

# NPFL108 – Bayesian inference

## Types of priors

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# Outline

- Types of priors
  - Conjugate prior vs. Non-conjugate prior
  - Proper prior vs. improper prior
  - Informative prior vs. uninformative prior

# Posterior probability

- Bayes rule

$$P(Y|X) = \frac{P(X|Y)P(Y)}{\sum_X P(X|Y)P(Y)}$$

$$P(Y|X) \propto P(X|Y)P(Y)$$

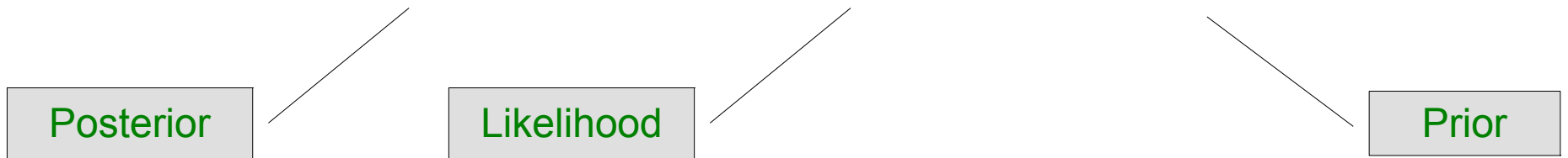
Posterior

Likelihood

Prior

# Conjugacy

$$P(Y|X) \propto P(X|Y) P(Y)$$



- We talk about conjugacy when the posterior has the same structure as the prior when combined with some likelihood
- This is convenient because a **posterior** can be **used** as **prior** when new data arrives.
- In Bayesian approach, processing data **one by one** (online) and in **batches** is **equivalent**.

# Conjugate priors #1

- Beta **is** conjugate with
  - Bernoulli and Binomial
- Dirichlet **is** conjugate with
  - Multinomial
- Normal **is** conjugate with
  - Normal when used for the mean and the variance is known
- Gamma and Inverse Gamma **is** conjugate with
  - Normal when used for the precision and the variance respectively when the mean is known

# Conjugate priors #2

- Normal and Gamma **is NOT** conjugate with

- Normal when used for the mean and the precision

$$p(\mu, \lambda | \mu_0, \lambda_0, a_0, b_0) = N(\mu | \mu_0, \lambda_0^{-1}) \text{Gam}(\lambda | a_0, b_0)$$

- Normal-Gamma (Gaussian-Gamma) **is** conjugate with

- Normal when used for the mean and the precision

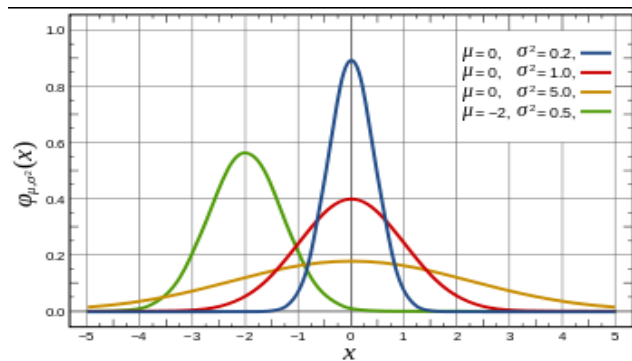
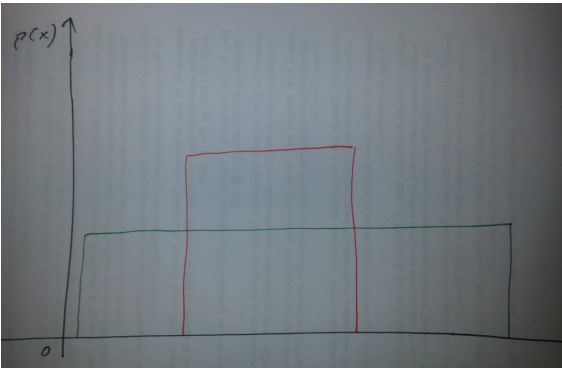
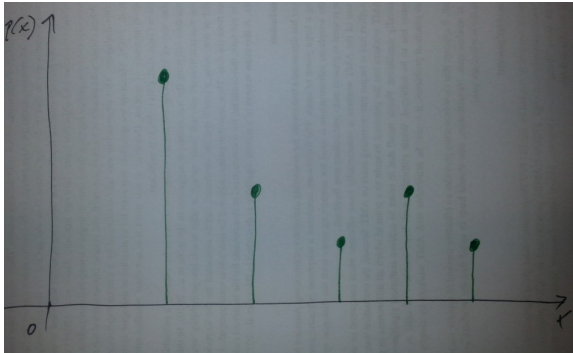
$$p(\mu, \lambda | \mu_0, \beta_0, a_0, b_0) = N(\mu | \mu_0, (\beta_0 \lambda)^{-1}) \text{Gam}(\lambda | a_0, b_0)$$

# Advantages of conjugate priors

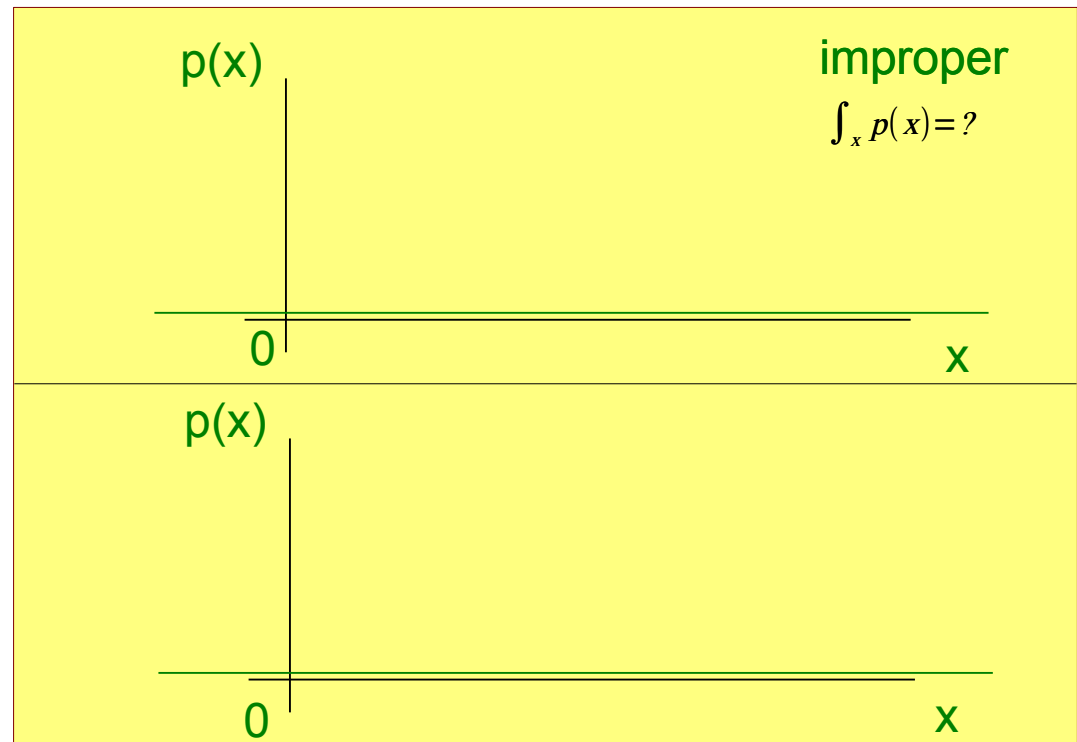
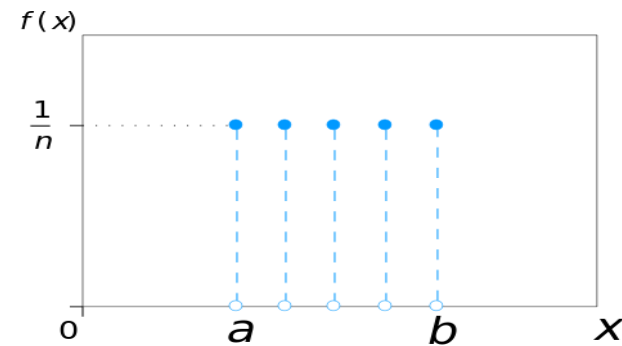
- This is convenient because a **posterior** can be **used** as **prior** when new data arrives.
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# Informative, non-informative, and improper priors

## informative



## non-informative





# Improper priors

- Improper priors are still useful
  - The posterior will not be of the form of the prior
  - but
  - The posterior will have a tractable form.
- Some times we just do not know what the prior should be!