1. Prove that for every cache size $C$ and every $\epsilon > 0$, there exists a sequence of requests for which $T_{LRU} \geq (1 - \epsilon) \cdot C \cdot T_{OPT}$.

2. Prove that if $H$ is $(k, c)$-independent for $k > 1$, then it is also $(k - 1, c)$-independent.

3. Prove that if $H$ is $(2, c)$-independent, then it is $c$-universal.