B

(1) $\lim\limits_{x\to\infty} x^2 \cdot \ln\left(1-\operatorname{arctg}\frac{2}{x}\right) = \text{"}\infty\cdot 0\text{"} =$

$= \lim\limits_{x\to\infty} \dfrac{\ln\left(1-\operatorname{arctg}\frac{2}{x}\right)}{\frac{1}{x^2}} = \text{"}\dfrac{0}{0}\text{"} \overset{l'H}{=} \lim\limits_{x\to\infty} \dfrac{\frac{1}{1-\operatorname{arctg}\frac{2}{x}}\cdot\frac{-1}{1+\left(\frac{2}{x}\right)^2}\cdot\frac{-2}{x^2}}{-\frac{2}{x^3}}$

spoj. fce mimo 0

$= \lim\limits_{x\to\infty} \dfrac{-2x^3}{\left(1-\operatorname{arctg}\frac{2}{x}\right)\left(1+\left(\frac{2}{x}\right)^2\right)\cdot 2x^2} = \lim\limits_{x\to\infty} \dfrac{-x}{\left(1+\operatorname{arctg}\frac{2}{x}\right)\left(1+\left(\frac{2}{x}\right)^2\right)}$

$\overset{AL}{=} \lim\limits_{x\to\infty} \dfrac{-x}{1+\operatorname{arctg}\frac{2}{x}} \cdot \lim\limits_{x\to\infty} \dfrac{1}{1+\left(\frac{2}{x}\right)^2} = \cancel{\lim} \text{"}\dfrac{-\infty}{1+0}\text{"}\cdot 1 = \boxed{-\infty}$

spoj. fce v 0

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(2) $f(x) = \begin{cases} \dfrac{1-\cos x}{x^2} & x \neq 0 \\ \dfrac{1}{2} & x = 0 \end{cases}$

$x \neq 0$ ... spoj. (skládání spoj. fcí)

$x = 0$ ... $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2} = \text{"}\dfrac{0}{0}\text{"} \overset{l'H}{=} \lim\limits_{x\to 0} \dfrac{\sin x}{2x} = \dfrac{1}{2}$

$\Rightarrow$ spoj. v 0

$\Rightarrow \boxed{f \text{ spoj. v } \mathbb{R}}$

$f'(x) = \dfrac{x^2\cdot\sin x - 2x\cdot(1-\cos x)}{x^4} = \dfrac{1}{x^3}\left(x\cdot\sin x - 2 + 2\cos x\right)$ mimo 0

$f'(0) = \lim\limits_{h\to 0} \dfrac{f(0+h)-f(0)}{h} = \lim\limits_{h\to 0} \dfrac{\frac{1-\cos h}{h^2} - \frac{1}{2}}{h} = \text{"}\dfrac{0}{0}\text{"} \overset{l'H}{=}$

NEBO podle VĚTY o derivaci jako limitě derivace!

$= \lim\limits_{h\to 0} \dfrac{\frac{1}{h^3}\left(h\cdot\sin h - 2 + 2\cos h\right)}{1} = \text{"}\dfrac{0}{0}\text{"} \overset{l'H}{=}$

$= \lim\limits_{h\to 0} \dfrac{\sin h + h\cdot\cos h - 2\cdot\sin h}{3h^2} = \lim\limits_{h\to 0} \dfrac{h\cdot\cos h - \sin h}{3h^2} = \text{"}\dfrac{0}{0}\text{"}$

$= \lim\limits_{h\to 0} \dfrac{\cancel{\cos h} + h\cdot\sin h - \cancel{\cos h}}{6h} \overset{AL}{=} \lim\limits_{h\to 0} \dfrac{h}{6} \cdot \lim\limits_{h\to 0} \dfrac{\sin h}{h} = 0\cdot 1 = 0$