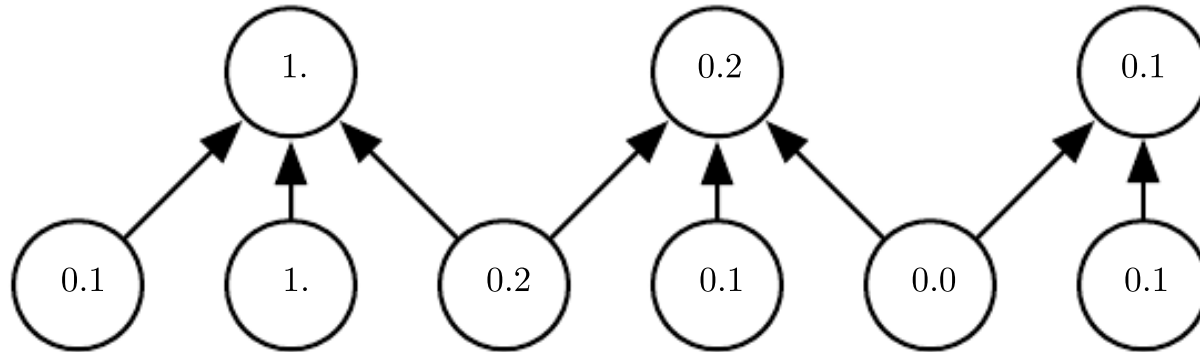
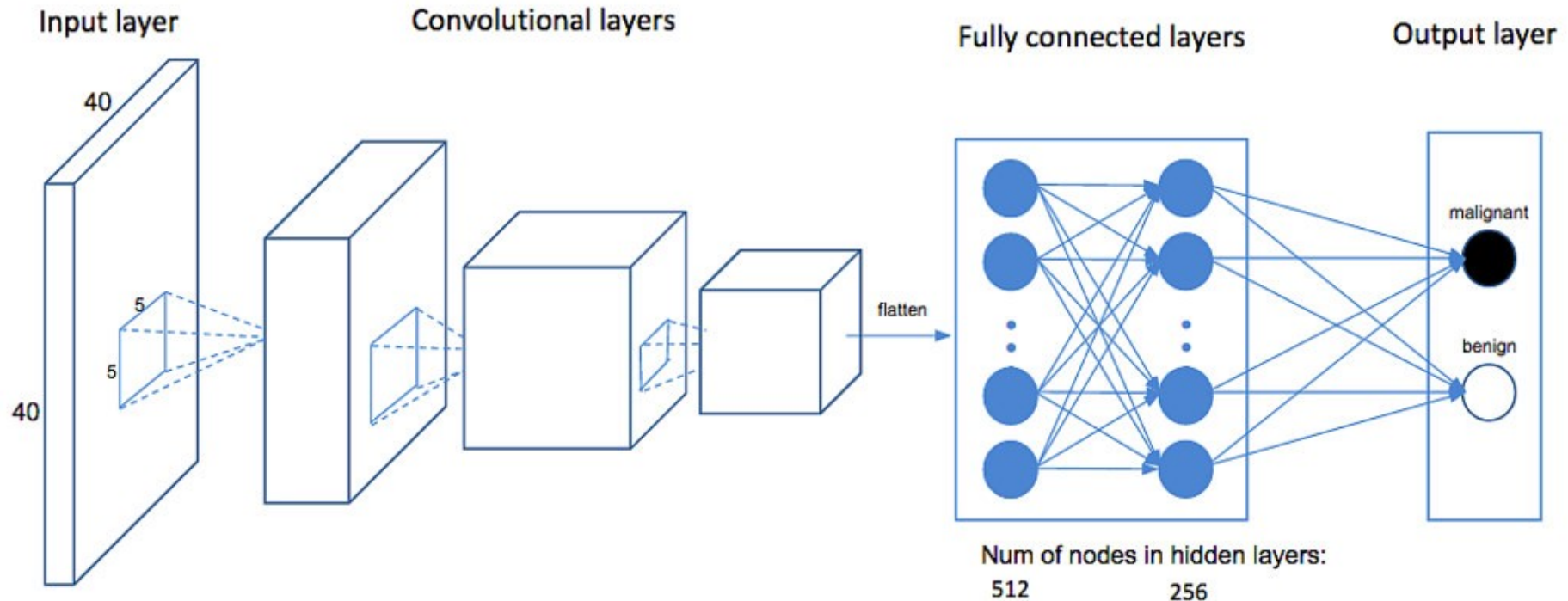


Figure 9.10 of "Deep Learning" book, <https://www.deeplearningbook.org>

# High-level CNN Architecture

We repeatedly use the following block:

1. Convolution operation
2. Non-linear activation (usually ReLU)
3. Pooling



[https://cdn-images-1.medium.com/max/1200/0\\*QyXSpqpm1wc\\_Dt6V](https://cdn-images-1.medium.com/max/1200/0*QyXSpqpm1wc_Dt6V)

# AlexNet – 2012 (16.4% error)

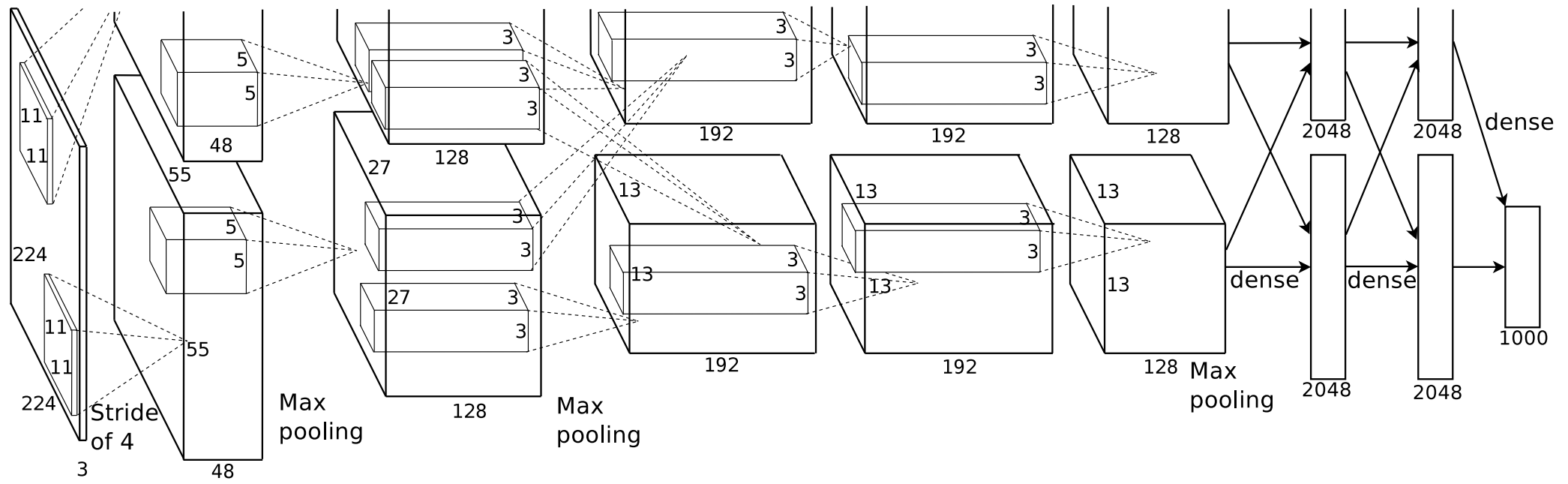


Figure 2: An illustration of the architecture of our CNN, explicitly showing the delineation of responsibilities between the two GPUs. One GPU runs the layer-parts at the top of the figure while the other runs the layer-parts at the bottom. The GPUs communicate only at certain layers. The network’s input is 150,528-dimensional, and the number of neurons in the network’s remaining layers is given by 253,440–186,624–64,896–64,896–43,264–4096–4096–1000.

Figure 2 of "ImageNet Classification with Deep Convolutional Neural Networks" by Alex Krizhevsky et al.



## Training details:

- 2 GPUs for 5-6 days
- SGD with batch size 128, momentum 0.9,  $L^2$  regularization strength (weight decay) 0.0005
  - $\mathbf{v} \leftarrow 0.9 \cdot \mathbf{v} - \alpha \cdot \frac{\partial L}{\partial \theta} - 0.0005 \cdot \alpha \cdot \theta$
  - $\theta \leftarrow \theta + \mathbf{v}$
- initial learning rate 0.01, manually divided by 10 when validation error rate stopped improving
- ReLU nonlinearities
- dropout with rate 0.5 on the fully-connected layers (except for the output layer)
- data augmentation using translations and horizontal reflections (choosing random  $224 \times 224$  patches from  $256 \times 256$  images)
  - during inference, 10 patches are used (four corner patches and a center patch, as well as their reflections)

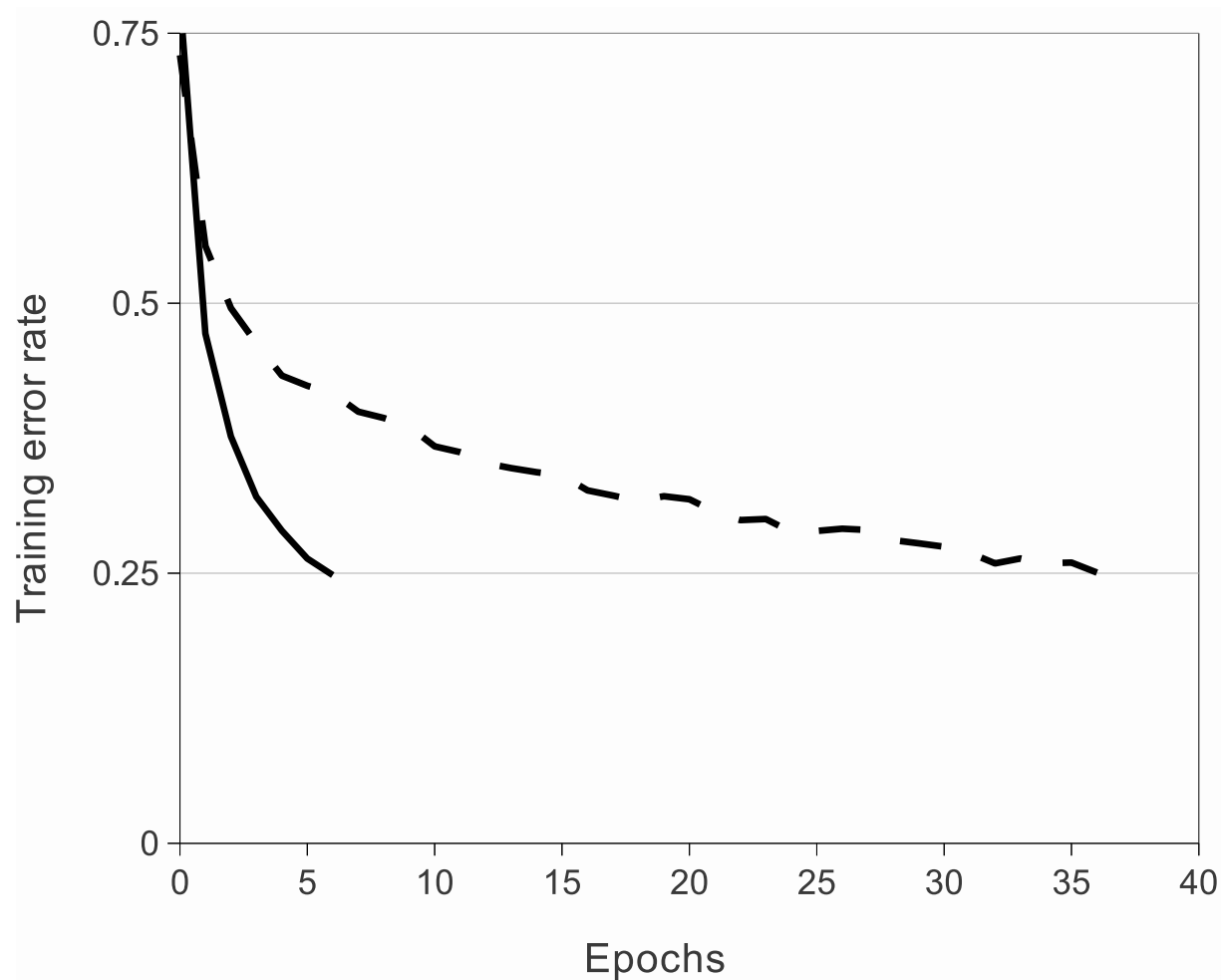


Figure 1 of "ImageNet Classification with Deep Convolutional Neural Networks" by Alex Krizhevsky et al.

AlexNet built on already existing CNN architectures, mostly on LeNet, which achieved 0.8% test error on MNIST.

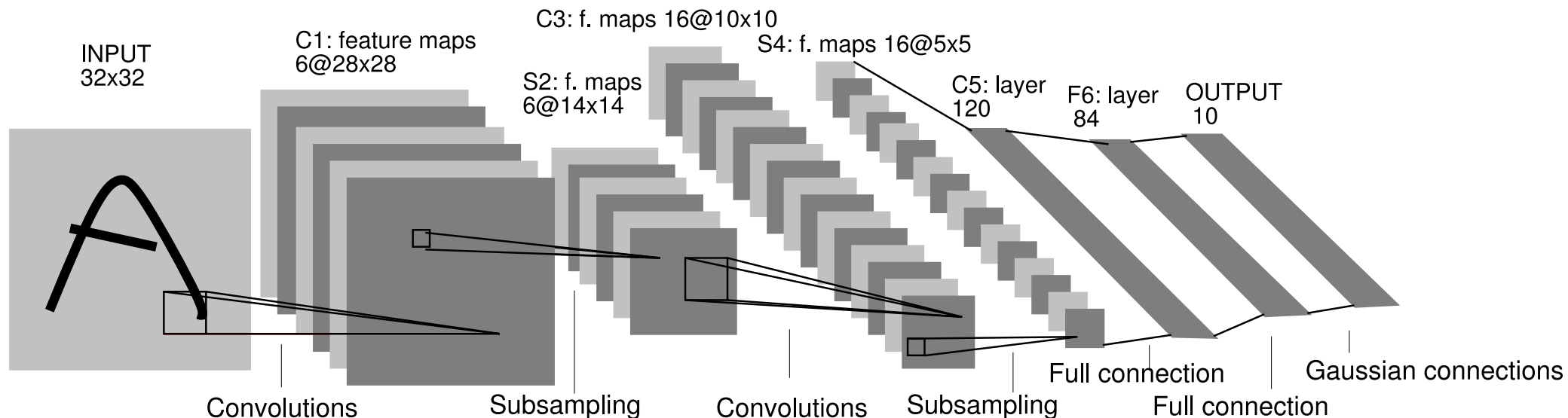


Figure 2 of "Gradient-Based Learning Applied to Document Recognition", <http://yann.lecun.com/exdb/publis/pdf/lecun-01a.pdf>

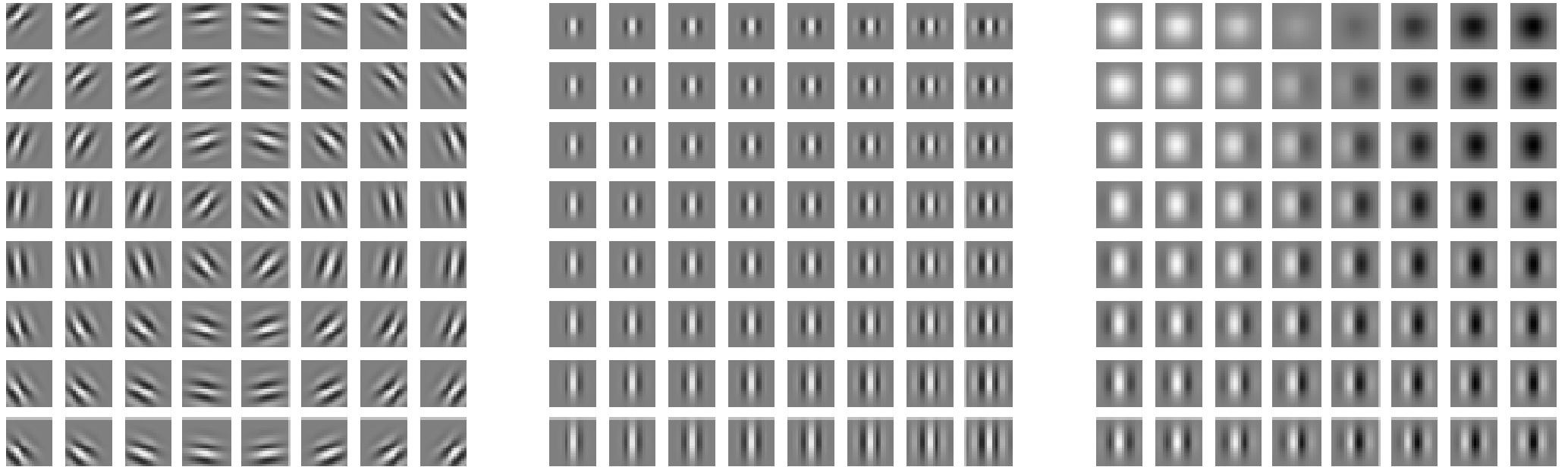


Figure 9.18 of "Deep Learning" book, <https://www.deeplearningbook.org>

The primary visual cortex recognizes Gabor functions.

# Similarities in Primary Visual Cortex (V1) and CNNs

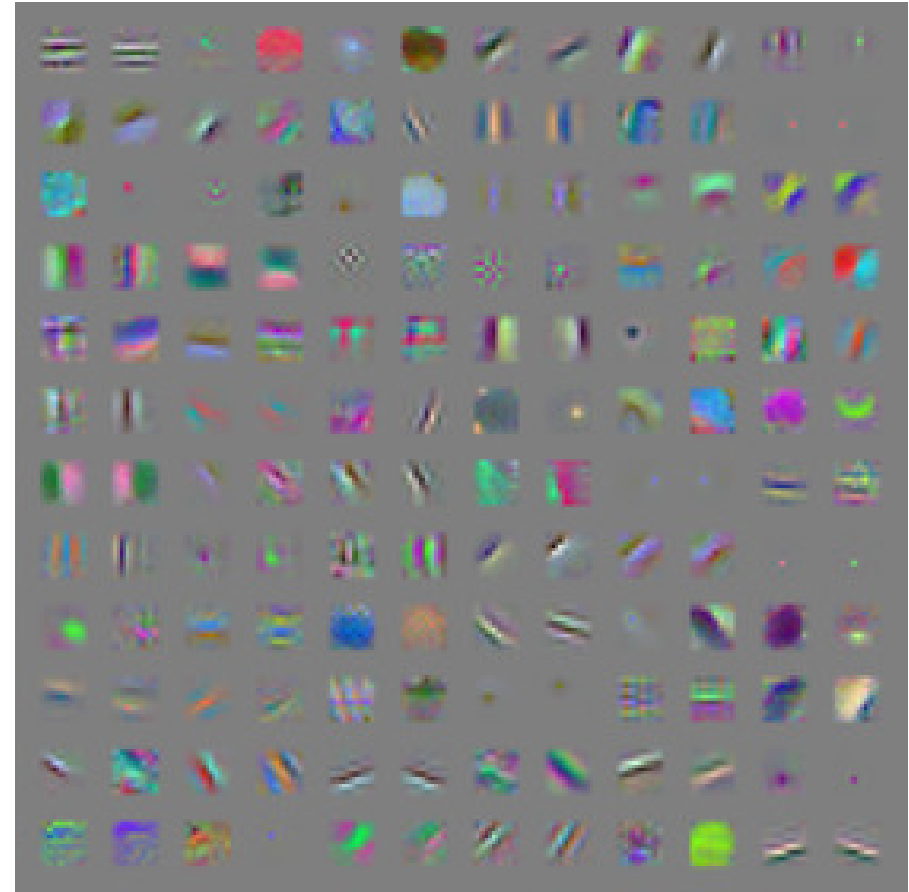
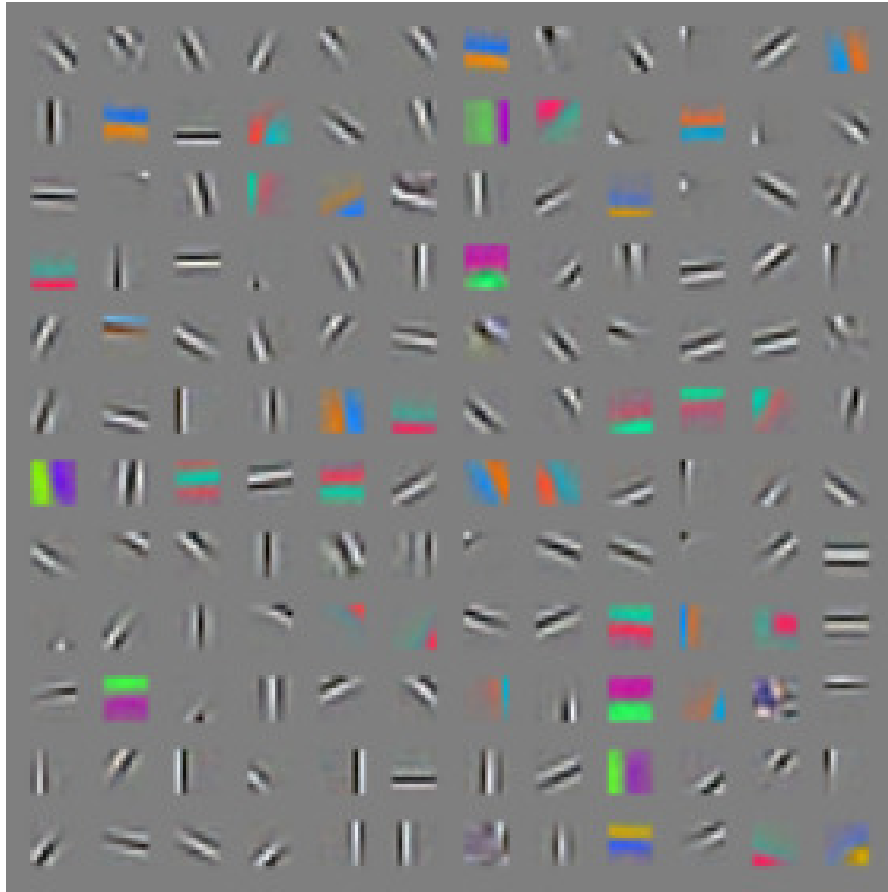
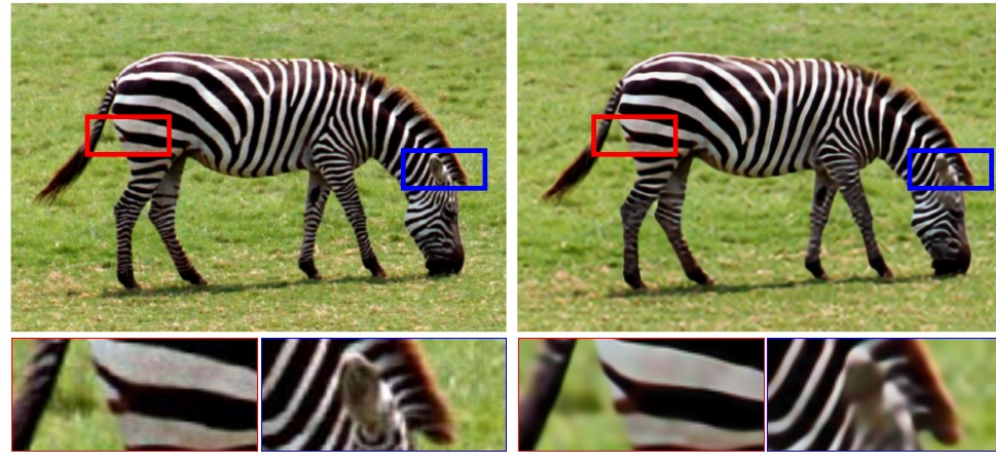


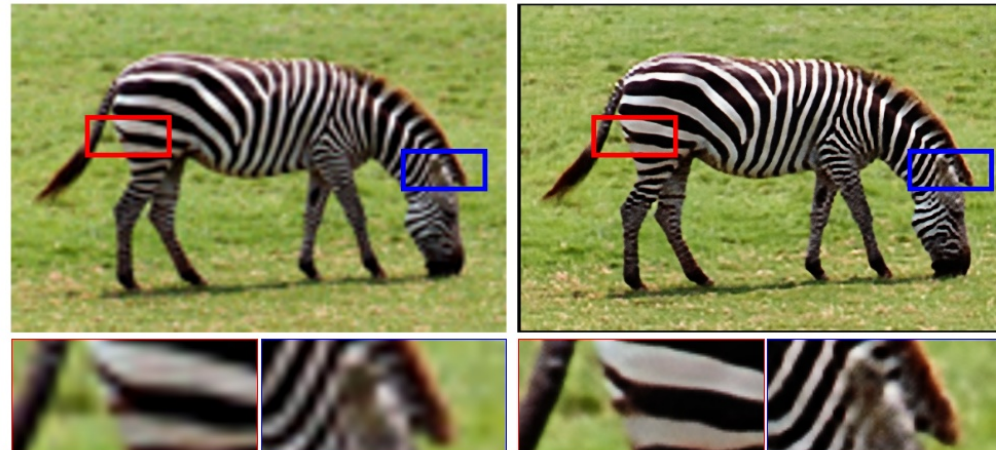
Figure 9.19 of "Deep Learning" book, <https://www.deeplearningbook.org>

Similar functions are recognized in the first layer of a CNN.



(a) Ground truth

(b) SRResNet [18], **Trained**



(c) Bicubic, **Not trained**

(d) Deep prior, **Not trained**

Figure 1 of "Deep Image Prior", <https://arxiv.org/abs/1711.10925>

# CNNs as Regularizers – Deep Prior

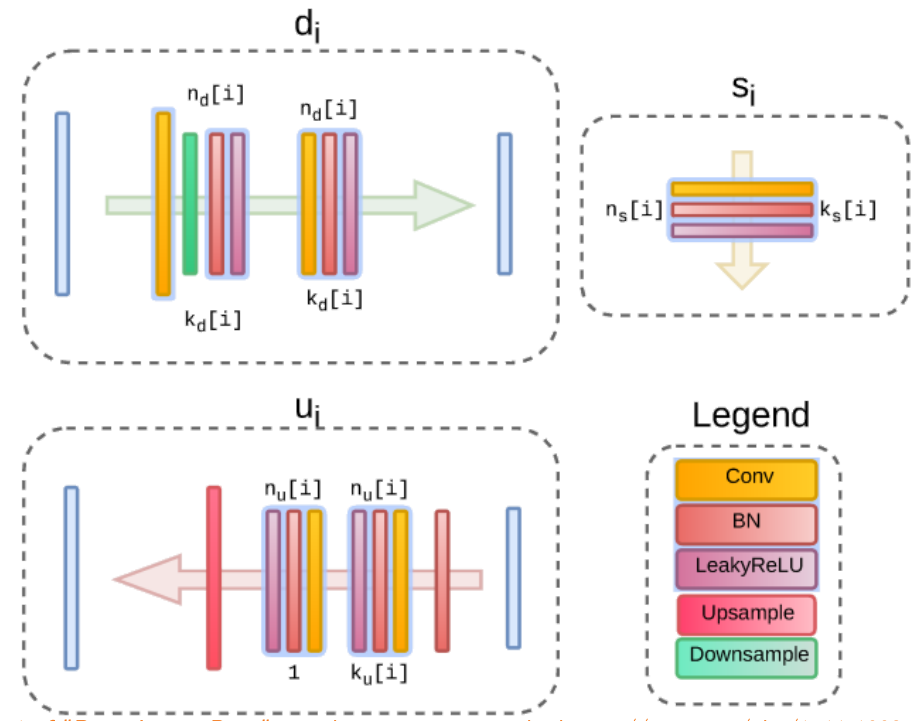
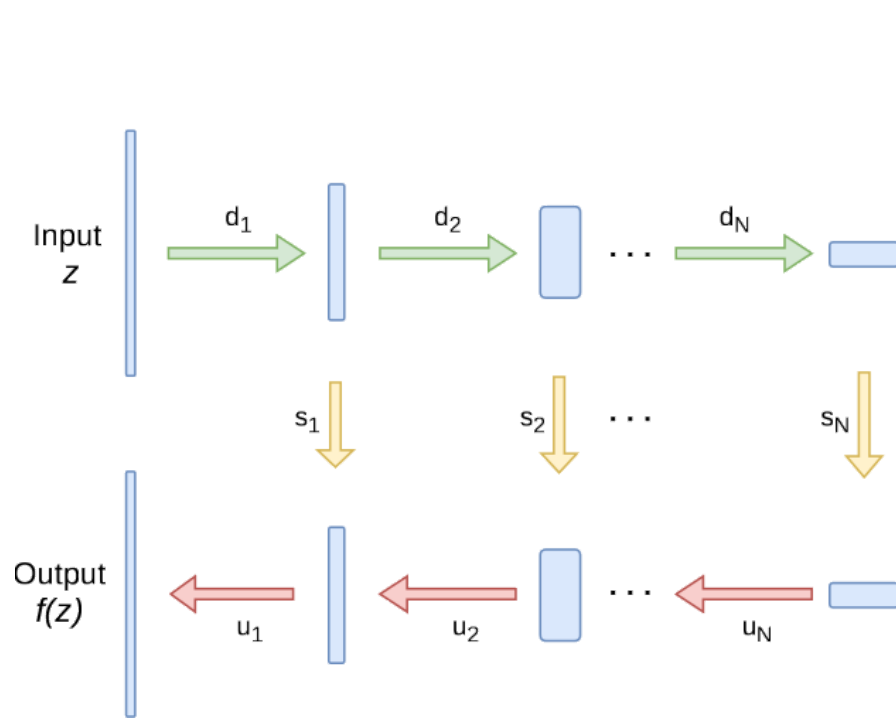


Figure 1 of "Deep Image Prior" supplementary materials, <https://arxiv.org/abs/1711.10925>

Random noise from  $U[0, \frac{1}{10}]$  used on input; in large inpainting, meshgrid is used instead and the skip-connections are not used.



Figure 2 of "Deep Image Prior" supplementary materials, <https://arxiv.org/abs/1711.10925>

# CNNs as Regularizers – Deep Prior

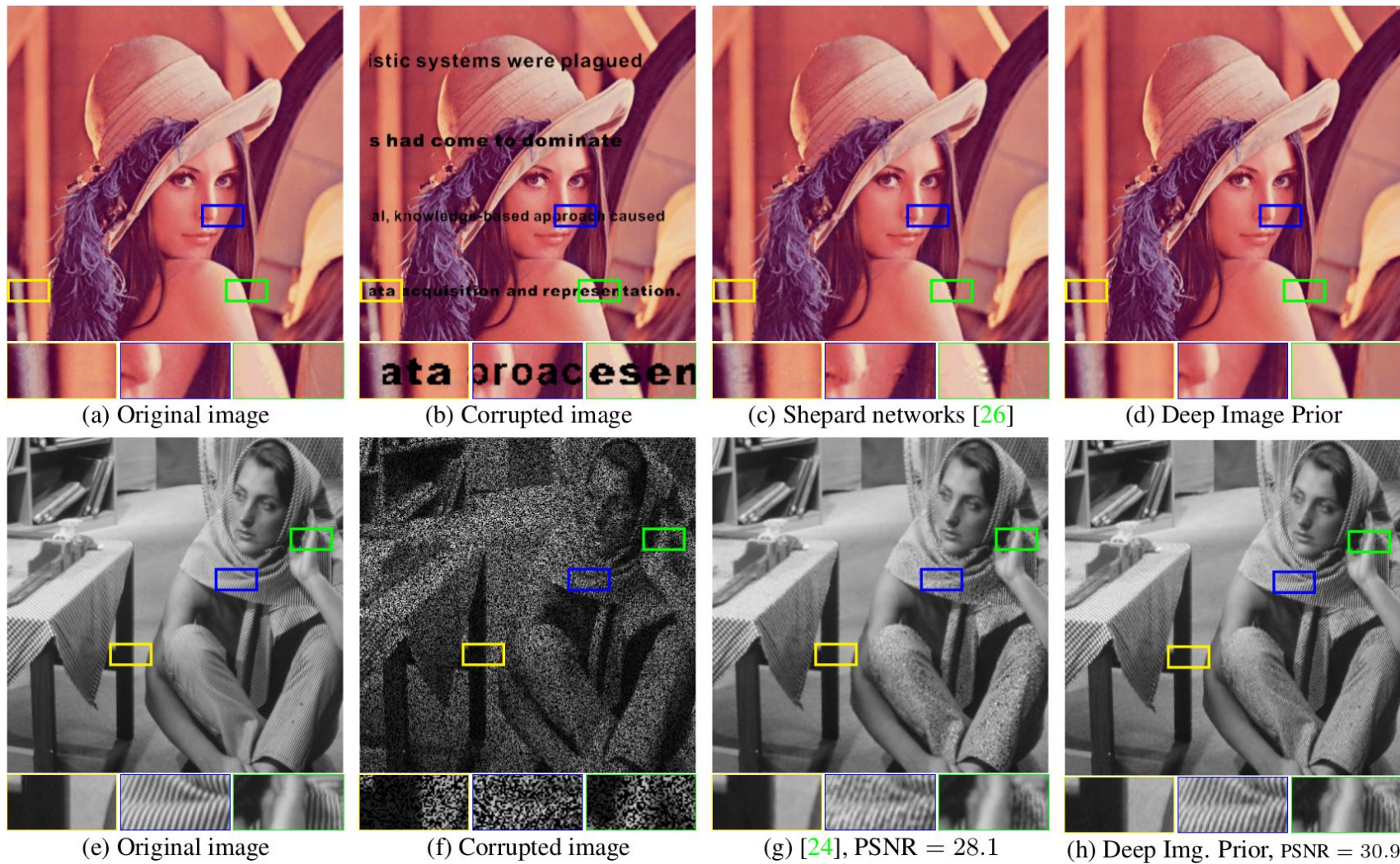


Figure 7 of "Deep Image Prior", <https://arxiv.org/abs/1711.10925v2>





Figure 5: **Inpainting diversity.** Left: original image (black pixels indicate holes). The remaining four images show results obtained using deep prior corresponding to different input vector  $z$ .

*Figure 5 of "Deep Image Prior" supplementary materials, <https://arxiv.org/abs/1711.10925>*

[Deep Prior paper website with supplementary material](#)

| ConvNet Configuration       |                        |                               |  |  |   |
|-----------------------------|------------------------|-------------------------------|--|--|---|
| A                           | A-LRN                  | B                             | C  | D  | E   |
| 11 weight layers            | 11 weight layers       | 13 weight layers              | 16 weight layers                           | 16 weight layers                           | 19 weight layers  |
| input (224 × 224 RGB image) |                        |                               |  |  |   |
| conv3-64                    | conv3-64<br><b>LRN</b> | conv3-64<br><b>conv3-64</b>   | conv3-64<br>conv3-64                       | conv3-64<br>conv3-64                       | conv3-64<br>conv3-64                                    |
| maxpool                     |                        |                               |  |  |   |
| conv3-128                   | conv3-128              | conv3-128<br><b>conv3-128</b> | conv3-128<br>conv3-128                     | conv3-128<br>conv3-128                     | conv3-128<br>conv3-128                                  |
| maxpool                     |                        |                               |  |  |   |
| conv3-256<br>conv3-256      | conv3-256<br>conv3-256 | conv3-256<br>conv3-256        | conv3-256<br>conv3-256<br><b>conv1-256</b> | conv3-256<br>conv3-256<br><b>conv3-256</b> | conv3-256<br>conv3-256<br>conv3-256<br><b>conv3-256</b> |
| maxpool                     |                        |                               |  |  |   |
| conv3-512<br>conv3-512      | conv3-512<br>conv3-512 | conv3-512<br>conv3-512        | conv3-512<br>conv3-512<br><b>conv1-512</b> | conv3-512<br>conv3-512<br><b>conv3-512</b> | conv3-512<br>conv3-512<br>conv3-512<br><b>conv3-512</b> |
| maxpool                     |                        |                               |  |  |   |
| conv3-512<br>conv3-512      | conv3-512<br>conv3-512 | conv3-512<br>conv3-512        | conv3-512<br>conv3-512<br><b>conv1-512</b> | conv3-512<br>conv3-512<br><b>conv3-512</b> | conv3-512<br>conv3-512<br>conv3-512<br><b>conv3-512</b> |
| maxpool                     |                        |                               |  |  |   |
| FC-4096                     |                        |                               |  |  |   |
| FC-4096                     |                        |                               |  |  |   |
| FC-1000                     |                        |                               |  |  |   |
| soft-max                    |                        |                               |  |  |   |

Figure 1 of "Very Deep Convolutional Networks For Large-Scale Image Recognition", <https://arxiv.org/abs/1409.1556>

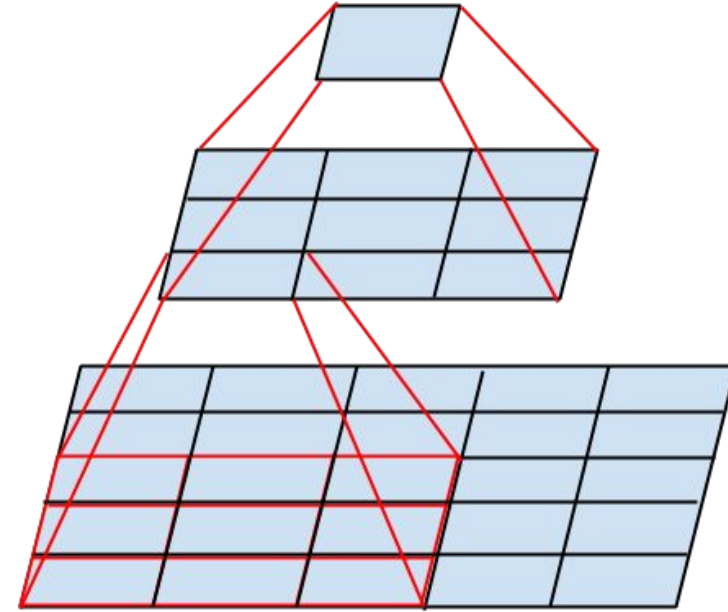


Figure 1 of "Rethinking the Inception Architecture for Computer Vision", <https://arxiv.org/abs/1512.00567>

Table 2: Number of parameters (in millions).

| Network              | A,A-LRN | B   | C   | D   | E   |
|----------------------|---------|-----|-----|-----|-----|
| Number of parameters | 133     | 133 | 134 | 138 | 144 |

Figure 2 of "Very Deep Convolutional Networks For Large-Scale Image Recognition", <https://arxiv.org/abs/1409.1556>

Training detail similar to AlexNet:

- SGD with batch size ~~128~~ 256, momentum 0.9, weight decay 0.0005
- initial learning rate 0.01, manually divided by 10 when validation error rate stopped improving
- ReLU nonlinearities
- dropout with rate 0.5 on the fully-connected layers (except for the output layer)
- data augmentation using translations and horizontal reflections (choosing random  $224 \times 224$  patches from  $256 \times 256$  images)
  - additionally, a multi-scale training and evaluation was performed. During training, each image was resized so that its smaller size was equal to  $S$ , which was sampled uniformly from  $[256, 512]$
  - during test time, the image was rescaled three times so that the smaller size was 256, 384, 512, respectively, and the results on the three images were averaged

Table 3: ConvNet performance at a single test scale.

| ConvNet config. (Table 1) | smallest image side |              | top-1 val. error (%) | top-5 val. error (%) |
|---------------------------|---------------------|--------------|----------------------|----------------------|
|                           | train ( $S$ )       | test ( $Q$ ) |                      |                      |
| A                         | 256                 | 256          | 29.6                 | 10.4                 |
| A-LRN                     | 256                 | 256          | 29.7                 | 10.5                 |
| B                         | 256                 | 256          | 28.7                 | 9.9                  |
| C                         | 256                 | 256          | 28.1                 | 9.4                  |
|                           | 384                 | 384          | 28.1                 | 9.3                  |
|                           | [256;512]           | 384          | 27.3                 | 8.8                  |
| D                         | 256                 | 256          | 27.0                 | 8.8                  |
|                           | 384                 | 384          | 26.8                 | 8.7                  |
|                           | [256;512]           | 384          | 25.6                 | 8.1                  |
| E                         | 256                 | 256          | 27.3                 | 9.0                  |
|                           | 384                 | 384          | 26.9                 | 8.7                  |
|                           | [256;512]           | 384          | <b>25.5</b>          | <b>8.0</b>           |

Table 3 of "Very Deep Convolutional Networks For Large-Scale Image Recognition", <https://arxiv.org/abs/1409.1556>

Table 4: ConvNet performance at multiple test scales.

| ConvNet config. (Table 1) | smallest image side |              | top-1 val. error (%) | top-5 val. error (%) |
|---------------------------|---------------------|--------------|----------------------|----------------------|
|                           | train ( $S$ )       | test ( $Q$ ) |                      |                      |
| B                         | 256                 | 224,256,288  | 28.2                 | 9.6                  |
| C                         | 256                 | 224,256,288  | 27.7                 | 9.2                  |
|                           | 384                 | 352,384,416  | 27.8                 | 9.2                  |
|                           | [256; 512]          | 256,384,512  | 26.3                 | 8.2                  |
| D                         | 256                 | 224,256,288  | 26.6                 | 8.6                  |
|                           | 384                 | 352,384,416  | 26.5                 | 8.6                  |
|                           | [256; 512]          | 256,384,512  | <b>24.8</b>          | <b>7.5</b>           |
| E                         | 256                 | 224,256,288  | 26.9                 | 8.7                  |
|                           | 384                 | 352,384,416  | 26.7                 | 8.6                  |
|                           | [256; 512]          | 256,384,512  | <b>24.8</b>          | <b>7.5</b>           |

Table 4 of "Very Deep Convolutional Networks For Large-Scale Image Recognition", <https://arxiv.org/abs/1409.1556>

| Method   | top-1 val. error (%) | top-5 val. error (%) | top-5 test error (%) |
|--|----------------------|----------------------|----------------------|
| VGG (2 nets, multi-crop & dense eval.)               | <b>23.7</b>          | <b>6.8</b>           | <b>6.8</b>           |
| VGG (1 net, multi-crop & dense eval.)                | 24.4                 | 7.1                  | 7.0                  |
| VGG (ILSVRC submission, 7 nets, dense eval.)         | 24.7                 | 7.5                  | 7.3                  |
| GoogLeNet (Szegedy et al., 2014) (1 net)             | -                    | 7.9                  |                      |
| GoogLeNet (Szegedy et al., 2014) (7 nets)            | -                    | <b>6.7</b>           |                      |
| MSRA (He et al., 2014) (11 nets)                     | -                    | -                    | 8.1                  |
| MSRA (He et al., 2014) (1 net)                       | 27.9                 | 9.1                  | 9.1                  |
| Clarifai (Russakovsky et al., 2014) (multiple nets)  | -                    | -                    | 11.7                 |
| Clarifai (Russakovsky et al., 2014) (1 net)          | -                    | -                    | 12.5                 |
| Zeiler & Fergus (Zeiler & Fergus, 2013) (6 nets)     | 36.0                 | 14.7                 | 14.8                 |
| Zeiler & Fergus (Zeiler & Fergus, 2013) (1 net)      | 37.5                 | 16.0                 | 16.1                 |
| OverFeat (Sermanet et al., 2014) (7 nets)            | 34.0                 | 13.2                 | 13.6                 |
| OverFeat (Sermanet et al., 2014) (1 net)             | 35.7                 | 14.2                 | -                    |
| Krizhevsky et al. (Krizhevsky et al., 2012) (5 nets) | 38.1                 | 16.4                 | 16.4                 |
| Krizhevsky et al. (Krizhevsky et al., 2012) (1 net)  | 40.7                 | 18.2                 | -                    |

Figure 2 of "Very Deep Convolutional Networks For Large-Scale Image Recognition", <https://arxiv.org/abs/1409.1556>

Inception block:

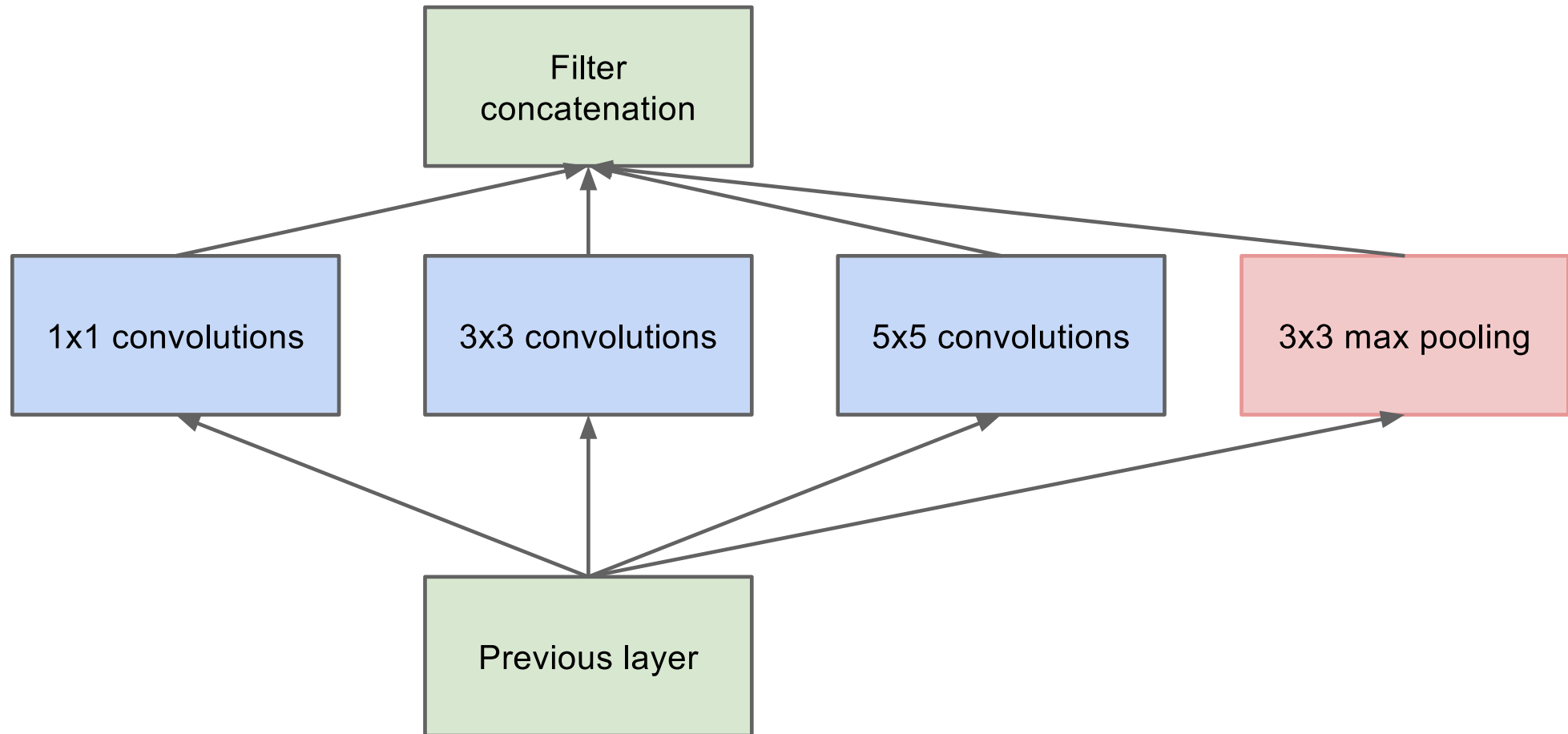


Figure 2 of "Going Deeper with Convolutions", <https://arxiv.org/abs/1409.4842>

Inception block with dimensionality reduction:

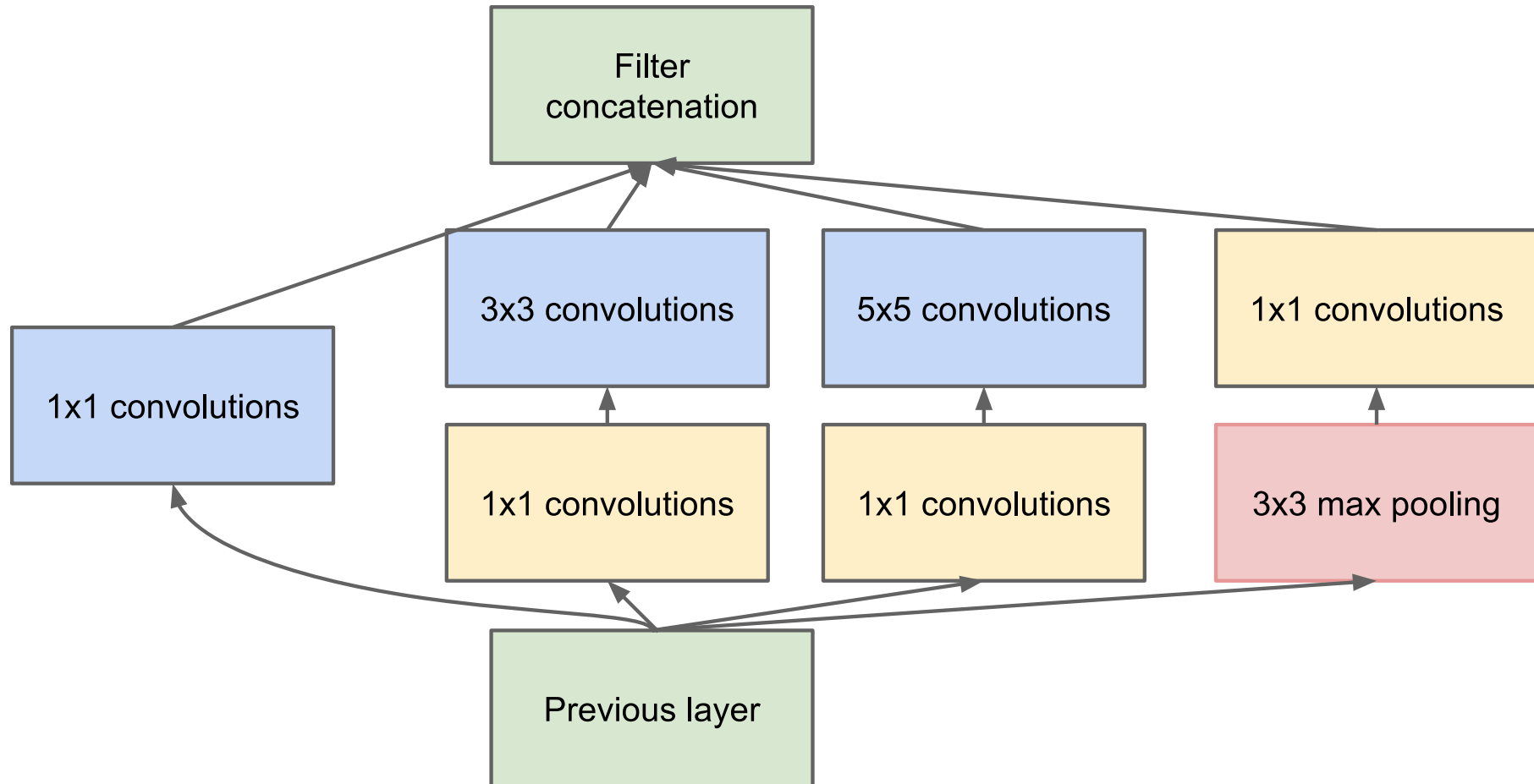


Figure 2 of "Going Deeper with Convolutions", <https://arxiv.org/abs/1409.4842>

# Inception (GoogLeNet) – 2014 (6.7% error)

| type           | patch size/<br>stride | output<br>size | depth | #1×1 | #3×3<br>reduce | #3×3 | #5×5<br>reduce | #5×5 | pool<br>proj | params | ops  |
|----------------|-----------------------|----------------|-------|------|----------------|------|----------------|------|--------------|--------|------|
| convolution    | 7×7/2                 | 112×112×64     | 1     |      |                |      |                |      |              | 2.7K   | 34M  |
| max pool       | 3×3/2                 | 56×56×64       | 0     |      |                |      |                |      |              |        |      |
| convolution    | 3×3/1                 | 56×56×192      | 2     |      | 64             | 192  |                |      |              | 112K   | 360M |
| max pool       | 3×3/2                 | 28×28×192      | 0     |      |                |      |                |      |              |        |      |
| inception (3a) |                       | 28×28×256      | 2     | 64   | 96             | 128  | 16             | 32   | 32           | 159K   | 128M |
| inception (3b) |                       | 28×28×480      | 2     | 128  | 128            | 192  | 32             | 96   | 64           | 380K   | 304M |
| max pool       | 3×3/2                 | 14×14×480      | 0     |      |                |      |                |      |              |        |      |
| inception (4a) |                       | 14×14×512      | 2     | 192  | 96             | 208  | 16             | 48   | 64           | 364K   | 73M  |
| inception (4b) |                       | 14×14×512      | 2     | 160  | 112            | 224  | 24             | 64   | 64           | 437K   | 88M  |
| inception (4c) |                       | 14×14×512      | 2     | 128  | 128            | 256  | 24             | 64   | 64           | 463K   | 100M |
| inception (4d) |                       | 14×14×528      | 2     | 112  | 144            | 288  | 32             | 64   | 64           | 580K   | 119M |
| inception (4e) |                       | 14×14×832      | 2     | 256  | 160            | 320  | 32             | 128  | 128          | 840K   | 170M |
| max pool       | 3×3/2                 | 7×7×832        | 0     |      |                |      |                |      |              |        |      |
| inception (5a) |                       | 7×7×832        | 2     | 256  | 160            | 320  | 32             | 128  | 128          | 1072K  | 54M  |
| inception (5b) |                       | 7×7×1024       | 2     | 384  | 192            | 384  | 48             | 128  | 128          | 1388K  | 71M  |
| avg pool       | 7×7/1                 | 1×1×1024       | 0     |      |                |      |                |      |              |        |      |
| dropout (40%)  |                       | 1×1×1024       | 0     |      |                |      |                |      |              |        |      |
| linear         |                       | 1×1×1000       | 1     |      |                |      |                |      |              | 1000K  | 1M   |
| softmax        |                       | 1×1×1000       | 0     |      |                |      |                |      |              |        |      |

Table 1 of "Going Deeper with Convolutions", <https://arxiv.org/abs/1409.4842>



# Inception (GoogLeNet) – 2014 (6.7% error)

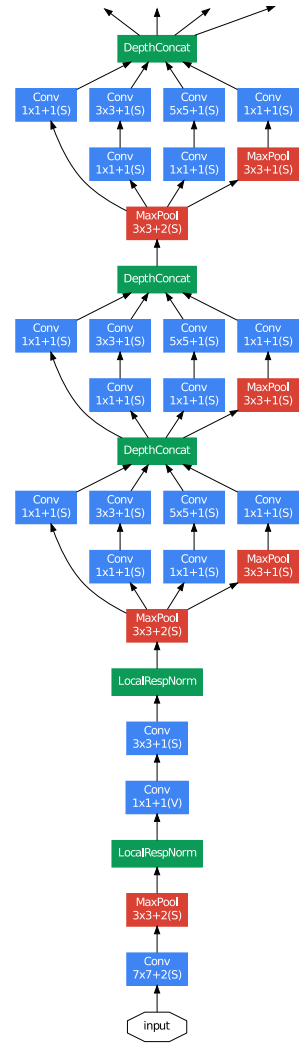


Figure 3 of "Going Deeper with Convolutions", <https://arxiv.org/abs/1409.4842>

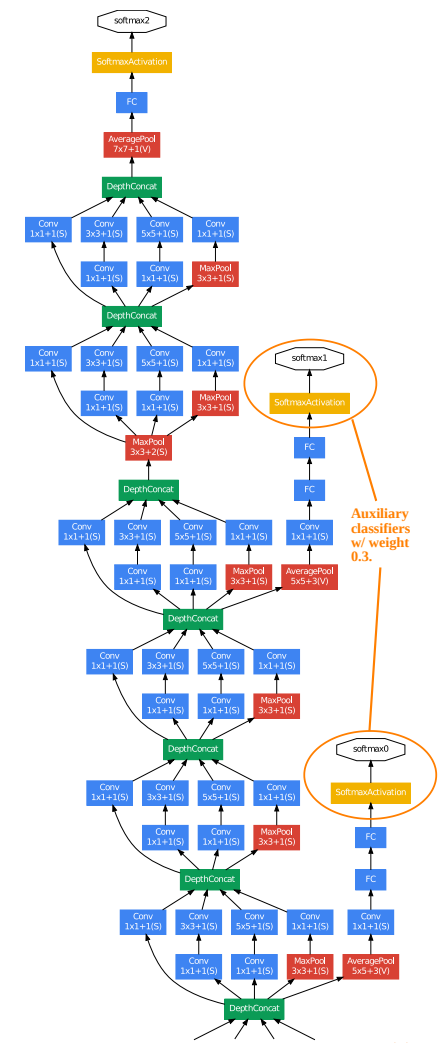


Figure 3 of "Going Deeper with Convolutions", <https://arxiv.org/abs/1409.4842>

Training details:

- SGD with momentum 0.9
- fixed learning rate schedule of decreasing the learning rate by 4% each 8 epochs
- during test time, the image was rescaled four times so that the smaller size was 256, 288, 320, 352, respectively.

For each image, the left, center and right square was considered, and from each square six crops of size  $224 \times 224$  were extracted (4 corners, middle crop and the whole scaled-down square) together with their horizontal flips, arriving at  $4 \cdot 3 \cdot 6 \cdot 2 = 144$  crops per image

- 7 independently trained models were ensembled

# Inception (GoogLeNet) – 2014 (6.7% error)

| <b>Number of models</b> | <b>Number of Crops</b> | <b>Cost</b> | <b>Top-5 error</b> | <b>compared to base</b> |
|-------------------------|------------------------|-------------|--------------------|-------------------------|
| 1                       | 1                      | 1           | 10.07%             | base                    |
| 1                       | 10                     | 10          | 9.15%              | -0.92%                  |
| 1                       | 144                    | 144         | 7.89%              | -2.18%                  |
| 7                       | 1                      | 7           | 8.09%              | -1.98%                  |
| 7                       | 10                     | 70          | 7.62%              | -2.45%                  |
| 7                       | 144                    | 1008        | 6.67%              | -3.45%                  |

Table 3 of "Going Deeper with Convolutions", <https://arxiv.org/abs/1409.4842>

# Batch Normalization

**Internal covariate shift** refers to the change in the distributions of hidden node activations due to the updates of network parameters during training.

Let  $\mathbf{x} = (x_1, \dots, x_d)$  be  $d$ -dimensional input. We would like to normalize each dimension as

$$\hat{x}_i = \frac{x_i - \mathbb{E}[x_i]}{\sqrt{\text{Var}[x_i]}}.$$

Furthermore, it may be advantageous to learn suitable scale  $\gamma_i$  and shift  $\beta_i$  to produce normalized value

$$y_i = \gamma_i \hat{x}_i + \beta_i.$$

# Batch Normalization

**Batch normalization** of a mini-batch of  $m$  examples  $(\mathbf{x}^{(1)}, \dots, \mathbf{x}^{(m)})$  is the following:

**Inputs:** Mini-batch  $(\mathbf{x}^{(1)}, \dots, \mathbf{x}^{(m)})$ ,  $\varepsilon \in \mathbb{R}$  with default value 0.001

**Parameters:**  $\beta$  initialized to  $\mathbf{0}$ ,  $\gamma$  initialized to  $\mathbf{1}$ ; both trained by the optimizer

**Outputs:** Normalized batch  $(\mathbf{y}^{(1)}, \dots, \mathbf{y}^{(m)})$

- $\boldsymbol{\mu} \leftarrow \frac{1}{m} \sum_{i=1}^m \mathbf{x}^{(i)}$
- $\boldsymbol{\sigma}^2 \leftarrow \frac{1}{m} \sum_{i=1}^m (\mathbf{x}^{(i)} - \boldsymbol{\mu})^2$
- $\hat{\mathbf{x}}^{(i)} \leftarrow (\mathbf{x}^{(i)} - \boldsymbol{\mu}) / \sqrt{\boldsymbol{\sigma}^2 + \varepsilon}$
- $\mathbf{y}^{(i)} \leftarrow \gamma \hat{\mathbf{x}}^{(i)} + \beta$

Batch normalization is added just before a nonlinearity  $f$ , and it is useless to add bias before it (because it will cancel out). Therefore, we replace  $f(\mathbf{W}\mathbf{x} + \mathbf{b})$  by

$$f(\text{BN}(\mathbf{W}\mathbf{x})).$$

# Batch Normalization during Inference

During inference,  $\mu$  and  $\sigma^2$  are fixed (so that prediction does not depend on other examples in a batch).

They could be precomputed after training on the whole training data, but in practice we estimate  $\hat{\mu}$  and  $\hat{\sigma}^2$  during training using an exponential moving average.

**Additional Inputs:** momentum  $\tau \in \mathbb{R}$  with default value of 0.99

**Additional Parameters:**  $\hat{\mu}$  initialized to  $\mathbf{0}$ ,  $\hat{\sigma}^2$  initialized to  $\mathbf{1}$ ; both updated manually

During training, also perform:

- $\hat{\mu} \leftarrow \tau \hat{\mu} + (1 - \tau) \mu$
- $\hat{\sigma}^2 \leftarrow \tau \hat{\sigma}^2 + (1 - \tau) \sigma^2$

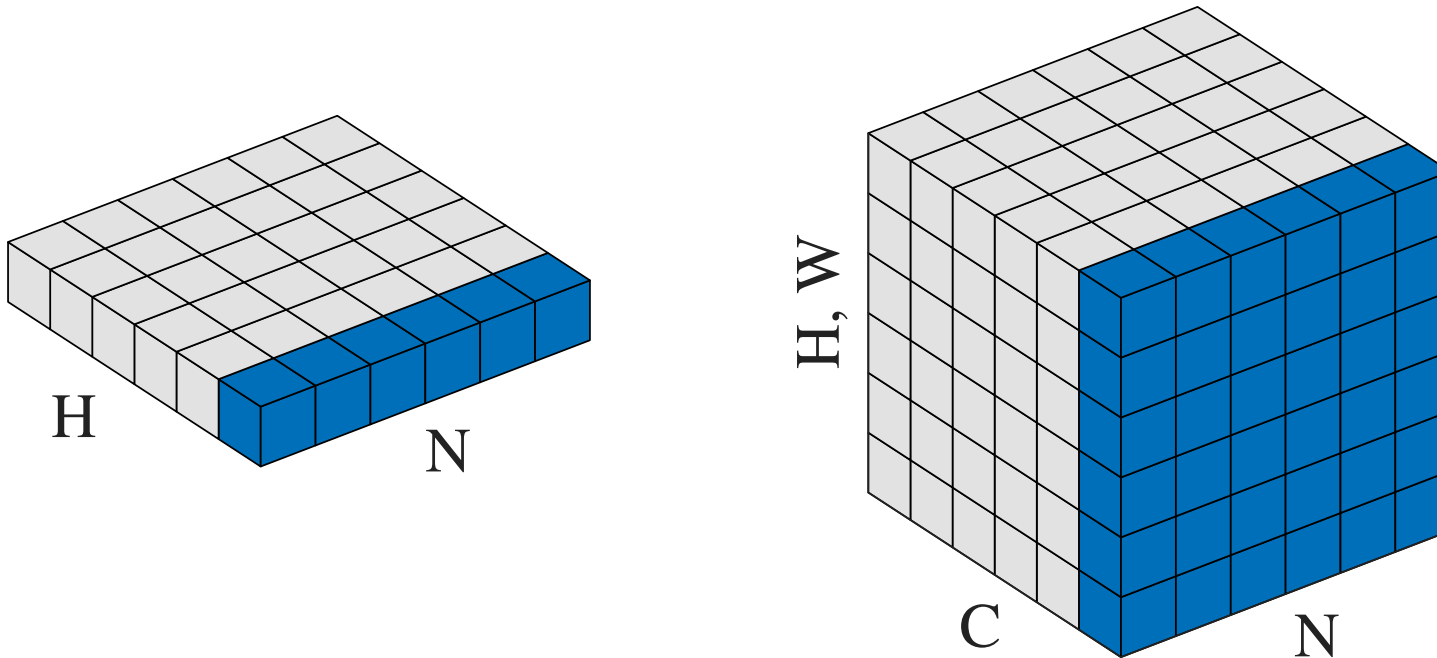
Batch normalization is then during inference computed as:

- $\hat{\mathbf{x}}^{(i)} \leftarrow (\mathbf{x}^{(i)} - \hat{\mu}) / \sqrt{\hat{\sigma}^2 + \epsilon}$
- $\mathbf{y}^{(i)} \leftarrow \gamma \hat{\mathbf{x}}^{(i)} + \beta$

# Batch Normalization

When a batch normalization is used on a fully connected layer, each neuron is normalized individually across the minibatch.

However, for convolutional networks we would like the normalization to honour their properties, most notably the shift invariance. We therefore normalize each channel across not only the minibatch, but also across all corresponding spacial/temporal locations.



Adapted from Figure 2 of "Group Normalization", <https://arxiv.org/abs/1803.08494>

# Inception with BatchNorm (4.8% error)

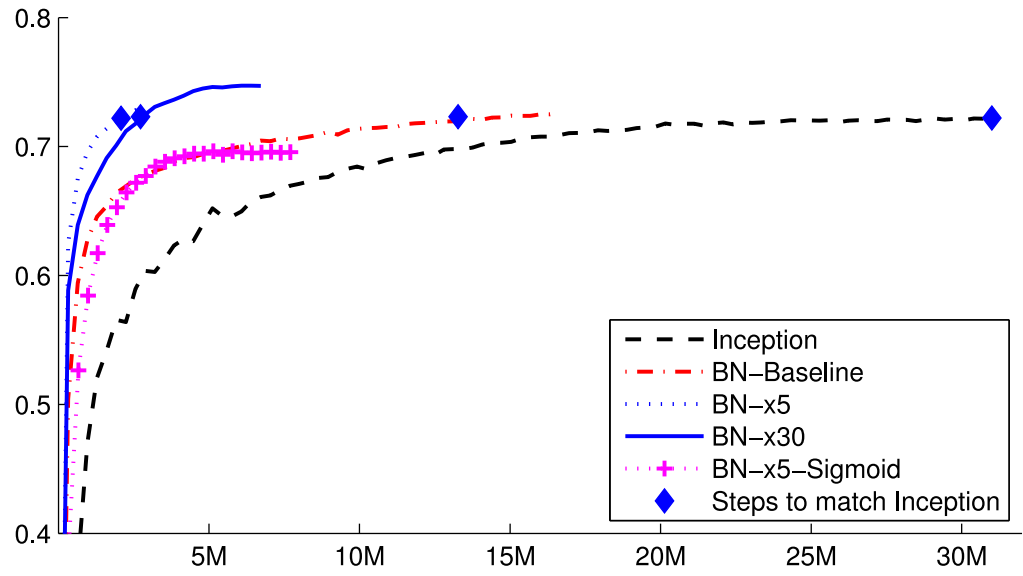


Figure 2: Single crop validation accuracy of Inception and its batch-normalized variants, vs. the number of training steps.

| Model         | Steps to 72.2%    | Max accuracy |
|---------------|-------------------|--------------|
| Inception     | $31.0 \cdot 10^6$ | 72.2%        |
| BN-Baseline   | $13.3 \cdot 10^6$ | 72.7%        |
| BN-x5         | $2.1 \cdot 10^6$  | 73.0%        |
| BN-x30        | $2.7 \cdot 10^6$  | 74.8%        |
| BN-x5-Sigmoid |                   | 69.8%        |

Figure 3: For Inception and the batch-normalized variants, the number of training steps required to reach the maximum accuracy of Inception (72.2%), and the maximum accuracy achieved by the network.

Figures 2 and 3 of "Batch Normalization: Accelerating Deep Network Training by Reducing Internal Covariate Shift", <https://arxiv.org/abs/1502.03167>

The BN-x5 and BN-x30 use 5/30 times larger initial learning rate, faster learning rate decay, no dropout, weight decay smaller by a factor of 5, and several more minor changes.



# Inception v2 and v3 – 2015 (3.6% error)

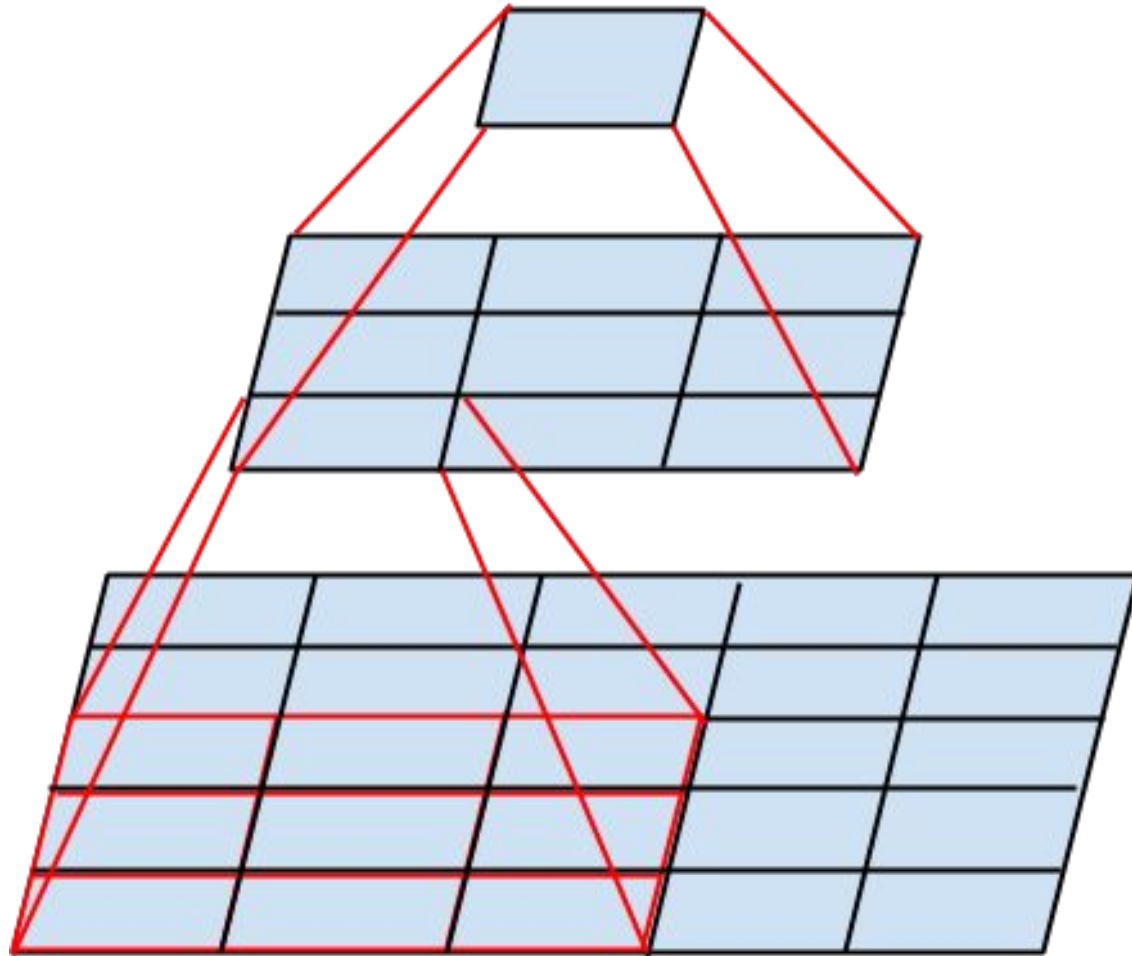


Figure 1 of "Rethinking the Inception Architecture for Computer Vision", <https://arxiv.org/abs/1512.00567>

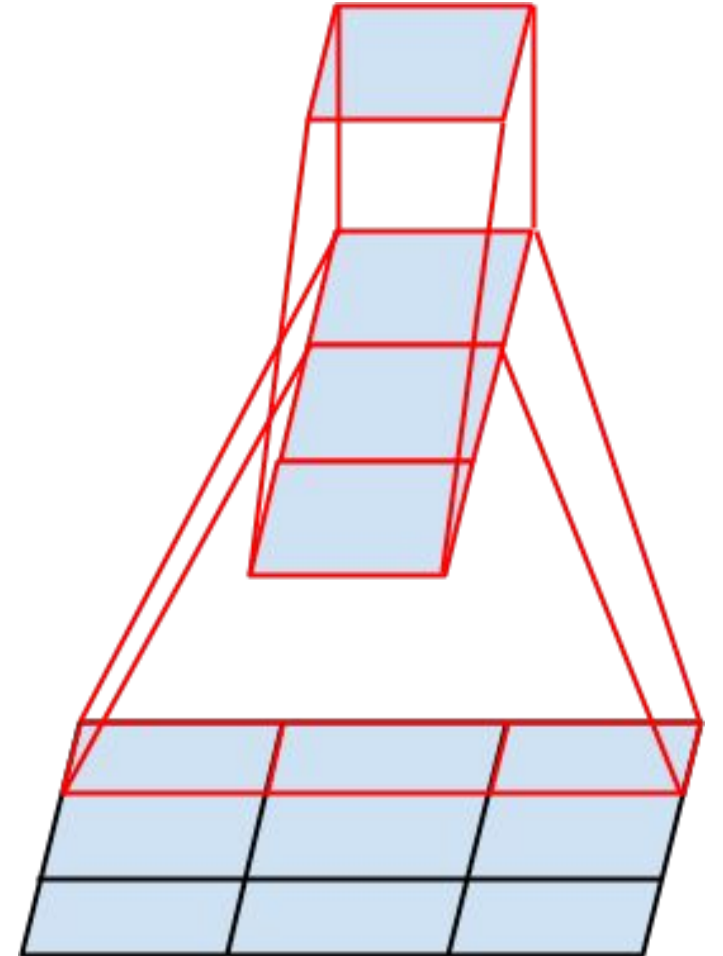


Figure 3 of "Rethinking the Inception Architecture for Computer Vision", <https://arxiv.org/abs/1512.00567>

# Inception v2 and v3 – 2015 (3.6% error)

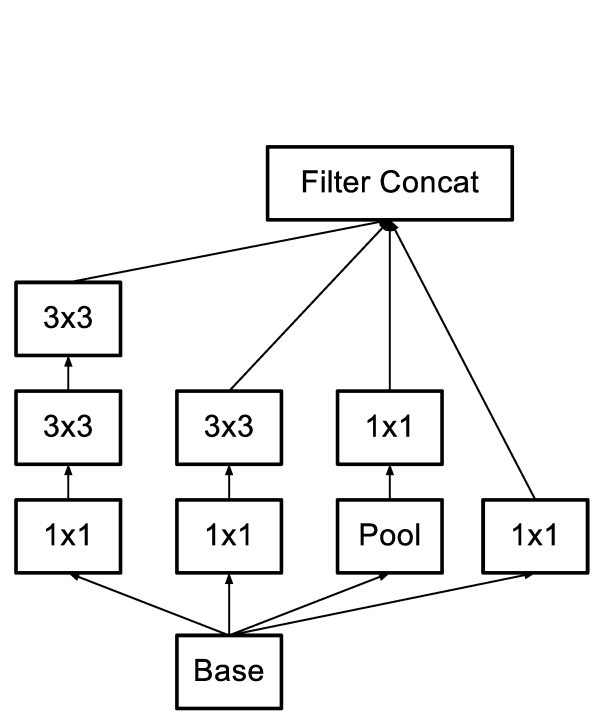


Figure 5. Inception modules where each  $5 \times 5$  convolution is replaced by two  $3 \times 3$  convolution, as suggested by principle 3 of Section 2.

Figure 5 of "Rethinking the Inception Architecture for Computer Vision", <https://arxiv.org/abs/1512.00567>

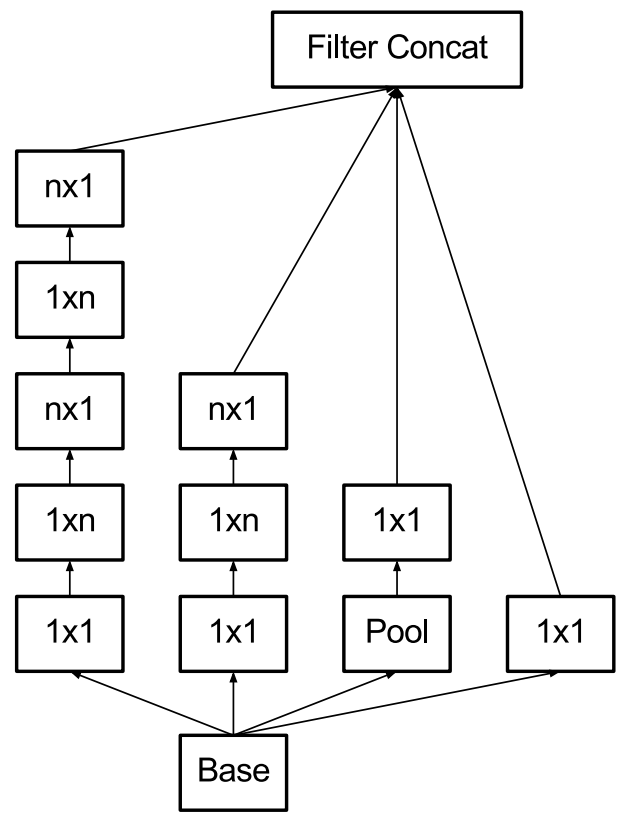


Figure 6. Inception modules after the factorization of the  $n \times n$  convolutions. In our proposed architecture, we chose  $n = 7$  for the  $17 \times 17$  grid. (The filter sizes are picked using principle 3)

Figure 6 of "Rethinking the Inception Architecture for Computer Vision", <https://arxiv.org/abs/1512.00567>

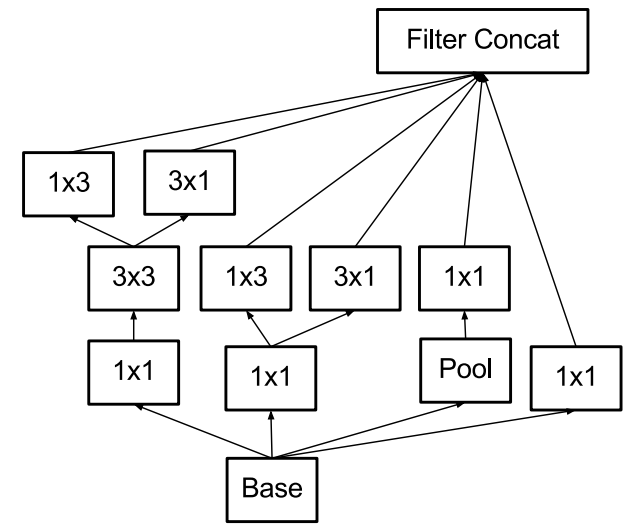


Figure 7. Inception modules with expanded the filter bank outputs. This architecture is used on the coarsest ( $8 \times 8$ ) grids to promote high dimensional representations, as suggested by principle 2 of Section 2. We are using this solution only on the coarsest grid, since that is the place where producing high dimensional sparse representation is the most critical as the ratio of local processing (by  $1 \times 1$  convolutions) is increased compared to the spatial aggregation.

Figure 7 of "Rethinking the Inception Architecture for Computer Vision", <https://arxiv.org/abs/1512.00567>

## Inception v2 and v3 – 2015 (3.6% error)

| type                 | patch size/stride<br>or remarks | input size                 |
|----------------------|---------------------------------|----------------------------|
| conv                 | $3 \times 3 / 2$                | $299 \times 299 \times 3$  |
| conv                 | $3 \times 3 / 1$                | $149 \times 149 \times 32$ |
| conv padded          | $3 \times 3 / 1$                | $147 \times 147 \times 32$ |
| pool                 | $3 \times 3 / 2$                | $147 \times 147 \times 64$ |
| conv                 | $3 \times 3 / 1$                | $73 \times 73 \times 64$   |
| conv                 | $3 \times 3 / 2$                | $71 \times 71 \times 80$   |
| conv                 | $3 \times 3 / 1$                | $35 \times 35 \times 192$  |
| $3 \times$ Inception | As in figure 5                  | $35 \times 35 \times 288$  |
| $5 \times$ Inception | As in figure 6                  | $17 \times 17 \times 768$  |
| $2 \times$ Inception | As in figure 7                  | $8 \times 8 \times 1280$   |
| pool                 | $8 \times 8$                    | $8 \times 8 \times 2048$   |
| linear               | logits                          | $1 \times 1 \times 2048$   |
| softmax              | classifier                      | $1 \times 1 \times 1000$   |

Table 1 of "Rethinking the Inception Architecture for Computer Vision", <https://arxiv.org/abs/1512.00567>

# Inception v2 and v3 – 2015 (3.6% error)

Training details:

- RMSProp with momentum of  $\beta = 0.9$  and  $\varepsilon = 1.0$
- batch size of 32 for 100 epochs
- initial learning rate of 0.045, decayed by 6% every two epochs
- gradient clipping with threshold 2.0 was used to stabilize the training
- label smoothing was first used in this paper, with  $\alpha = 0.1$
- input image size enlarged to  $299 \times 299$

## Inception v2 and v3 – 2015 (3.6% error)

| Network                                 | Top-1 Error  | Top-5 Error | Cost Bn Ops |
|---|--------------|-------------|-------------|
| GoogLeNet [20]                          | 29%          | 9.2%        | 1.5         |
| BN-GoogLeNet                            | 26.8%        | -           | <b>1.5</b>  |
| BN-Inception [7]                        | 25.2%        | 7.8         | 2.0         |
| Inception-v2                            | 23.4%        | -           | 3.8         |
| Inception-v2<br>RMSProp                 | 23.1%        | 6.3         | 3.8         |
| Inception-v2<br>Label Smoothing         | 22.8%        | 6.1         | 3.8         |
| Inception-v2<br>Factorized $7 \times 7$ | 21.6%        | 5.8         | 4.8         |
| Inception-v2<br>BN-auxiliary            | <b>21.2%</b> | <b>5.6%</b> | 4.8         |

Table 3 of "Rethinking the Inception Architecture for Computer Vision", <https://arxiv.org/abs/1512.00567>

# Inception v2 and v3 – 2015 (3.6% error)

| Network          | Crops Evaluated | Top-5 Error   | Top-1 Error |
|------------------|-----------------|---------------|-------------|
| GoogLeNet [20]   | 10              | -             | 9.15%       |
| GoogLeNet [20]   | 144             | -             | 7.89%       |
| VGG [18]         | -               | 24.4%         | 6.8%        |
| BN-Inception [7] | 144             | 22%           | 5.82%       |
| PReLU [6]        | 10              | 24.27%        | 7.38%       |
| PReLU [6]        | -               | 21.59%        | 5.71%       |
| Inception-v3     | 12              | 19.47%        | 4.48%       |
| Inception-v3     | 144             | <b>18.77%</b> | <b>4.2%</b> |

Table 4 of "Rethinking the Inception Architecture for Computer Vision", <https://arxiv.org/abs/1512.00567>

| Network          | Models Evaluated | Crops Evaluated | Top-1 Error  | Top-5 Error   |
|------------------|------------------|-----------------|--------------|---------------|
| VGGNet [18]      | 2                | -               | 23.7%        | 6.8%          |
| GoogLeNet [20]   | 7                | 144             | -            | 6.67%         |
| PReLU [6]        | -                | -               | -            | 4.94%         |
| BN-Inception [7] | 6                | 144             | 20.1%        | 4.9%          |
| Inception-v3     | 4                | 144             | <b>17.2%</b> | <b>3.58%*</b> |

Table 5 of "Rethinking the Inception Architecture for Computer Vision", <https://arxiv.org/abs/1512.00567>

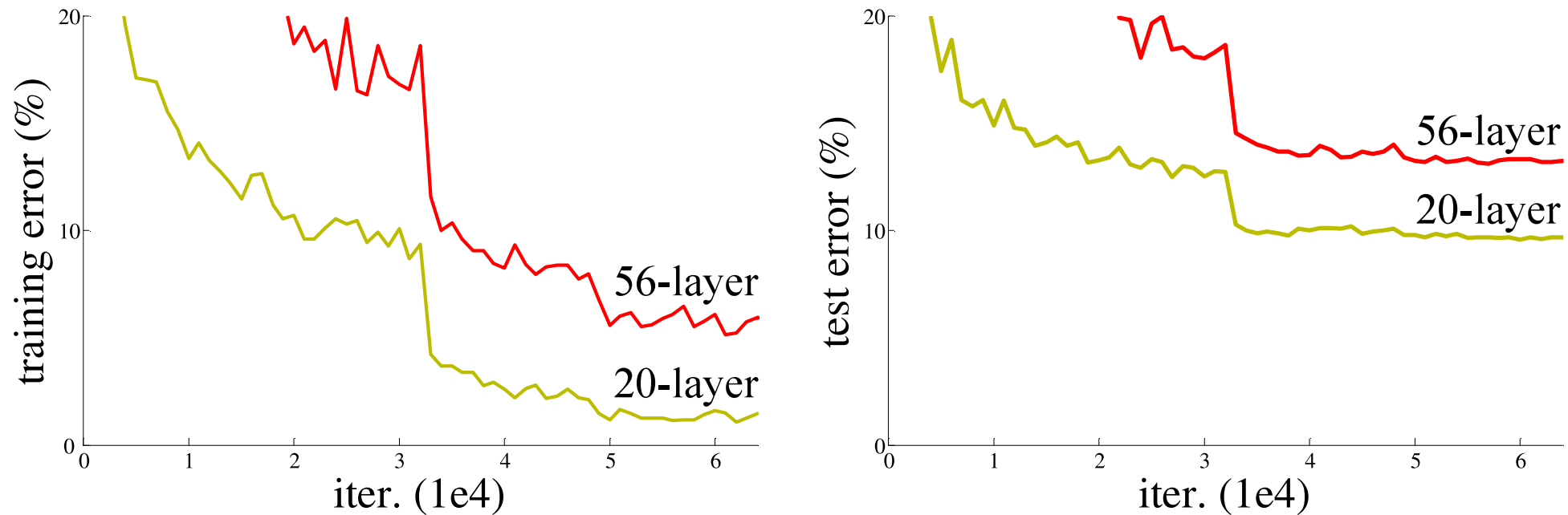


Figure 1. Training error (left) and test error (right) on CIFAR-10 with 20-layer and 56-layer “plain” networks. The deeper network has higher training error, and thus test error. Similar phenomena on ImageNet is presented in Fig. 4.

Figure 1 of "Deep Residual Learning for Image Recognition", <https://arxiv.org/abs/1512.03385>

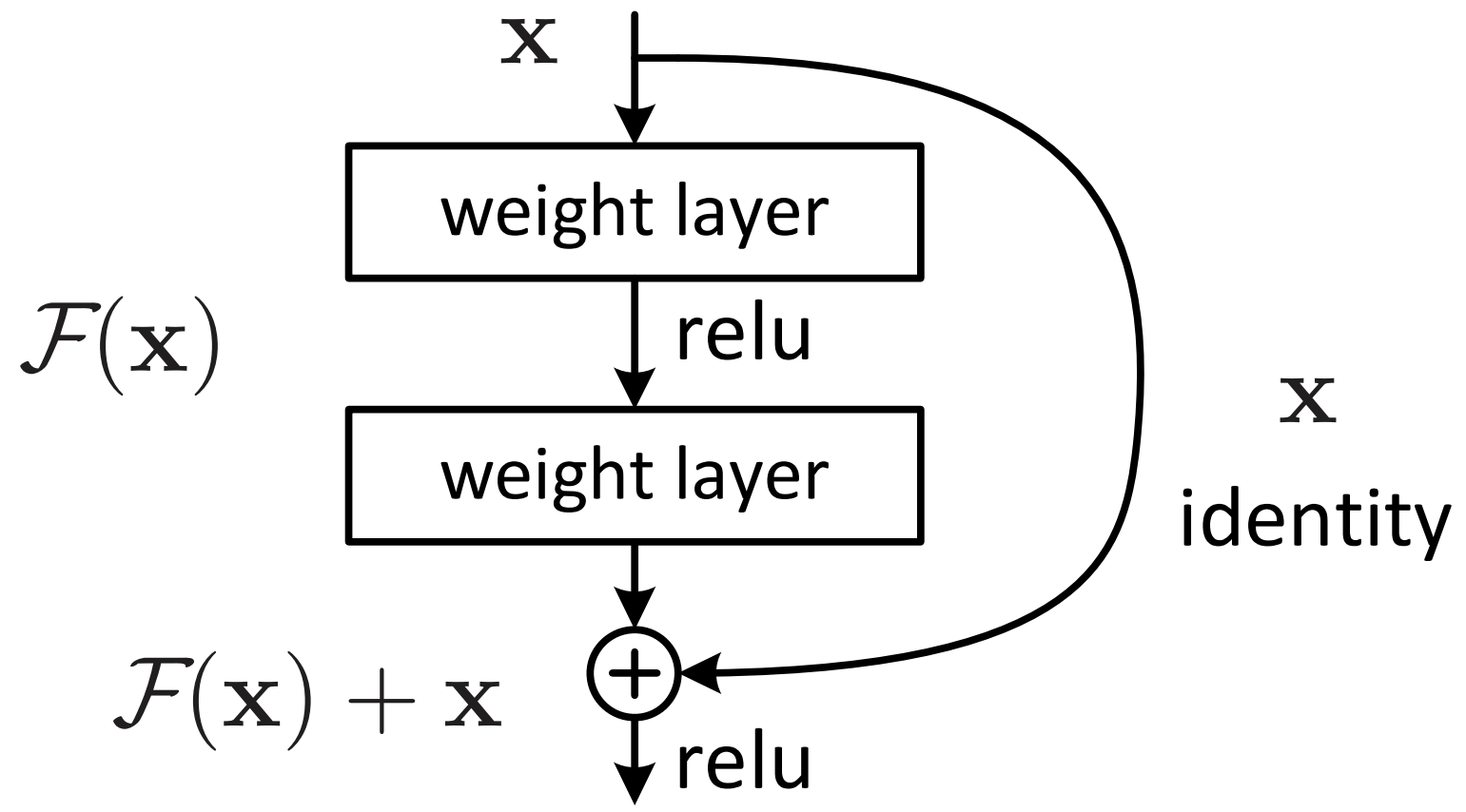


Figure 2. Residual learning: a building block.

Figure 2 of "Deep Residual Learning for Image Recognition", <https://arxiv.org/abs/1512.03385>



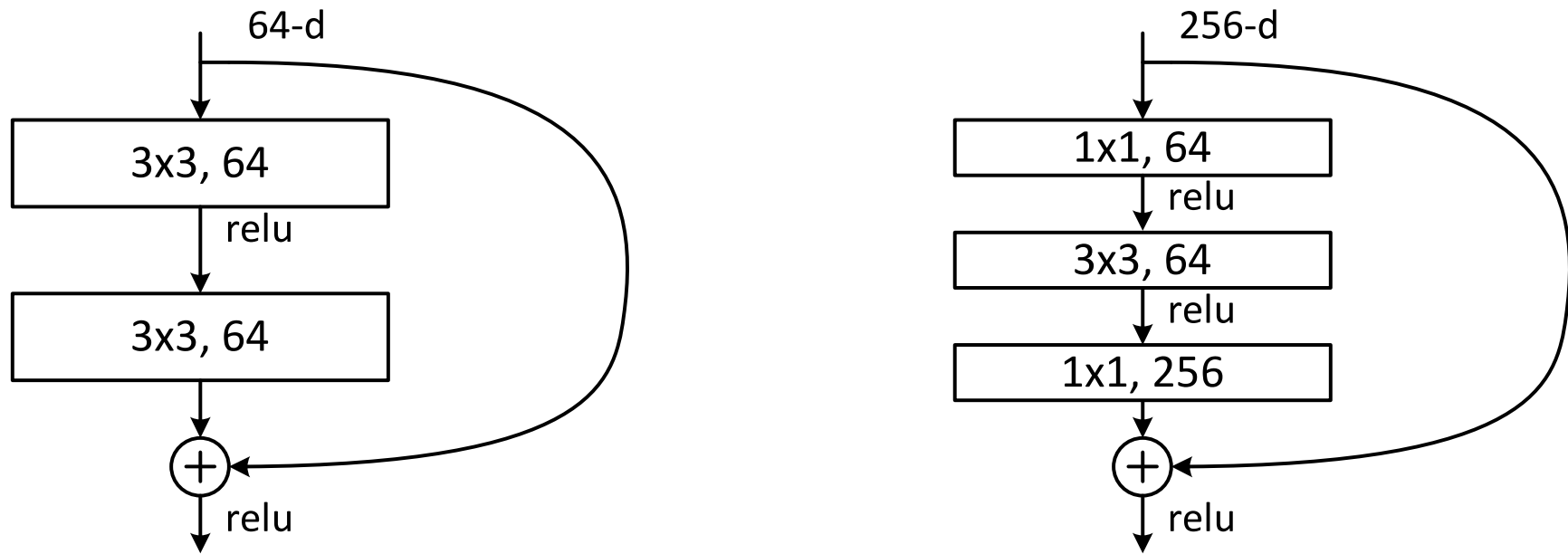


Figure 5. A deeper residual function  $\mathcal{F}$  for ImageNet. Left: a building block (on  $56 \times 56$  feature maps) as in Fig. 3 for ResNet-34. Right: a “bottleneck” building block for ResNet-50/101/152.

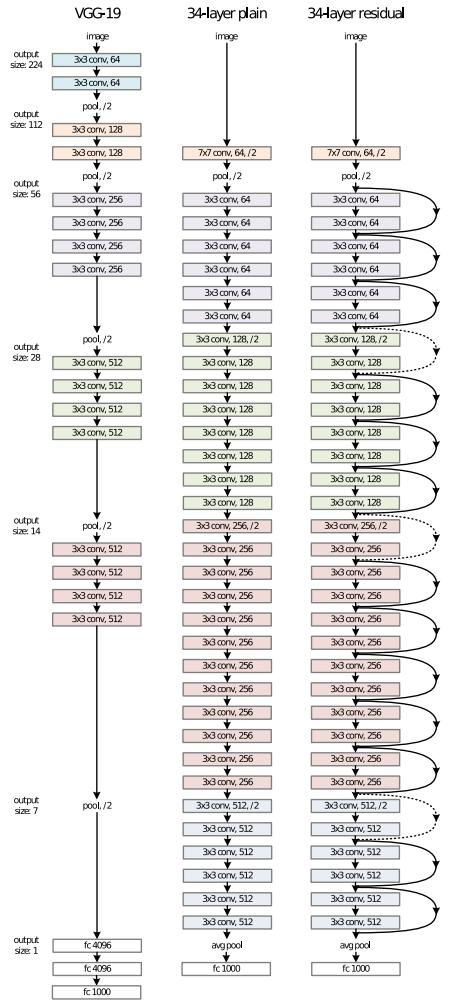
Figure 5 of “Deep Residual Learning for Image Recognition”, <https://arxiv.org/abs/1512.03385>

# ResNet – 2015 (3.6% error)

| layer name | output size | 18-layer  | 34-layer  | 50-layer  | 101-layer  | 152-layer  |
|------------|-------------|---|---|---|--|--|
| conv1      | 112×112     | 7×7, 64, stride 2   |   |   |  |  |
| conv2_x    | 56×56       | 3×3 max pool, stride 2  |   |   |  |  |
|            |             | $\begin{bmatrix} 3 \times 3, 64 \\ 3 \times 3, 64 \end{bmatrix} \times 2$   | $\begin{bmatrix} 3 \times 3, 64 \\ 3 \times 3, 64 \end{bmatrix} \times 3$   | $\begin{bmatrix} 1 \times 1, 64 \\ 3 \times 3, 64 \\ 1 \times 1, 256 \end{bmatrix} \times 3$    | $\begin{bmatrix} 1 \times 1, 64 \\ 3 \times 3, 64 \\ 1 \times 1, 256 \end{bmatrix} \times 3$     | $\begin{bmatrix} 1 \times 1, 64 \\ 3 \times 3, 64 \\ 1 \times 1, 256 \end{bmatrix} \times 3$     |
| conv3_x    | 28×28       | $\begin{bmatrix} 3 \times 3, 128 \\ 3 \times 3, 128 \end{bmatrix} \times 2$ | $\begin{bmatrix} 3 \times 3, 128 \\ 3 \times 3, 128 \end{bmatrix} \times 4$ | $\begin{bmatrix} 1 \times 1, 128 \\ 3 \times 3, 128 \\ 1 \times 1, 512 \end{bmatrix} \times 4$  | $\begin{bmatrix} 1 \times 1, 128 \\ 3 \times 3, 128 \\ 1 \times 1, 512 \end{bmatrix} \times 4$   | $\begin{bmatrix} 1 \times 1, 128 \\ 3 \times 3, 128 \\ 1 \times 1, 512 \end{bmatrix} \times 8$   |
| conv4_x    | 14×14       | $\begin{bmatrix} 3 \times 3, 256 \\ 3 \times 3, 256 \end{bmatrix} \times 2$ | $\begin{bmatrix} 3 \times 3, 256 \\ 3 \times 3, 256 \end{bmatrix} \times 6$ | $\begin{bmatrix} 1 \times 1, 256 \\ 3 \times 3, 256 \\ 1 \times 1, 1024 \end{bmatrix} \times 6$ | $\begin{bmatrix} 1 \times 1, 256 \\ 3 \times 3, 256 \\ 1 \times 1, 1024 \end{bmatrix} \times 23$ | $\begin{bmatrix} 1 \times 1, 256 \\ 3 \times 3, 256 \\ 1 \times 1, 1024 \end{bmatrix} \times 36$ |
| conv5_x    | 7×7         | $\begin{bmatrix} 3 \times 3, 512 \\ 3 \times 3, 512 \end{bmatrix} \times 2$ | $\begin{bmatrix} 3 \times 3, 512 \\ 3 \times 3, 512 \end{bmatrix} \times 3$ | $\begin{bmatrix} 1 \times 1, 512 \\ 3 \times 3, 512 \\ 1 \times 1, 2048 \end{bmatrix} \times 3$ | $\begin{bmatrix} 1 \times 1, 512 \\ 3 \times 3, 512 \\ 1 \times 1, 2048 \end{bmatrix} \times 3$  | $\begin{bmatrix} 1 \times 1, 512 \\ 3 \times 3, 512 \\ 1 \times 1, 2048 \end{bmatrix} \times 3$  |
|            | 1×1         | average pool, 1000-d fc, softmax  |   |   |  |  |
| FLOPs      |             | $1.8 \times 10^9$   | $3.6 \times 10^9$   | $3.8 \times 10^9$   | $7.6 \times 10^9$  | $11.3 \times 10^9$   |

Table 1 of "Deep Residual Learning for Image Recognition", <https://arxiv.org/abs/1512.03385>

# ResNet – 2015 (3.6% error)



The residual connections cannot be applied directly when number of channels increases.

The authors considered several alternatives, and chose the one where in case of channels increase a  $1 \times 1$  convolution + BN is used on the projections to match the required number of channels. The required spacial resolution is achieved by using stride 2.

Figure 3 of "Deep Residual Learning for Image Recognition", <https://arxiv.org/abs/1512.03385>

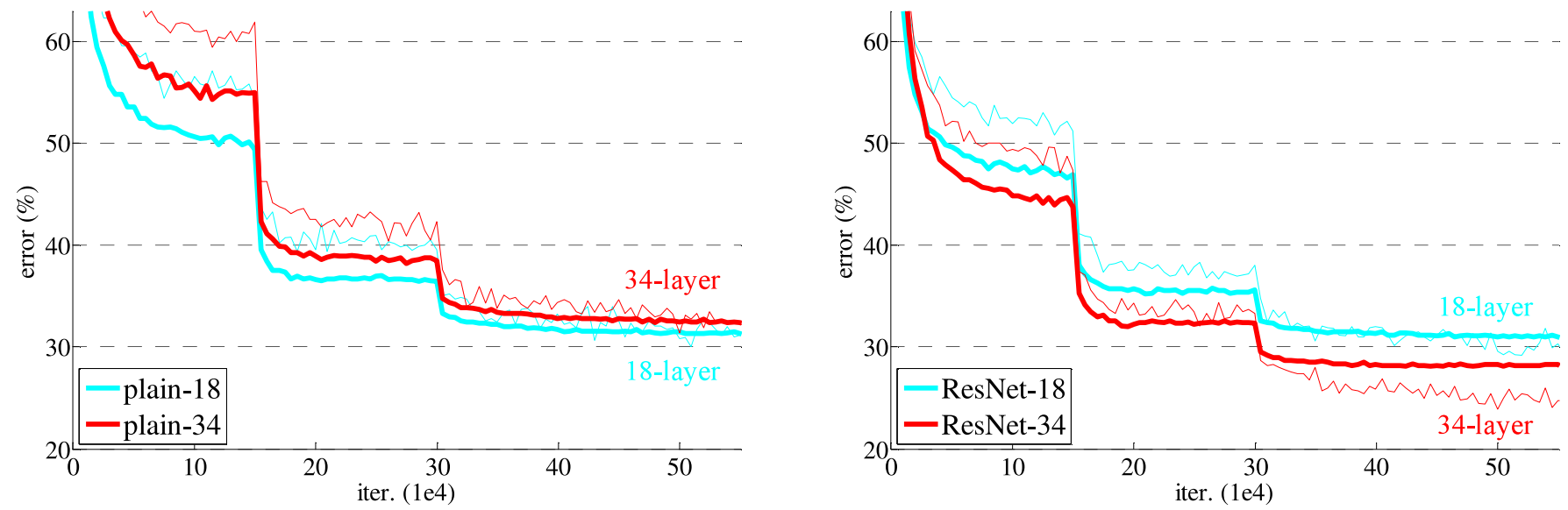


Figure 4. Training on **ImageNet**. Thin curves denote training error, and bold curves denote validation error of the center crops. Left: plain networks of 18 and 34 layers. Right: ResNets of 18 and 34 layers. In this plot, the residual networks have no extra parameter compared to their plain counterparts.

Figure 4 of "Deep Residual Learning for Image Recognition", <https://arxiv.org/abs/1512.03385>

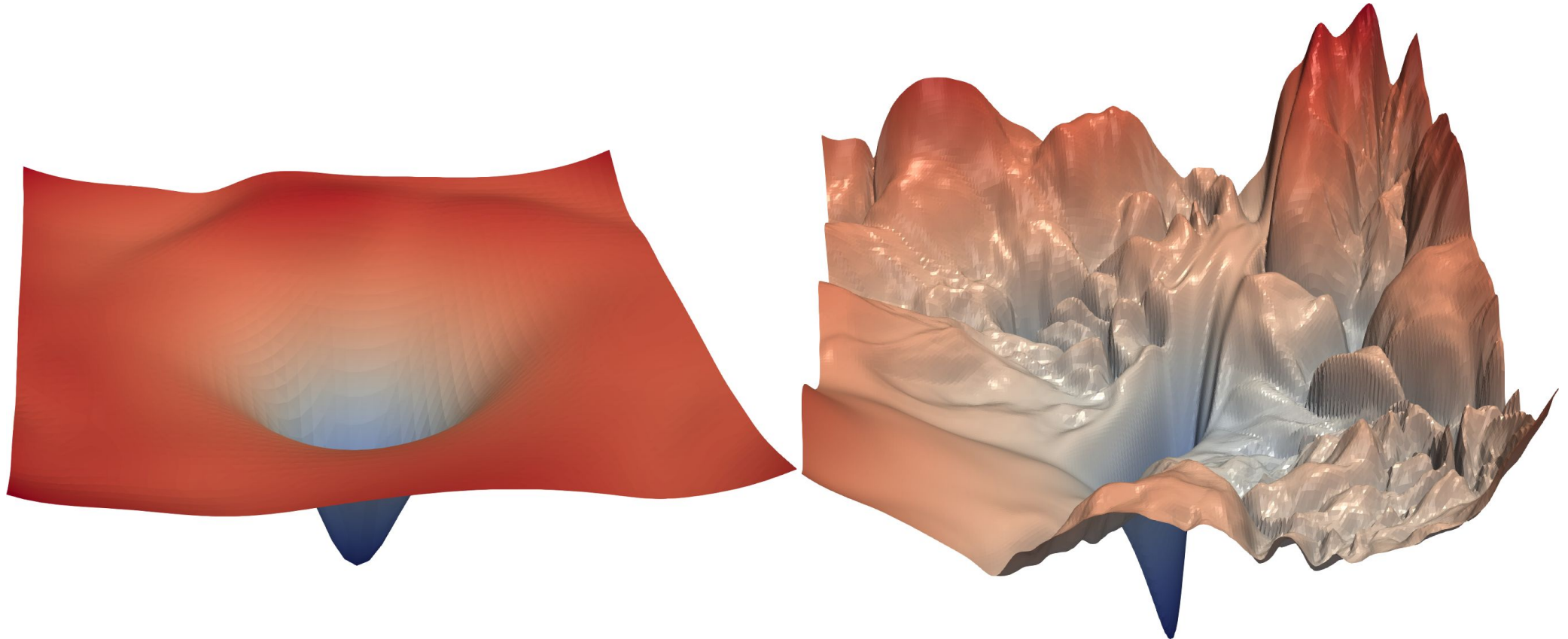


Figure 1 of "Visualizing the Loss Landscape of Neural Nets", <https://arxiv.org/abs/1712.09913>

## Training details:

- batch normalizations after each convolution and before activation
- SGD with batch size 256 and momentum of 0.9
- learning rate starts with 0.1 and is divided by 10 when error plateaus
- no dropout, weight decay 0.0001
- during testing, 10-crop evaluation strategy is used, averaging scores across multiple scales – the images are resized so that their smaller size is in  $\{224, 256, 384, 480, 640\}$

# ResNet – 2015 (3.6% error)

| method                     | top-1 err.   | top-5 err.        |
|----------------------------|--------------|-------------------|
| VGG [41] (ILSVRC'14)       | -            | 8.43 <sup>†</sup> |
| GoogLeNet [44] (ILSVRC'14) | -            | 7.89              |
| VGG [41] (v5)              | 24.4         | 7.1               |
| PRReLU-net [13]            | 21.59        | 5.71              |
| BN-inception [16]          | 21.99        | 5.81              |
| ResNet-34 B                | 21.84        | 5.71              |
| ResNet-34 C                | 21.53        | 5.60              |
| ResNet-50                  | 20.74        | 5.25              |
| ResNet-101                 | 19.87        | 4.60              |
| ResNet-152                 | <b>19.38</b> | <b>4.49</b>       |

Table 4. Error rates (%) of **single-model** results on the ImageNet validation set (except <sup>†</sup> reported on the test set).  
*Table 4 of "Deep Residual Learning for Image Recognition", <https://arxiv.org/abs/1512.03385>*

| method                     | top-5 err. (test) |
|----------------------------|-------------------|
| VGG [41] (ILSVRC'14)       | 7.32              |
| GoogLeNet [44] (ILSVRC'14) | 6.66              |
| VGG [41] (v5)              | 6.8               |
| PRReLU-net [13]            | 4.94              |
| BN-inception [16]          | 4.82              |
| <b>ResNet (ILSVRC'15)</b>  | <b>3.57</b>       |

Table 5. Error rates (%) of **ensembles**. The top-5 error is on the test set of ImageNet and reported by the test server.  
*Table 5 of "Deep Residual Learning for Image Recognition", <https://arxiv.org/abs/1512.03385>*

The ResNet-34 B uses the  $1 \times 1$  convolution on residual connections with different number of input and output channels; ResNet-34 C uses this convolution on all residual connections. Variant B is used for ResNet-50/101/152.

- Convolutions can provide
  - local interactions in spacial/temporal dimensions
  - shift invariance
  - *much* less parameters than a fully connected layer
- Usually repeated  $3 \times 3$  convolutions are enough, no need for larger filter sizes.
- When pooling is performed, double the number of channels (i.e., the first convolution following the pooling layer will have twice as many output channels).
- If your network is deep enough (the last hidden neurons have a large receptive fields), final fully connected layers are not needed, and global average pooling is enough.
- Batch normalization is a great regularization method for CNNs, allowing removal/decrease of dropout and  $L^2$  regularization.
- Small weight decay (i.e.,  $L^2$  regularization) of usually  $1e-4$  is still useful for regularizing convolutional kernels.