Introduction to Natural Language Processing I [Statistické metody zpracování přirozených jazyků I] (NPFL067)

http://ufal.mff.cuni.cz/courses/npfl067

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HMM Algorithms: Trellis and Viterbi

HMM: The Two Tasks

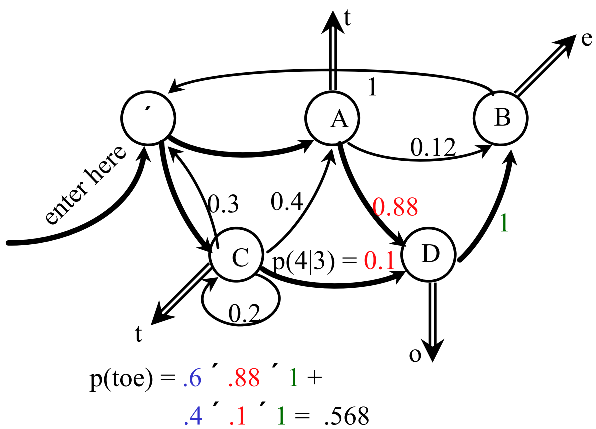
- HMM (the general case):
  - five-tuple (S, S_0, Y, P_S, P_Y), where:
    - **$S = \{s_1,s_2,\ldots,s_T\}$ is the set of states, S_0 is the initial state,**
    - **$Y = \{y_1,y_2,\ldots,y_V\}$ is the output alphabet,**
    - **$P_S(s_j|s_i)$ is the set of prob. distributions of transitions,**
    - **$P_Y(y_k|s_i,s_j)$ is the set of output (emission) probability distributions.**
- Given an HMM & an output sequence $Y = \{y_1,y_2,\ldots,y_k\}$:

  (Task 1) compute the probability of Y;

  (Task 2) compute the most likely sequence of states which has generated Y.

Trellis - Deterministic Output

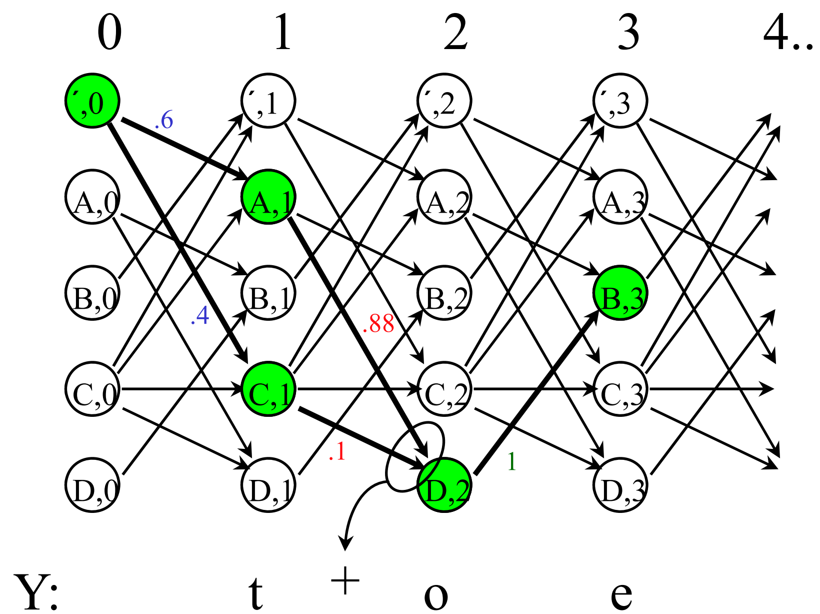
HMM:



“rollout”

Trellis:

time/position *t*



α(´,0) = 1 α(A,1) = .6 α(D,2) = .568 α(B,3) = .568

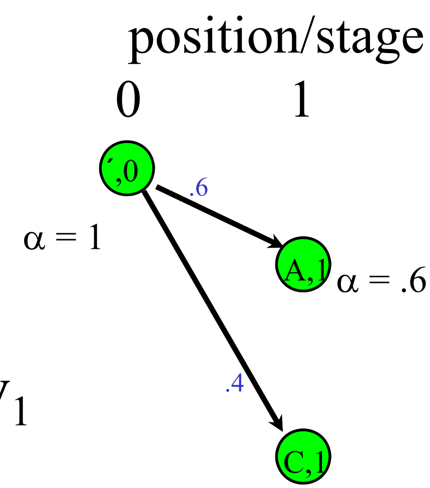
α(C,1) = .4

- trellis state: (HMM state, position)
- each state: holds ***one*** number (prob): α
- probability or Y: Σα in the last state

Creating the Trellis: The Start

- Start in the start state ($\times$),
  - set its $\alpha(\times,0)$ to 1.
- Create the first stage:
  - get the first "output" symbol y_1
  - create the first stage (column)
  - but only those trellis states

    which generate y_1
  - set their $\alpha(state,1)$ to the $P_S(state|\times)\ \underbrace{\alpha(\times,0)}_{1}$
- ...and forget about the *0*-th stage



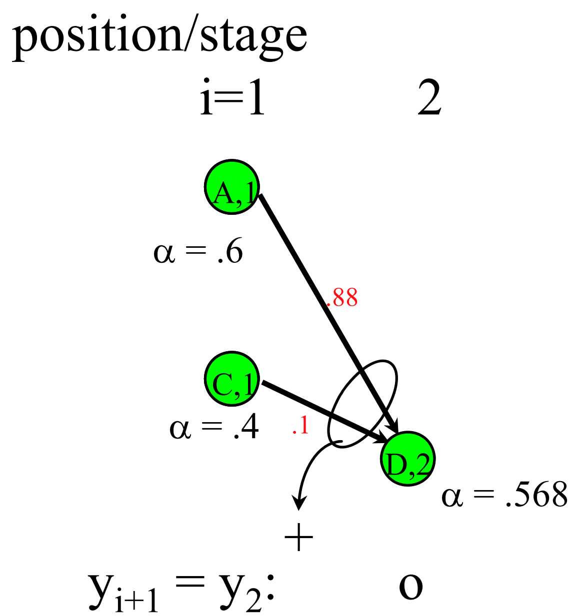
y_1: t

Trellis: The Next Step

- Suppose we are in stage *i*
- Creating the next stage:
  - create all trellis states in the next stage which generate y_{i+1}, but only those reachable from any of the stage-*i* states
  - set their $\alpha(state, i+1)$ to:

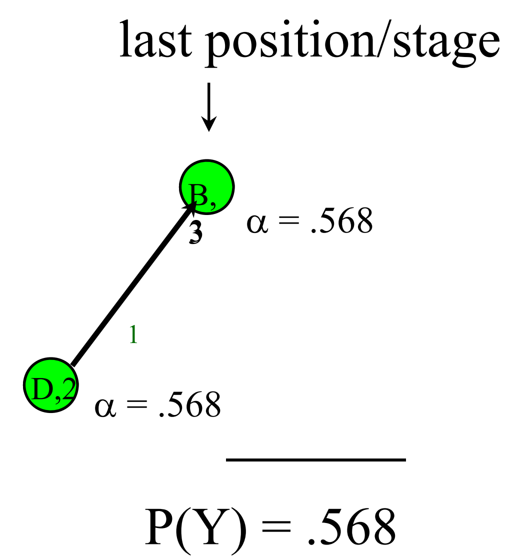
    $$P_S(state|prev.state) \times \alpha(prev.state, i)$$

    (add up all such numbers on arcs going to a common trellis state)
  - ...and forget about stage *i*



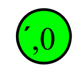
Trellis: The Last Step

- Continue until "output" exhausted
  - $|Y| = 3$: until stage 3
- Add together all the $\alpha(state,|Y|)$
- That's the <u>P(Y)</u>.
- Observation (pleasant):
  - memory usage max: $2|S|$
  - multiplications max: $|S|^2|Y|$

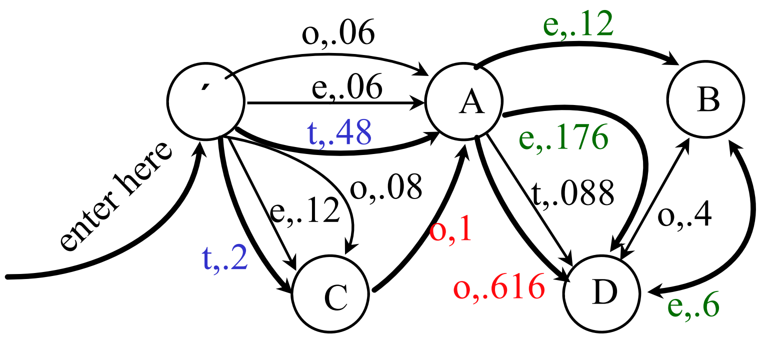


Trellis: The General Case (still, bigrams)

- Start as usual:
  - start state (´), set its α(´,*0*) to 1.



α = 1



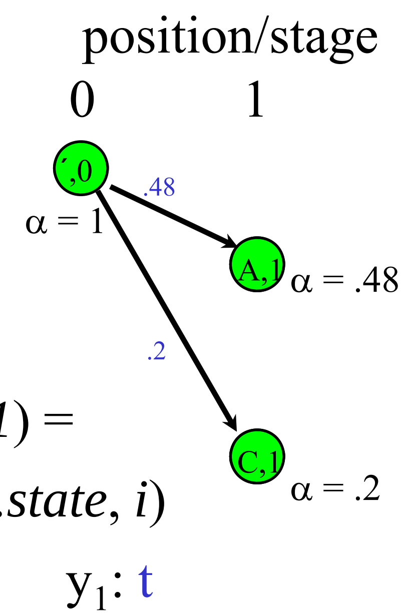
p(toe) = .48´.616´.6+

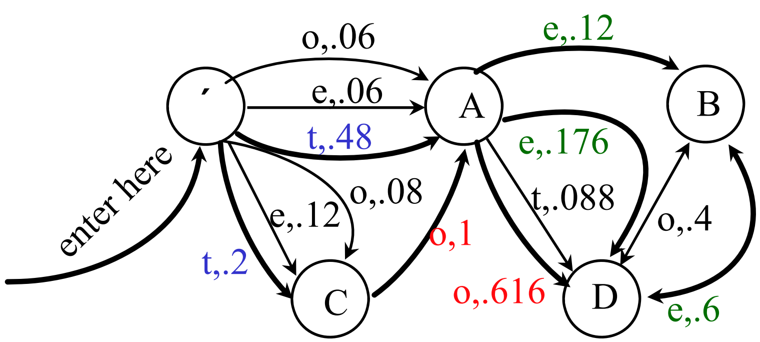
.2´1´.176 +

.2´1´.12 ≅ .237

General Trellis: The Next Step

- We are in stage *i* :
  - Generate the next stage *i+1* as before (except now <u>arcs</u> generate output, thus use only those arcs marked by the output symbol y_{i+1})
  - For each generated *state*, compute $\alpha(state, i+1) =$
    $= \Sigma_{\text{incoming arcs}} P_Y(y_{i+1}|state, prev.state) \times \alpha(prev.state, i)$

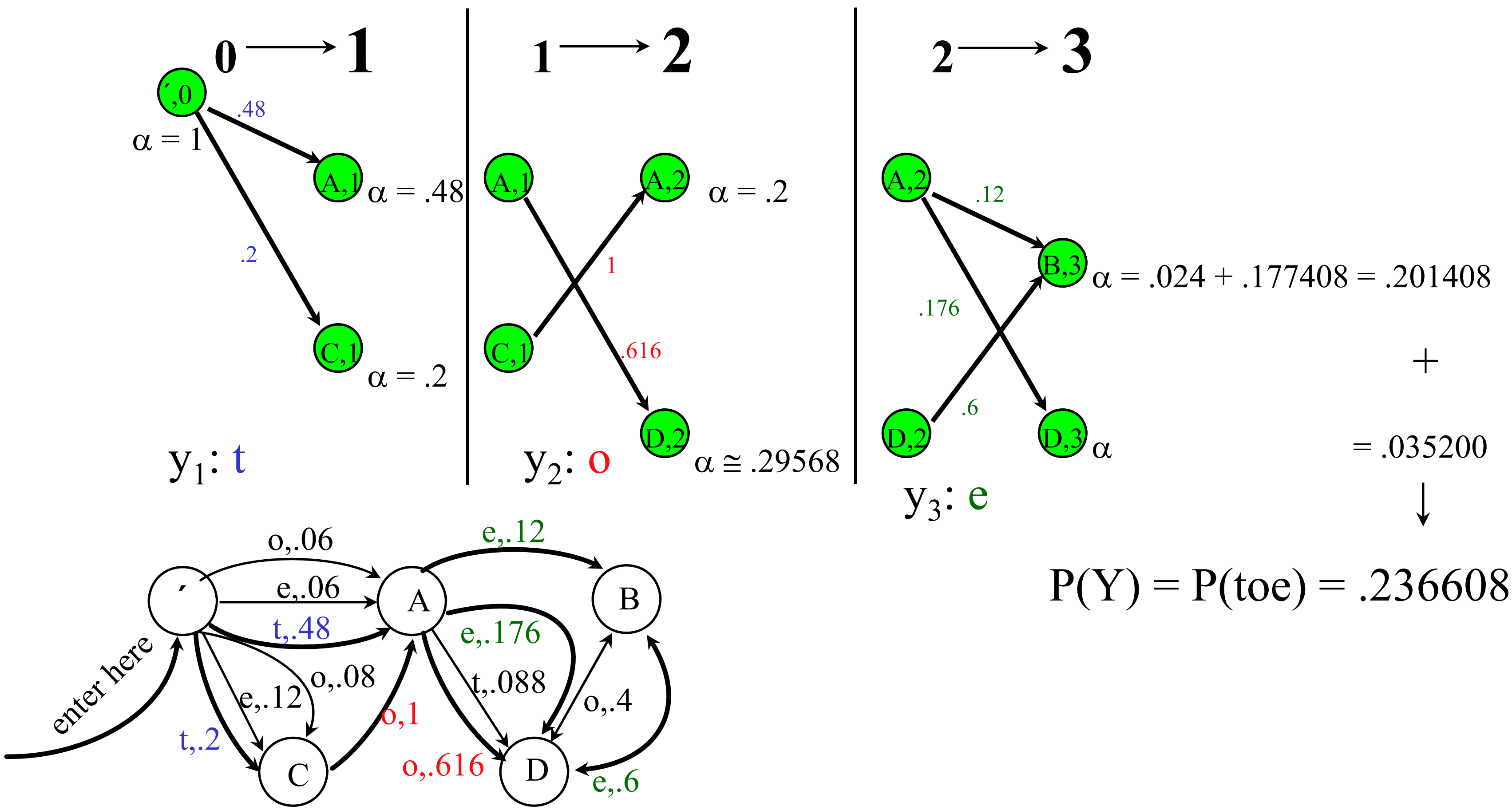




...and forget about stage *i* as usual.

Trellis: The Complete Example

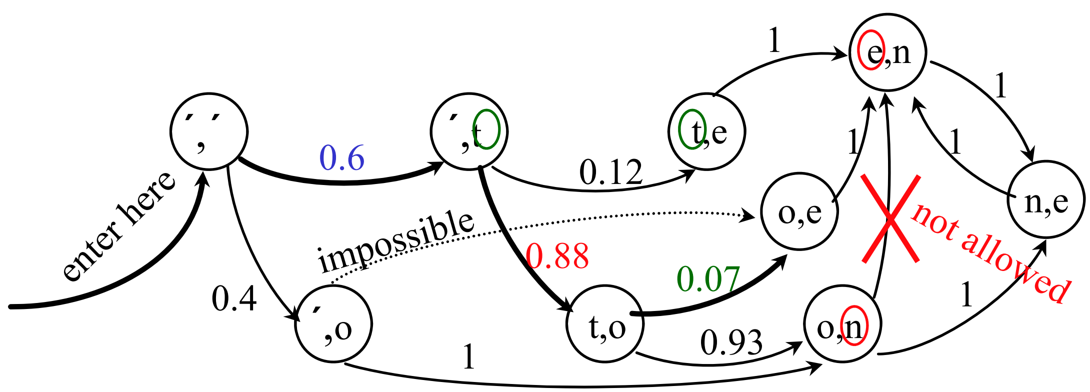
Stage:

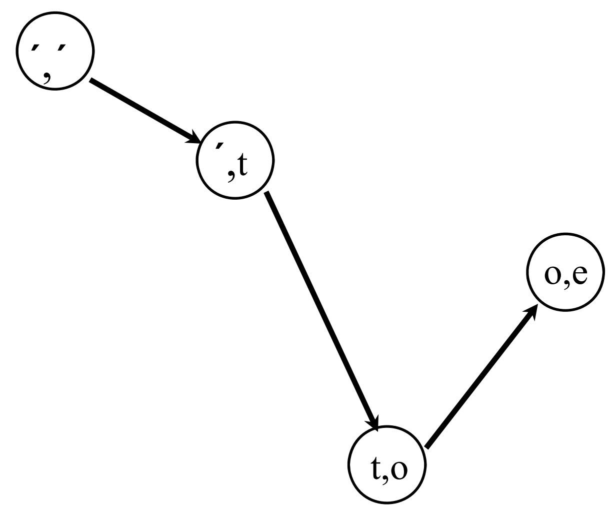


P(Y) = P(toe) = .236608

The Case of Trigrams

- Like before, but:
  - states correspond to bigrams,
  - output function always emits the second output symbol of the pair (state) to which the arc goes:





p(toe) = .6 ´ .88 ´ .07 ≅ .037

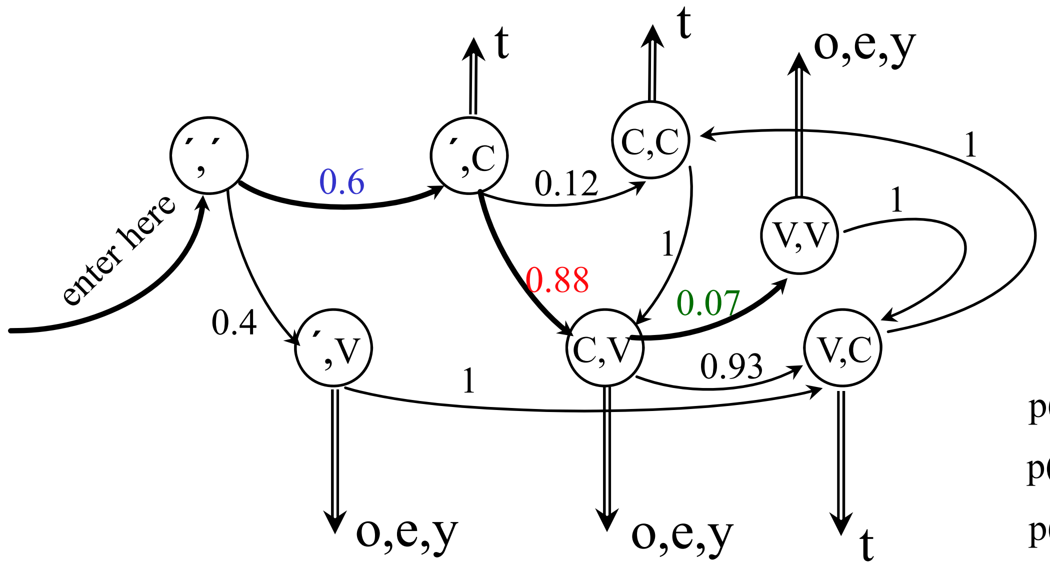
Multiple paths not possible → trellis not really needed

Trigrams with Classes

- More interesting:
  - n-gram class LM: $p(w_i|w_{i-2},w_{i-1}) = p(w_i|c_i)\, p(c_i|c_{i-2},c_{i-1})$

    $\rightarrow$ states are pairs of classes (c_{i-1},c_i), and emit "words":

    (letters in our example)



p(t|C) = 1 usual,

p(o|V) = .3 non-

p(e|V) = .6 overlapping

p(y|V) = .1 classes

p(toe) = .6 ´ 1 ´ .88 ´ .3 ´ .07 ´ .6 ≅ .00665

p(teo) = .6 ´ 1 ´ .88 ´ .6 ´ .07 ´ .3 ≅ .00665

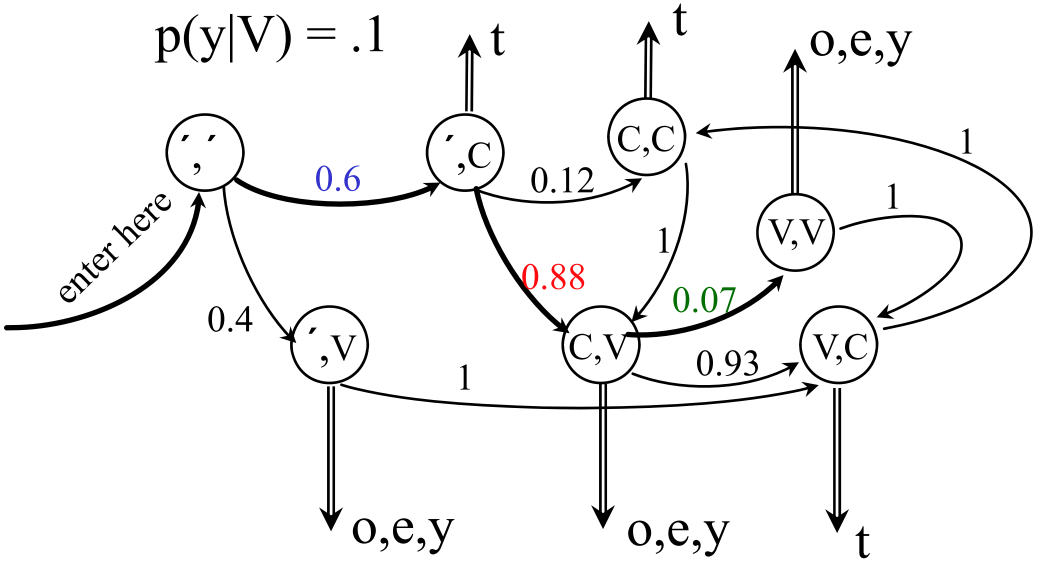
p(toy) = .6 ´ 1 ´ .88 ´ .3 ´ .07 ´ .1 ≅ .00111

p(tty) = .6 ´ 1 ´ .12 ´ 1 ´ 1 ´ .1 ≅ .0072

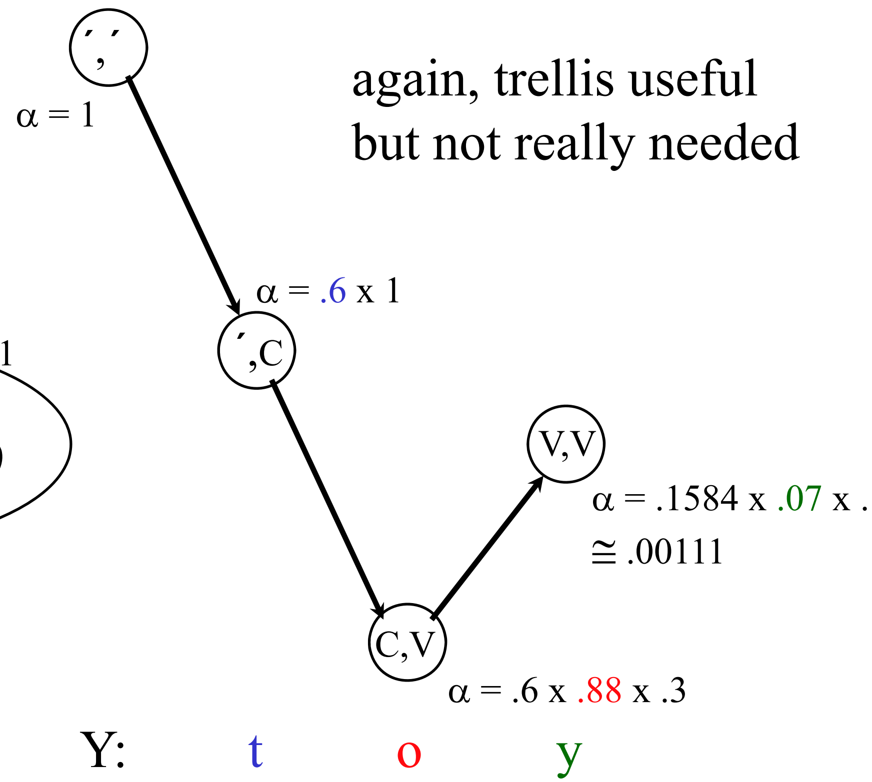
Class Trigrams: the Trellis

- Trellis generation (Y = “toy”):

p(t|C) = 1
p(o|V) = .3
p(e|V) = .6
p(y|V) = .1

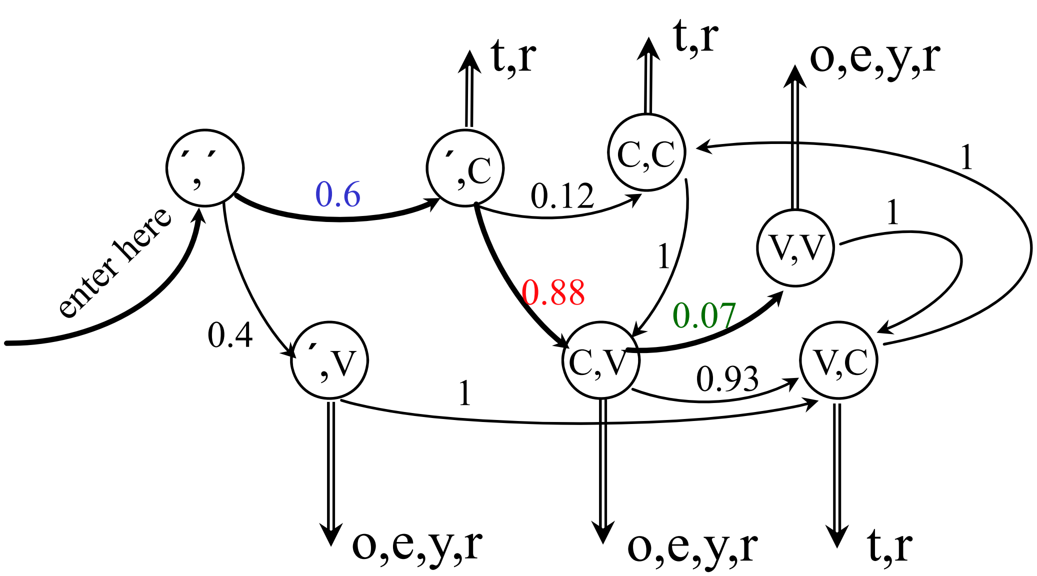


again, trellis useful
but not really needed



Overlapping Classes

- Imagine that classes may overlap
  - e.g. ‘r’ is sometimes vowel sometimes consonant, belongs to V as well as C:

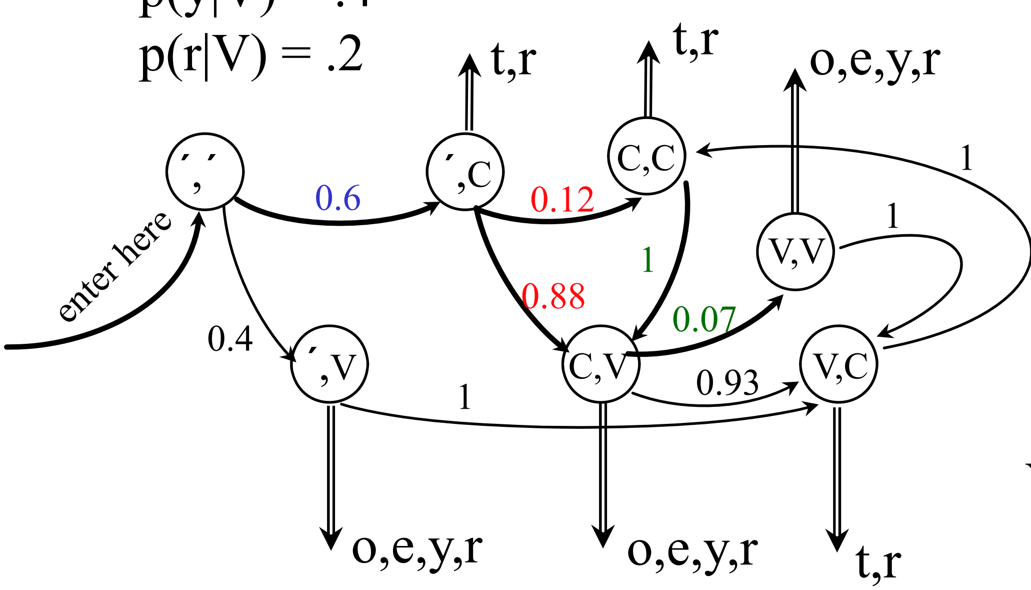


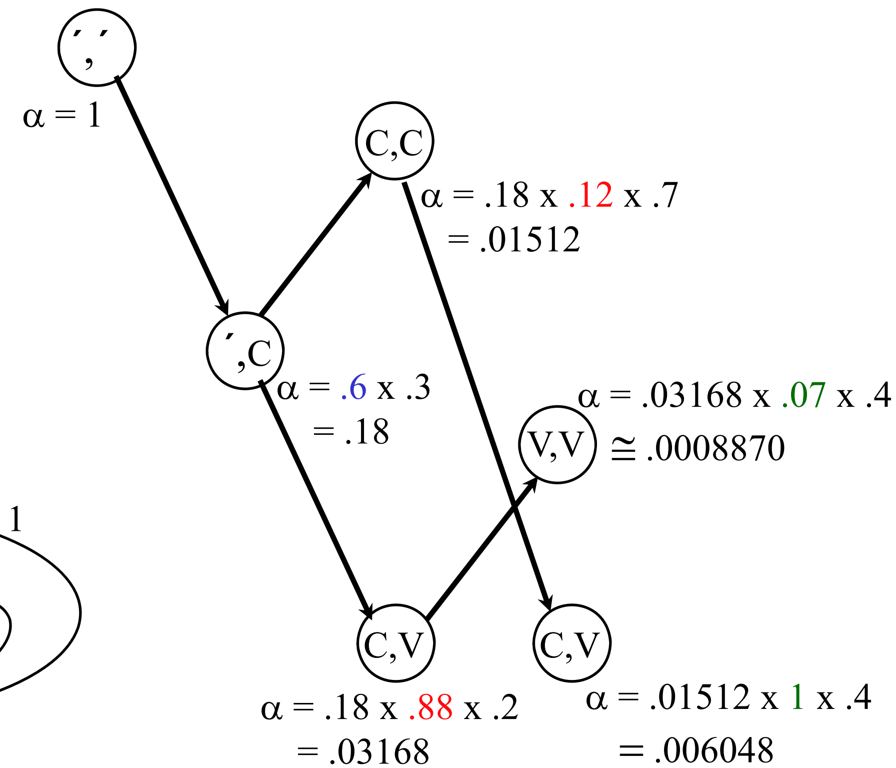
p(t|C) = .3
p(r|C) = .7
p(o|V) = .1
p(e|V) = .3
p(y|V) = .4
p(r|V) = .2

p(try) = ?

Overlapping Classes: Trellis Example

$p(t|C) = .3$
$p(r|C) = .7$
$p(o|V) = .1$
$p(e|V) = .3$
$p(y|V) = .4$
$p(r|V) = .2$





Y: t r y $p(Y) = \underline{.006935}$

Trellis: Remarks

- So far, we went left to right (computing α)
- Same result: going right to left (computing β)
  - supposed we know where to start (finite data)
- In fact, we might start in the middle going left and right
- Important for parameter estimation

  (Forward-Backward Algortihm alias Baum-Welch)
- Implementation issues:
  - scaling/normalizing probabilities, to avoid too small numbers & addition problems with many transitions

The Viterbi Algorithm

- Solving the task of finding the most likely sequence of states which generated the observed data
- i.e., finding

$$S_{best} = \text{argmax}_S P(S|Y)$$

which is equal to (Y is constant and thus P(Y) is fixed):

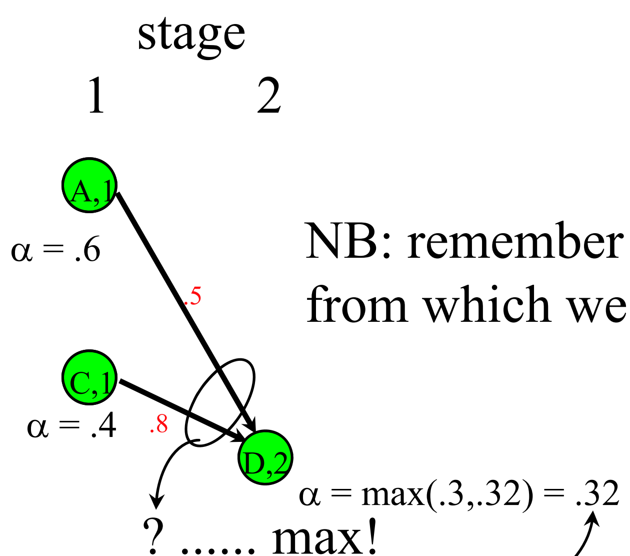
$$S_{best} = \text{argmax}_S P(S,Y) =$$

$$= \text{argmax}_S P(s_0,s_1,s_2,\ldots,s_k,y_1,y_2,\ldots,y_k) =$$

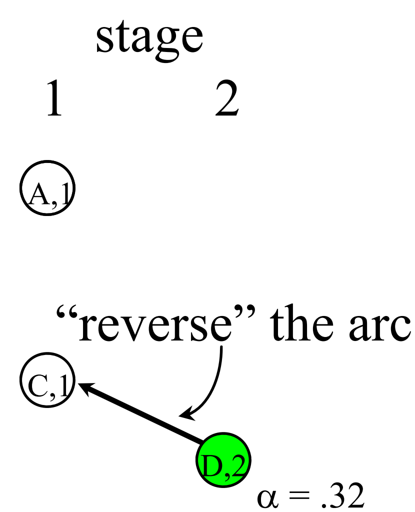
$$= \text{argmax}_S \Pi_{i=1..k}\ p(y_i|s_i,s_{i-1})p(s_i|s_{i-1})$$

The Crucial Observation

- Imagine the trellis build as before (but do not compute the αs yet; assume they are o.k.); stage *i*:



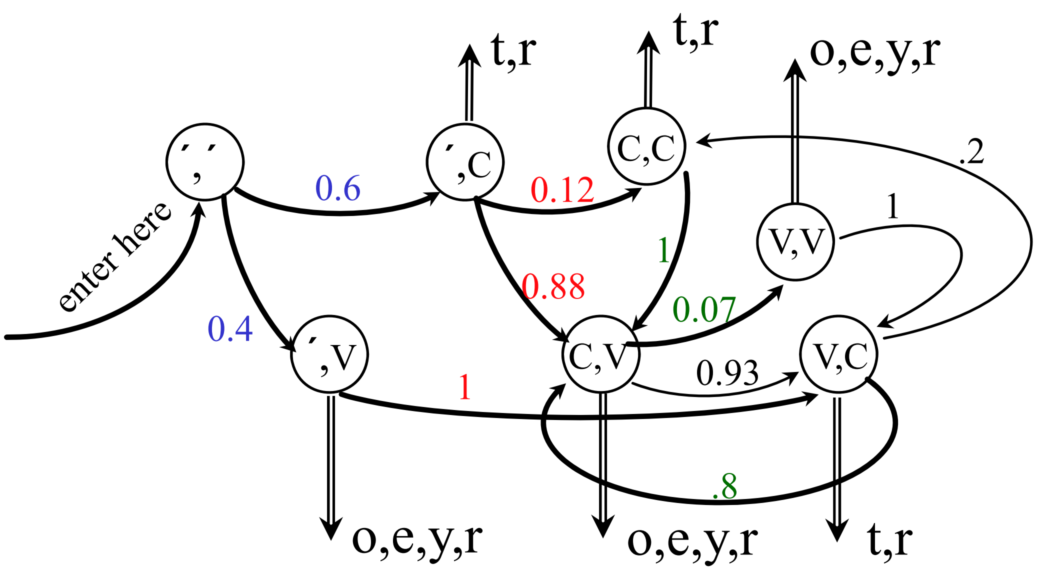
NB: remember previous state from which we got the maximum:



this is certainly the "backwards" maximum to (D,2)... but
it <u>cannot change even whenever we go forward</u> (M. Property: Limited History)

Viterbi Example

- ‘r’ classification (C or V?, sequence?):



$p(t|C) = .3$
$p(r|C) = .7$
$p(o|V) = .1$
$p(e|V) = .3$
$p(y|V) = .4$
$p(r|V) = .2$

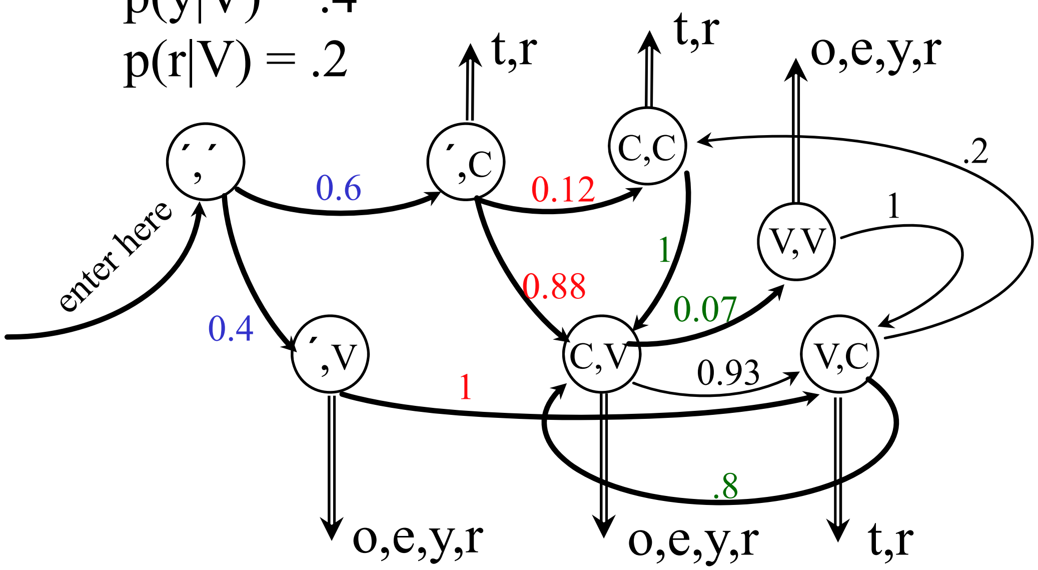
$\text{argmax}_{XYZ}\ p(rry|XYZ) = ?$

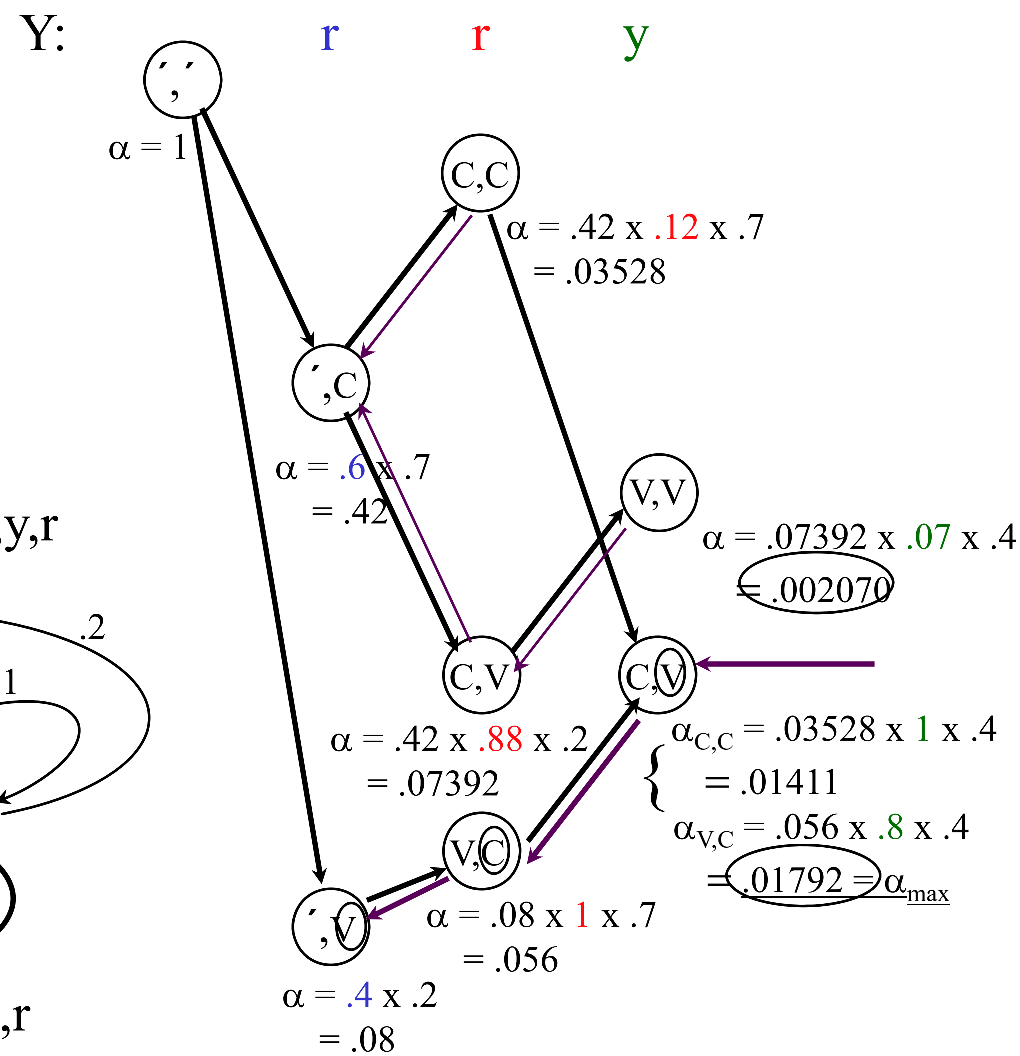
Possible state seq.: (´,v)(v,c)(c,v)[VCV], (´,c)(c,c)(c,v)[CCV], (´,c)(c,v)(v,v) [CVV]

Viterbi Computation

$p(t|C) = .3$
$p(r|C) = .7$
$p(o|V) = .1$
$p(e|V) = .3$
$p(y|V) = .4$
$p(r|V) = .2$

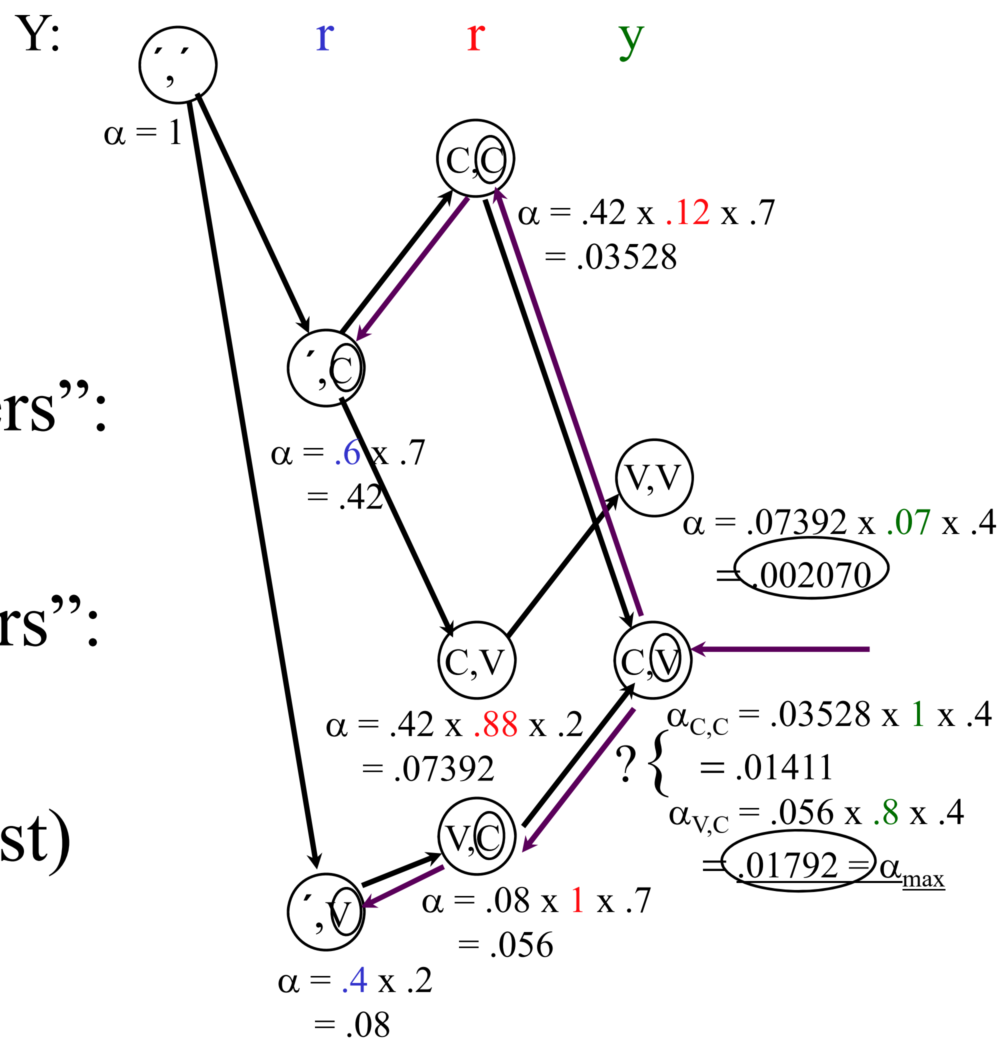
α in trellis state: best prob from start to here





n-best State Sequences

- Keep track of n best "back pointers":
- Ex.: n= 2: Two "winners": VCV (best) CCV (2nd best)

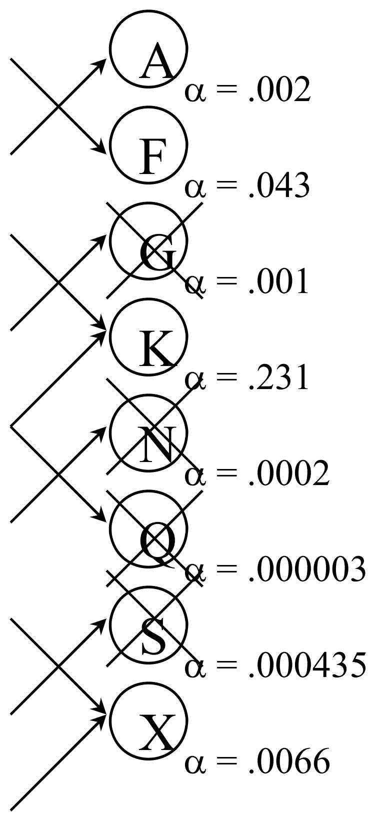


Tracking Back the n-best paths

- Backtracking-style algorithm:
    - **Start at the end, in the best of the n states (s_{best})**
    - **Put the other n-1 best nodes/back pointer pairs on stack, except those leading from s_{best} to the same best-back state.**
- Follow the back "beam" towards the start of the data, spitting out nodes on the way (backwards of course) using always only the <u>best</u> back pointer.
- At every beam split, push the diverging node/back pointer pairs onto the stack (node/beam width is sufficient!).
- When you reach the start of data, close the path, and pop the top-most node/back pointer(width) pair from the stack.
- Repeat until the stack is empty; expand the result tree if necessary.

Pruning

- Sometimes, too many trellis states in a stage:



criteria: (a) α < threshold
(b) $\Sigma\pi$ < threshold
(c) # of states > threshold
(get rid of smallest α)