

11. Cvičení z MA I. (12.12.2012)

1. Spočítejte následující limity:

$$(a) \quad \lim_{x \rightarrow 0} \frac{\cos x + 1}{\cos x - 1} \quad [-\infty] \quad (b) \quad \lim_{x \rightarrow +\infty} \sin \left(\pi \cdot \frac{4\sqrt{x} - 3\sqrt[3]{x}}{2\sqrt[4]{x^2 + 1}} \right) \quad [0]$$

$$(c) \quad \lim_{x \rightarrow -\infty} \operatorname{tg} \left(\frac{\pi}{4} x \cdot (\sqrt{x^2 + 1} - \sqrt{x^2 - 1}) \right) \quad [-1]$$

$$(d) \quad \lim_{x \rightarrow 16} \sqrt{\frac{4 - \sqrt{x}}{64 - \sqrt{x^3}}} \quad \left[\frac{1}{4\sqrt{3}} \right] \quad (e) \quad \lim_{x \rightarrow 0} e^{\frac{\sqrt[3]{1-x^2} - 1}{5x^2}} \quad [e^{-\frac{1}{15}}]$$

$$(g) \quad \lim_{x \rightarrow +\infty} \sqrt{\frac{\cos x + 2}{x^2 + x}} \quad [0]$$

$$(h) \quad \lim_{x \rightarrow -\infty} \frac{\operatorname{arccotg} x}{x} \quad [0] \quad (i) \quad \lim_{x \rightarrow 2} \left(\operatorname{arctg} \left(\frac{1}{2-x} \right) \right)^2 \quad \left[\frac{\pi^2}{4} \right]$$

2. Ukažte, že platí (a zapamatujte si !!!):

$$(a) \quad \lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1 \quad (b) \quad \lim_{x \rightarrow 0} \frac{\ln(x+1)}{x} = 1 \quad (c) \quad \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

3. Spočítejte následující limity:

$$(a) \quad \lim_{x \rightarrow 0} \frac{\operatorname{tg}(x^2)}{x \cdot \sin 3x} \quad \left[\frac{1}{3} \right] \quad (b) \quad \lim_{x \rightarrow 0} \frac{4^{3x} - 1}{\ln(x+1)} \quad [3 \ln 4]$$

$$(c) \quad \lim_{x \rightarrow 0} \frac{\sin x + 1}{\sin x} \quad [\text{neex.}] \quad (d) \quad \lim_{x \rightarrow +\infty} x \left(2^{\frac{1}{x}} - 1 \right) \quad [\ln 2]$$

$$(e) \quad \lim_{x \rightarrow 0} \frac{\ln(1+x^2)}{\ln(1-x^2)} \quad [-1] \quad (f) \quad \lim_{x \rightarrow 3} \frac{\arcsin(x-3)}{x^2 - 3x} \quad \left[\frac{1}{3} \right]$$

$$(g) \quad \lim_{x \rightarrow +\infty} \left(\frac{3x+2}{2x+3} \right)^{2x-1} \quad [+ \infty] \quad (h) \quad \lim_{x \rightarrow +\infty} \left(\frac{x^2+3}{x^2+7} \right)^x \quad [1]$$

$$(i) \quad \lim_{x \rightarrow 0} \frac{1-4^x}{\sin 2x} \quad [-\ln 2] \quad (j) \quad \lim_{x \rightarrow 0} \left(\frac{2^x + 8^x}{2} \right)^{\frac{1}{x}} \quad [4]$$

$$(k) \quad \lim_{x \rightarrow -\infty} \frac{\ln(1+e^x)}{x} \quad [0]$$

Dů. Spočítejte limity (1) a (2) a najděte $a, b \in \mathbb{R}$ tak, aby platilo (3):

$$(1) \quad \lim_{x \rightarrow \frac{\pi}{6}} \frac{2 \sin^2 x + \sin x - 1}{2 \sin^2 x - 3 \sin x + 1} \quad [-3]$$

$$(2) \quad \lim_{x \rightarrow 0} \left(\frac{1 + \operatorname{tg} x}{1 + \sin x} \right)^{\frac{1}{\sin^2 x}} \quad [1]$$

$$(3) \quad \lim_{x \rightarrow -\infty} (\sqrt{x^2 - x + 1} - ax - b) = 0 \quad [a = -1, b = \frac{1}{2}]$$