



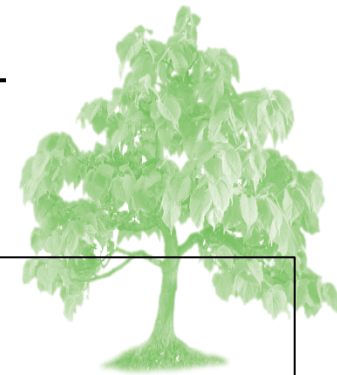
# Prague Dependency Treebank: Introduction – (Non-)Projectivity

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# Dependency tree (definition)

$$T = \langle N, D, Q, WO, L \rangle$$

$\langle N, D \rangle$  ... **rooted tree**

Q ... lexical and grammatical categories

L ... labeling function  $N \rightarrow Q$

D ... oriented edges  $\sim$  relation on lex. and gram. categories

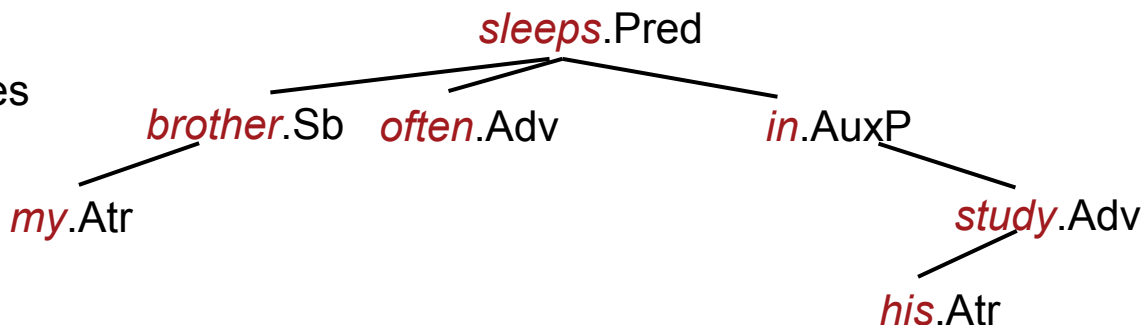
*'dependency' relation*

WO ... relation on N  $\sim$  (strong total ordering on N) ...

*word order*

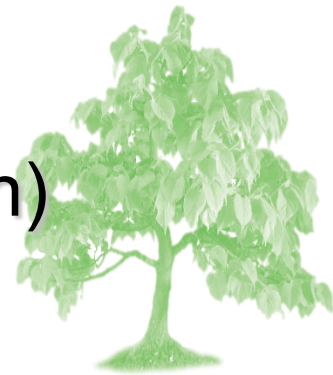
## **rooted tree**

- finite graph
- no cycles, no loops
- max 1 edge between 2 diff. nodes
- root



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# Projectivity and non-projectivity (definition)

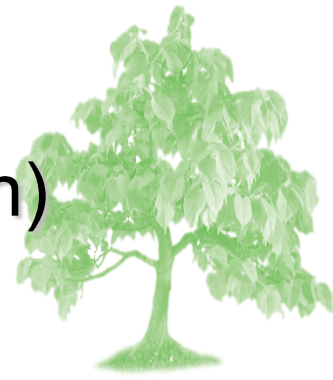


Whom Mark decided to marry?

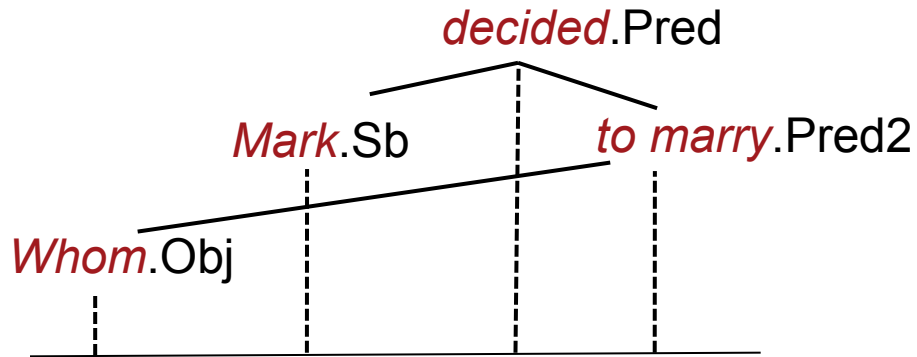
*Soubor se nepodařilo otevřít. (Oliva)*

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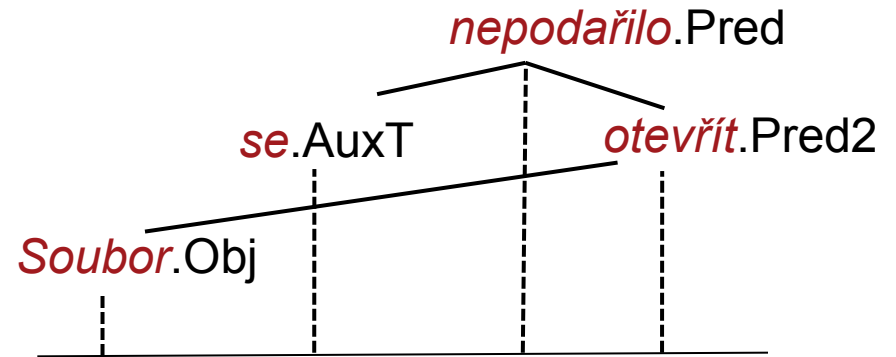
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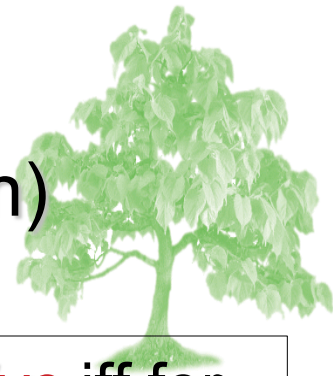
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# Projectivity and non-projectivity (definition)



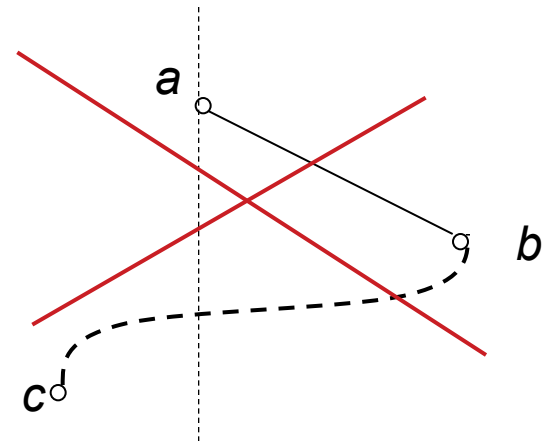
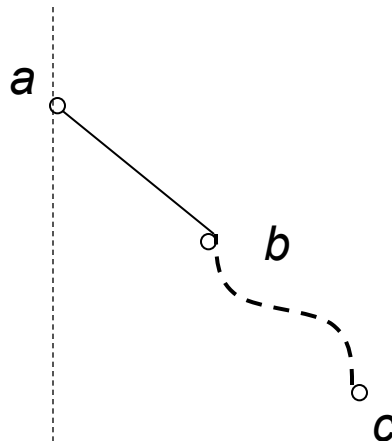
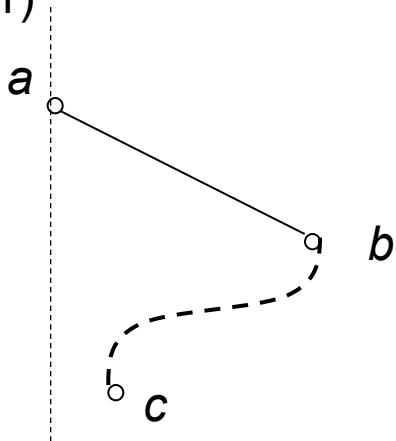
A subtree  $S$  of a rooted dependency tree  $T$  is *projective* iff for all nodes  $a$ ,  $b$  and  $c$  of the subtree  $S$  the condition holds:

$$(1) (a \leq_D b) \ \& \ (a <_{WO} b) \ \& \ (b \leq_D^* c) \Rightarrow (a <_{WO} c)$$

and

$$(2) (a \leq_D b) \ \& \ (b <_{WO} a) \ \& \ (b \leq_D^* c) \Rightarrow (c <_{WO} a)$$

(1)



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# Projectivity and free word order



## free word order:

- freedom of word order of dependents within a continuous 'head domain' (i.e., substring of head + its dependents)

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# Projectivity and free word order



## free word order:

- freedom of word order of dependents within a continuous ‘head domain’ (i.e., substring of head + its dependents)
- relaxation of continuity of a head domain

German:

*Maria hat einen Mann kennengelernt der Schmetterlinge sammelt.*

Mary - has - a man - met - the butterflies - collects

Mary has met a man who collects butterflies

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# Projectivity and free word order



English: long-distance unbounded dependency

*John, Peter thought that Sue said that Mary loves.*



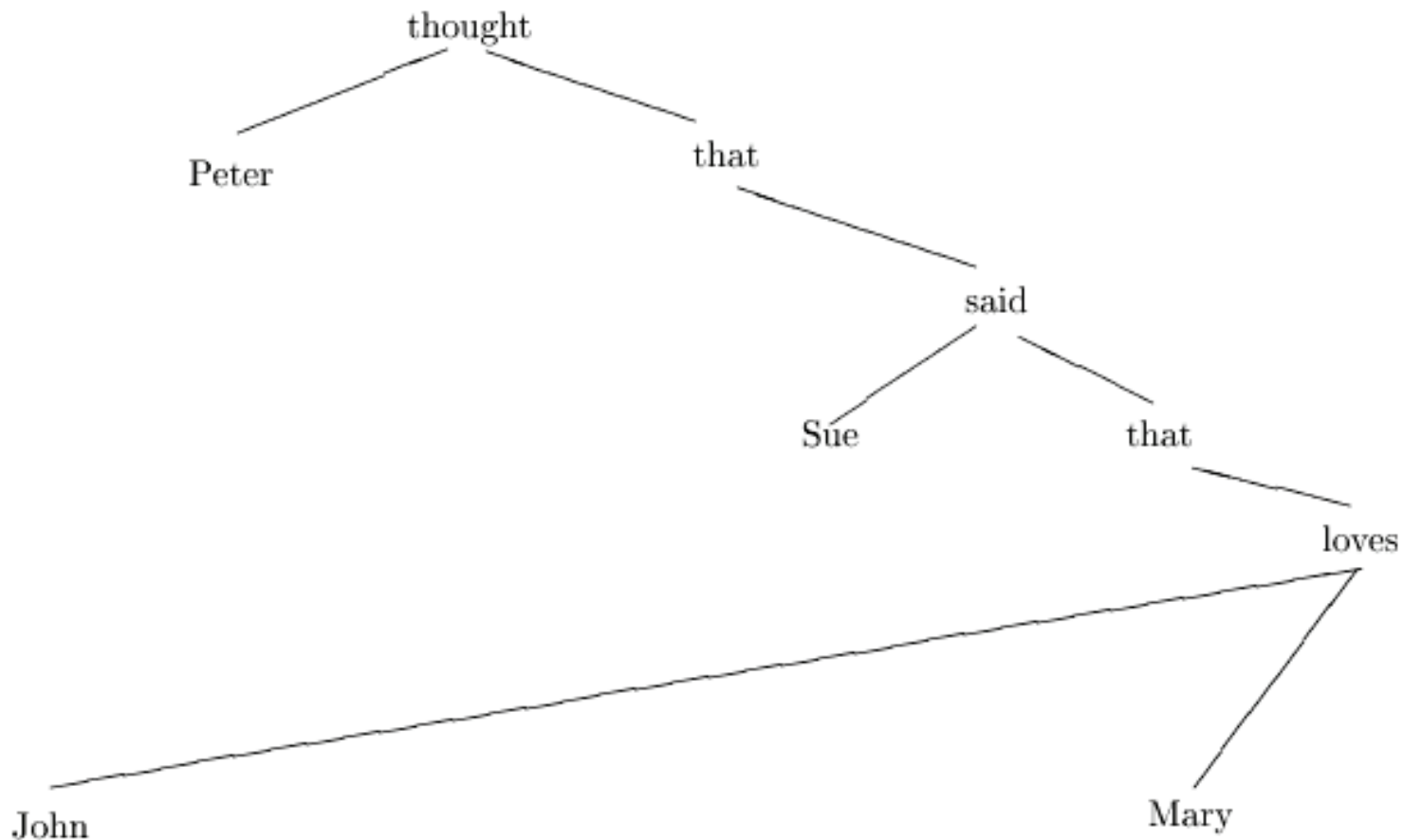
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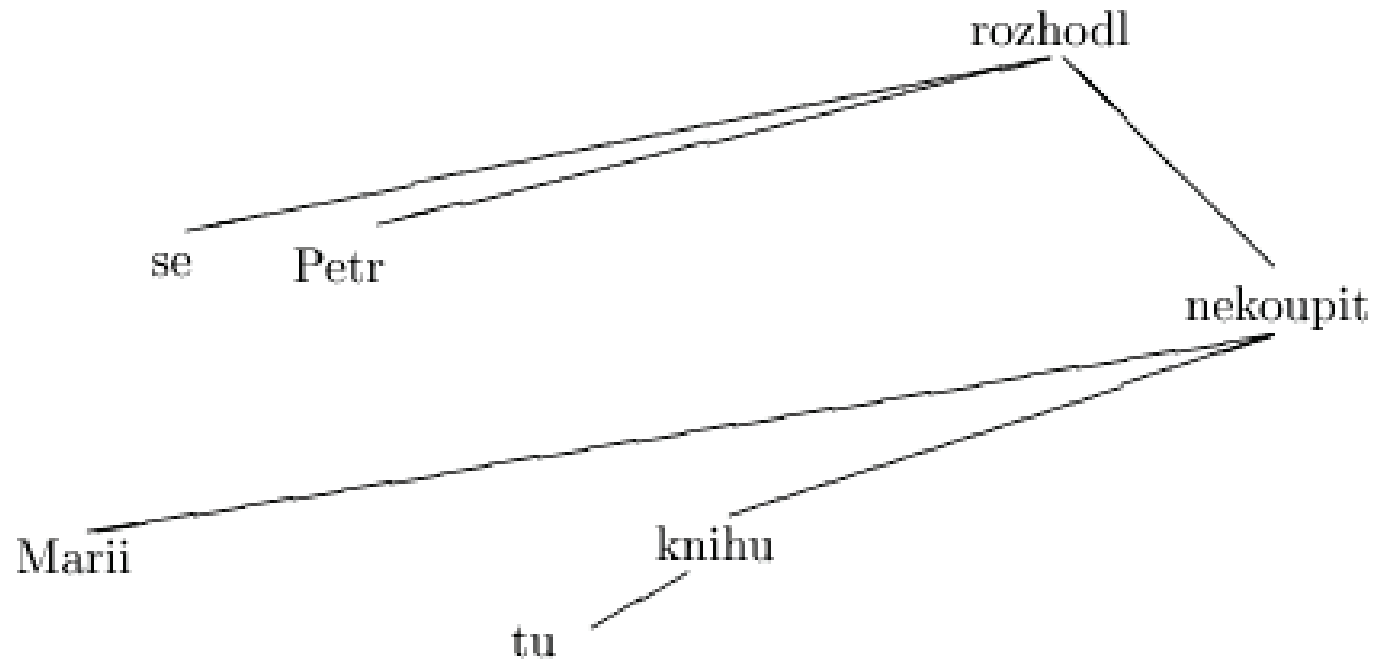


Czech:

*Marii se Petr tu knihu rozhodl nekoupit.*

to-Mary PART Peter that book decided not-buy

[Peter decided not to buy that book to Mary.]

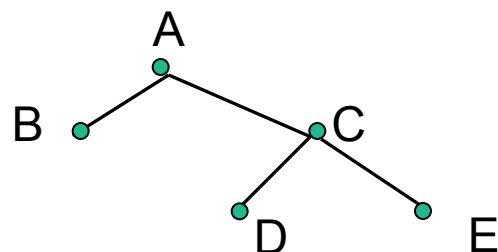




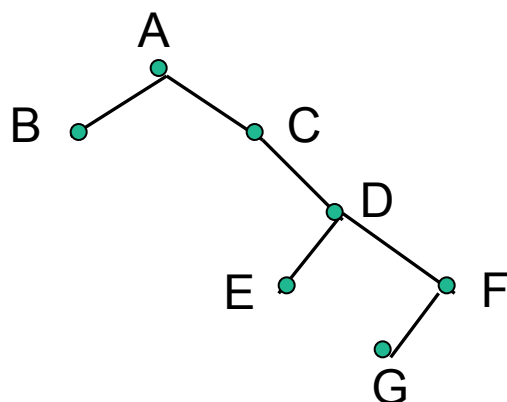
# Projectivity and non-projectivity

Projective dependency trees can be encoded by *linearization*:

- string of nodes, edges ~ brackets



$A ( B C ( D ) )$  without WO ordering  
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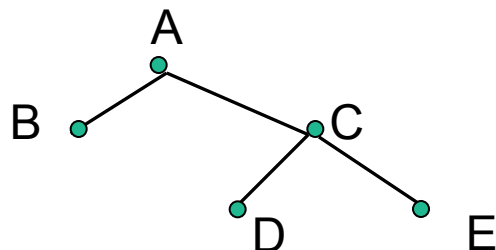




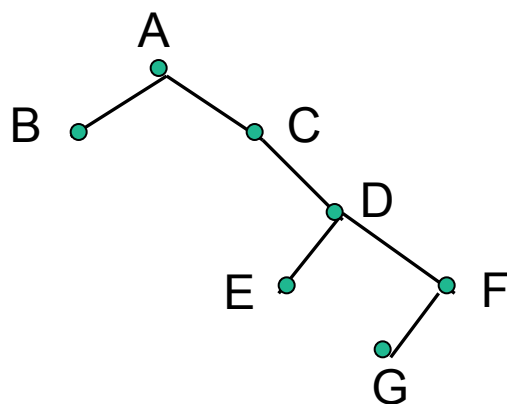
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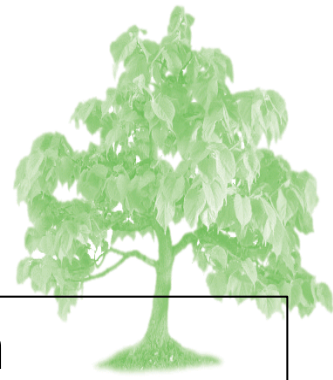


$A ( B C ( D ) )$  without WO ordering  
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$A ( B C ( D ( E F ( G ) ) ) )$  without WO  
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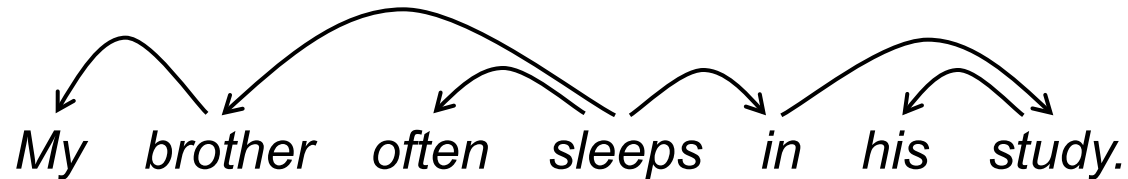
# Planarity



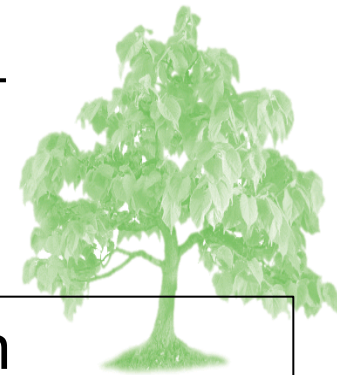
A dependency graph  $T$  is *planar*, if it does not contain nodes  $a, b, c, d$  such that:

$$\text{linked}(a,c) \ \& \ \text{linked}(b,d) \ \& \ a <_{\text{WO}} b <_{\text{WO}} c <_{\text{WO}} d$$

*linked*( $i,j$ ) ... ‘there is an edge in  $T$  from  $i$  to  $j$ , or vice versa’



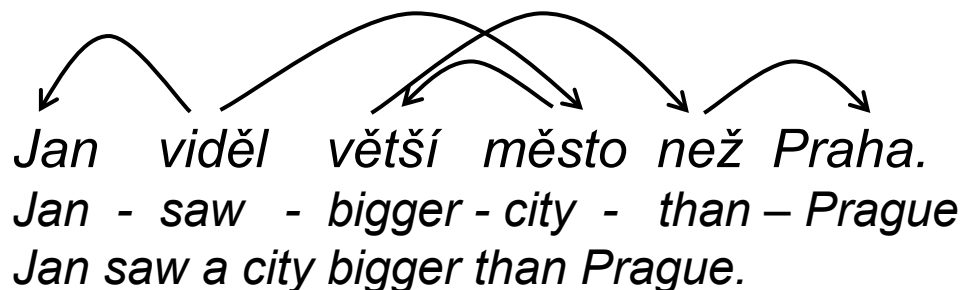
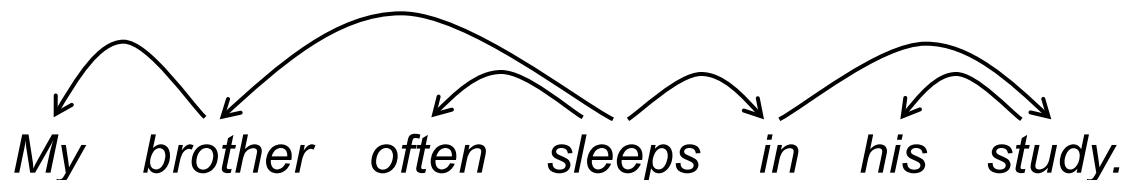
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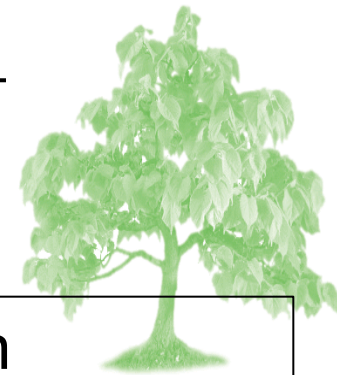
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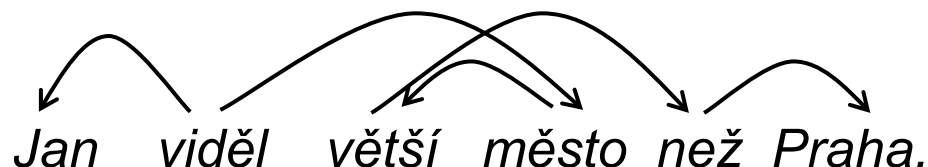
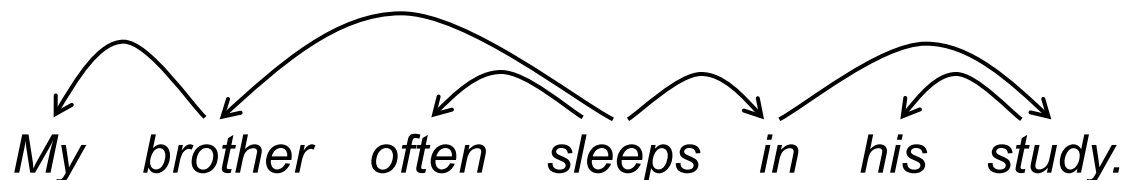
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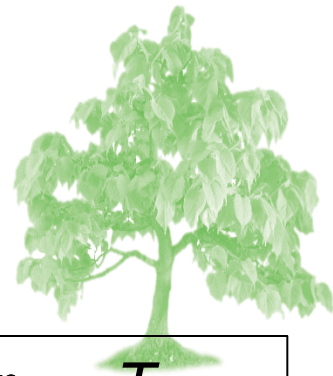
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Informally, a dependency graph is planar, if its edges can be drawn above the sentence without crossing.

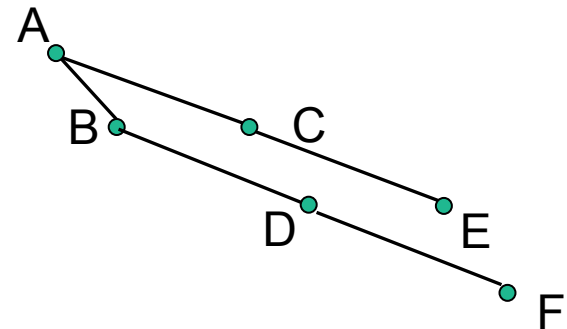
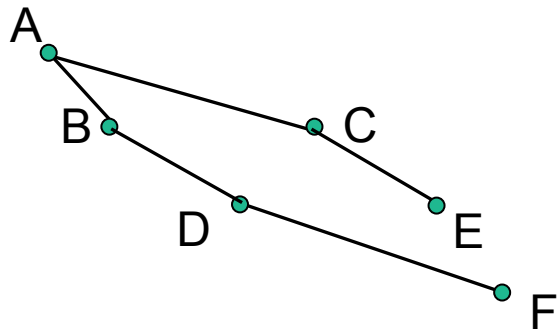
# 'Well-Nestedness'



Two subtrees  $T_1, T_2$  *interleave*, if there are nodes  $l_1, r_1 \in T_1$  and  $l_2, r_2 \in T_2$  such that

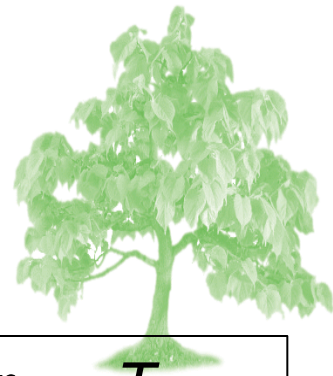
$$l_1 <_{WO} l_2 <_{WO} r_1 <_{WO} r_2$$

A dependency graph is *well-nested*, if no two of its disjoint subtrees interleave.'





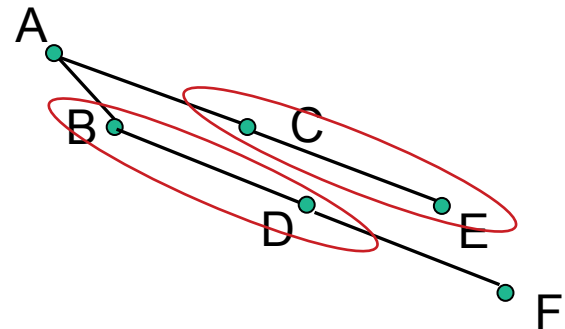
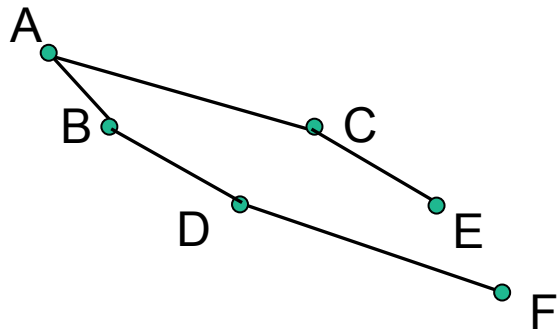
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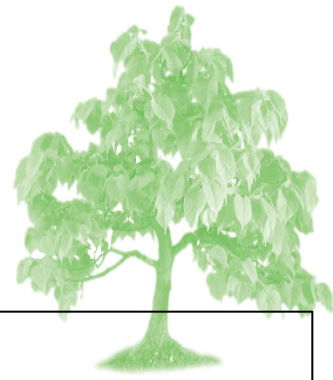
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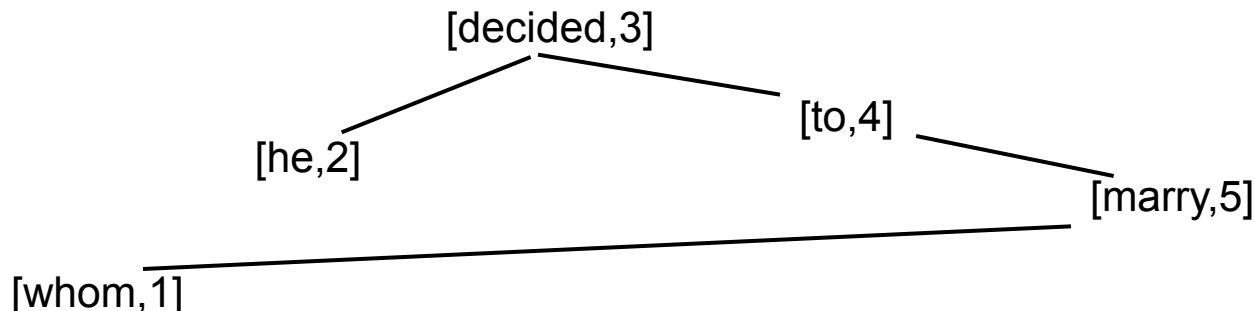
# Gap Degree $dNh(T)$



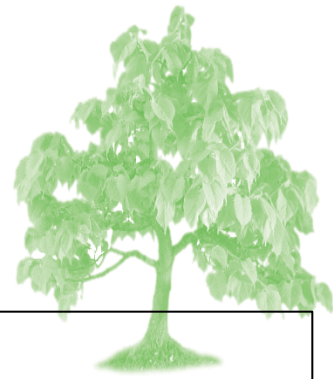
**Coverage** of a node  $u \in T$

$Cov(u, T) = \{ i \mid i - \text{word order position of } v \in T \text{ such that, } u \leq_D v \}$

$Cov(u_1, T) = \{ 1 \}; \quad Cov(u_2, T) = \{ 2 \}; \quad Cov(u_3, T) = \{ 1, 2, 3, 4, 5 \}; \quad Cov(u_4, T) = \{ 1, 4, 5 \}; \quad Cov(u_5, T) = \{ 1, 5 \}$



# Gap Degree $dNh(T)$

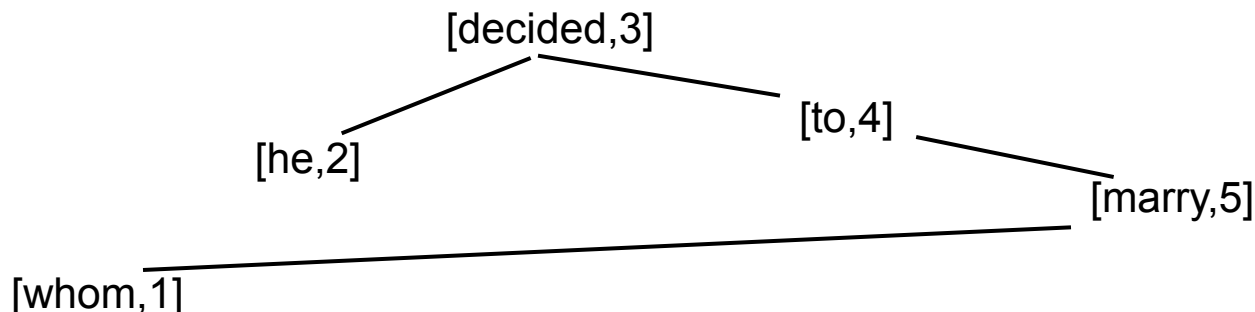


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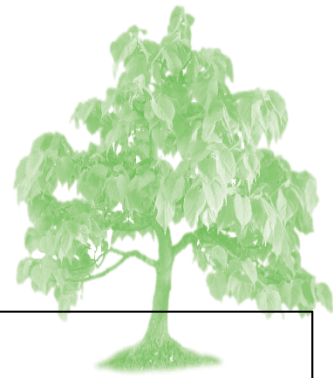
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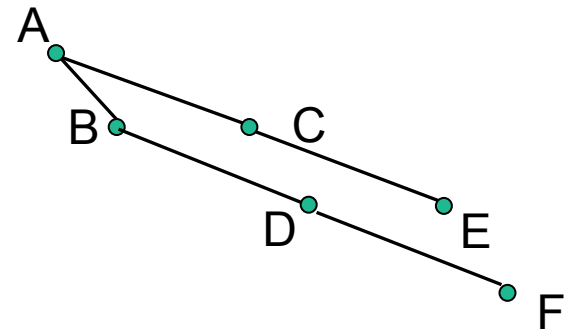
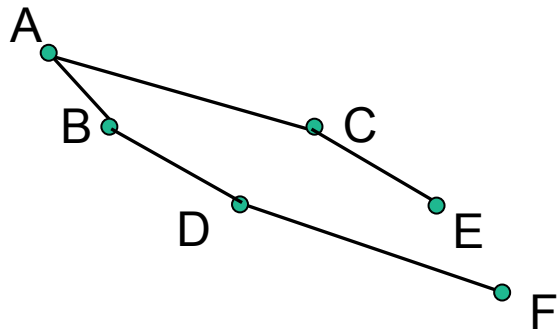
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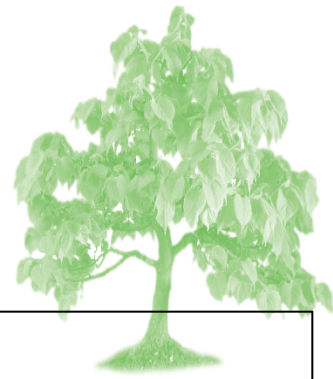
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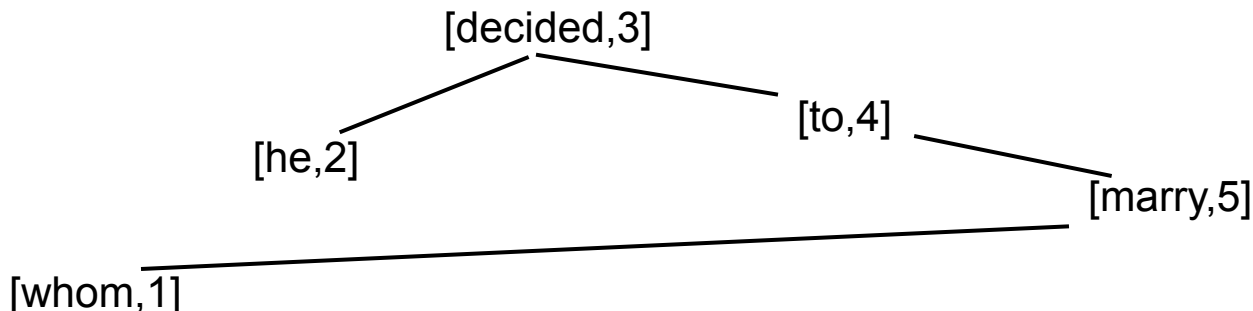
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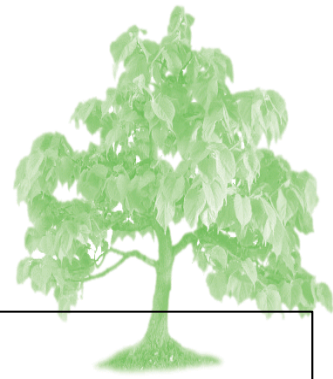
$dNh(u, T)$  ... **number of Gaps** in  $Cov(u, T)$

**Tree Gegree Degree**  $dNh(T) = \max \{ dNh(u, T) \mid u \in T \}$

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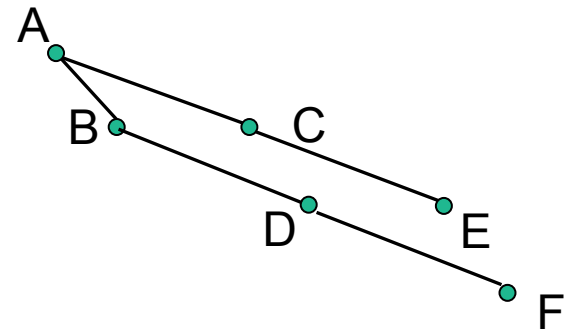
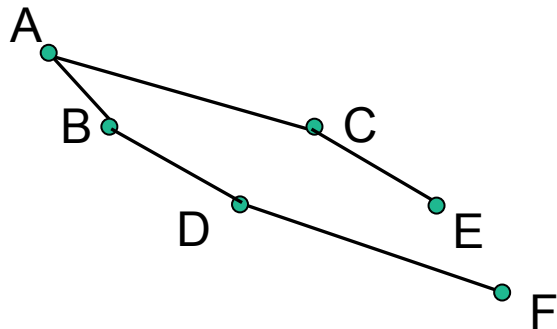
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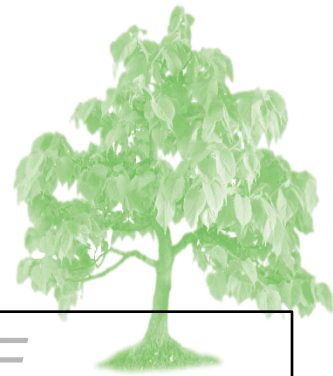
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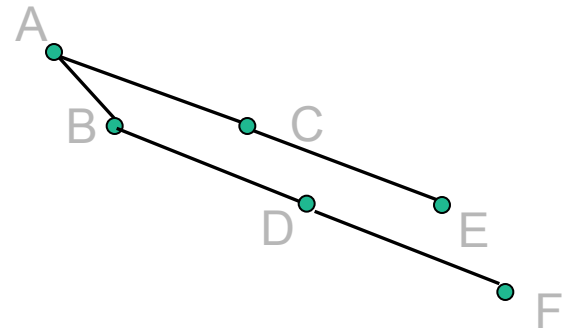
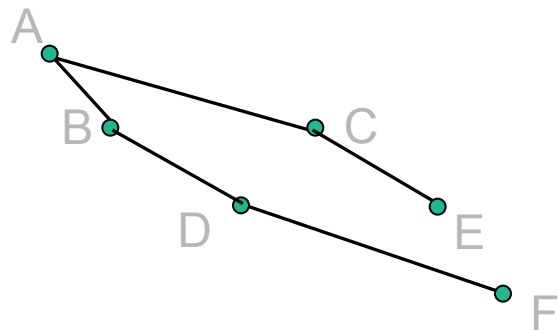
# Edge Degree



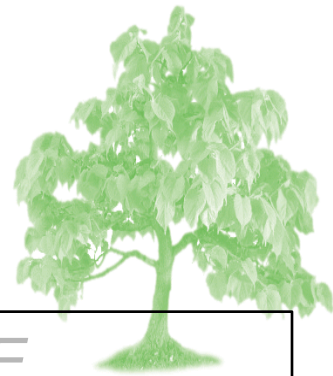
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**Degree of an edge**  $e \in E$ ,  $ed(e)$ , is the number of connected components  $c$  in  $T_e$  such that the root of  $c$  is not dominated by the head of  $e$ .

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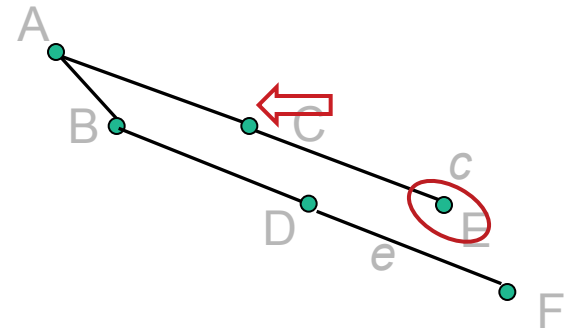
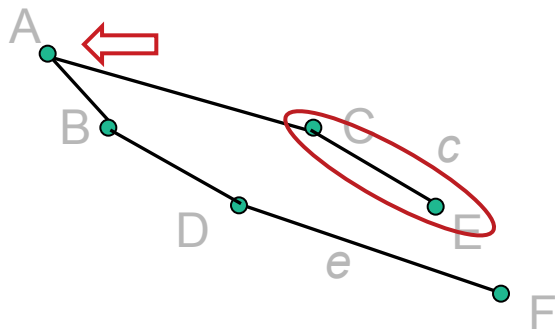
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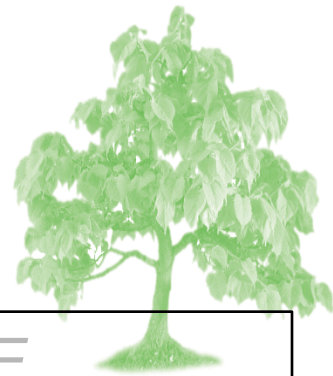
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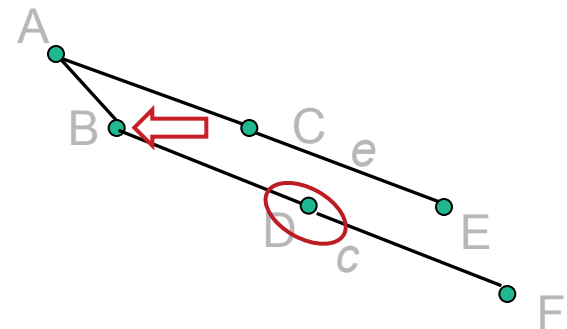
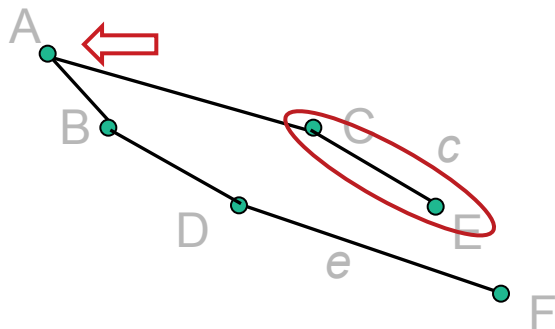
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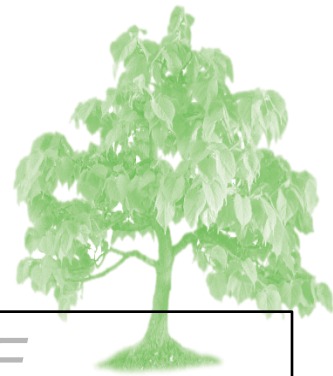
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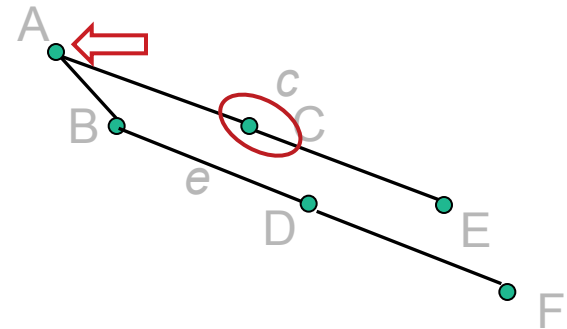
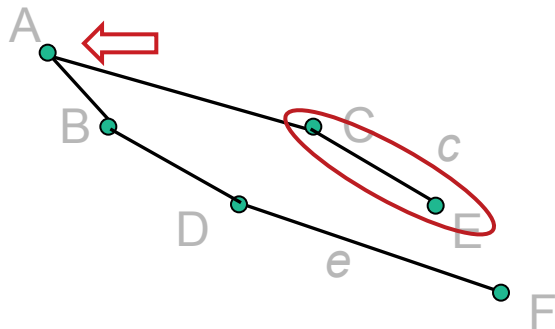
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property	DDT		PDT	
<i>all structures</i>	<i>n</i> = 4393		<i>n</i> = 73088	
gap degree 0	3732	84.95%	56168	76.85%
gap degree 1	654	14.89%	16608	22.72%
gap degree 2	7	0.16%	307	0.42%
gap degree 3	—	—	4	0.01%
gap degree 4	—	—	1	< 0.01%
edge degree 0	3732	84.95%	56168	76.85%
edge degree 1	584	13.29%	16585	22.69%
edge degree 2	58	1.32%	259	0.35%
edge degree 3	17	0.39%	63	0.09%
edge degree 4	2	0.05%	10	0.01%
edge degree 5	—	—	2	< 0.01%
edge degree 6	—	—	1	< 0.01%
projective	3732	84.95%	56168	76.85%
planar	3796	86.41%	60048	82.16%
well-nested	4388	99.89%	73010	99.89%
<i>non-projective structures only</i>	<i>n</i> = 661		<i>n</i> = 16920	
planar	64	9.68%	3880	22.93%
well-nested	656	99.24%	16842	99.54%





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# References

- Hajičová, E., Havelka, J., Sgall, P., Veselá, K., Zeman, D. (2004) Issues of Projectivity in the Prague Dependency Treebank. *PBML*, vol. 81
- Holan, T., Kuboň, V., Oliva, K., Plátek, M. (2000) On Complexity of Word Order. *Les grammaires de dépendance – Traitement automatique des langues*, vol. 41, no. 1, 273-300
- Kuhlmann, M., Nivre, J. (2006) Mildly Non-Projective Dependency Structures. In COLING/ACL Main Conference Poster Sessions, 507–514.
- Mel'čuk, I. (1988) *Dependency Syntax: Theory and Practice*. State University of New York Press, Albany
- Partee, B. H.; ter Meulen, A.; Wall, R. E. (1990) *Mathematical Methods in Linguistics*. Kluwer Academic Publishers
- Petkevič, V. (1995) A New Formal Specification of Underlying Structure. *Theoretical Linguistics*, vol. 21, No.1
- Sgall, P., Hajičová, E., Panevová, J. (1986) *The Meaning of the Sentence in Its Semantic and Pragmatic Aspects*. D. Reidel Publishing Company, Dordrecht/Academia, Prague
- Štěpánek, J. (2006) *Závislostní zachycení větné struktury v anotovaném syntaktickém korpusu*. PhD Thesis, MFF UK