NPFL103: Information Retrieval (11)

Latent semantic indexing

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Based on slides by Hinrich Schütze, University of Stuttgart.

Contents

Latent semantic indexing

Recall: Term-document matrix

	Anthony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
anthony	5.25	3.18	0.0	0.0	0.0	0.35
brutus	1.21	6.10	0.0	1.0	0.0	0.0
caesar	8.59	2.54	0.0	1.51	0.25	0.0
calpurnia	0.0	1.54	0.0	0.0	0.0	0.0
cleopatra	2.85	0.0	0.0	0.0	0.0	0.0
mercy	1.51	0.0	1.90	0.12	5.25	0.88
worser	1.37	0.0	0.11	4.15	0.25	1.95
...						

This matrix is the basis for computing the similarity between documents and queries.

Today: Can we transform this matrix, so that we get a better measure of similarity between documents and queries?

Latent semantic indexing: Overview

- We decompose the term-document matrix into a product of matrices.
- The particular decomposition: singular value decomposition (SVD).
- SVD: $C = U\Sigma V^T$ (where C = term-document matrix)
- We use SVD to compute a new, improved term-document matrix C'.
- We get better similarity values out of C' (compared to C).
- Using SVD for this purpose is called latent semantic indexing or LSI.

Example of $C = U\Sigma V^T$: The matrix C

C	d_1	d_2	d_3	d_4	d_5	d_6
ship	1	0	1	0	0	0
boat	0	1	0	0	0	0
ocean	1	1	0	0	0	0
wood	1	0	0	1	1	0
tree	0	0	0	1	0	1

- This is a standard term-document matrix.
- Actually, we use a non-weighted matrix here to simplify the example.

Example of $C = U\Sigma V^T$: The matrix U

U	1	2	3	4	5
ship	−0.44	−0.30	0.57	0.58	0.25
boat	−0.13	−0.33	−0.59	0.00	0.73
ocean	−0.48	−0.51	−0.37	0.00	−0.61
wood	−0.70	0.35	0.15	−0.58	0.16
tree	−0.26	0.65	−0.41	0.58	−0.09

- One row per term, one column per $\min(M, N)$ where M is the number of terms and N is the number of documents.
- This is an orthonormal matrix: (i) Row vectors have unit length. (ii) Any two distinct row vectors are orthogonal to each other.
- Think of the dimensions as "semantic" dimensions that capture distinct topics like politics, sports, economics. 2 = land/water
- Each number u_{ij} in the matrix indicates how strongly related term i is to the topic represented by semantic dimension j.

Example of $C = U\Sigma V^T$: The matrix Σ

Σ	1	2	3	4	5
1	2.16	0.00	0.00	0.00	0.00
2	0.00	1.59	0.00	0.00	0.00
3	0.00	0.00	1.28	0.00	0.00
4	0.00	0.00	0.00	1.00	0.00
5	0.00	0.00	0.00	0.00	0.39

- This is a square, diagonal matrix of dimensionality $\min(M, N) \times \min(M, N)$.
- The diagonal consists of the singular values of C.
- The magnitude of the singular value measures the importance of the corresponding semantic dimension.
- We'll make use of this by omitting unimportant dimensions.

Example of $C = U\Sigma V^T$: The matrix V^T

V^T	d_1	d_2	d_3	d_4	d_5	d_6
1	-0.75	-0.28	-0.20	-0.45	-0.33	-0.12
2	-0.29	-0.53	-0.19	0.63	0.22	0.41
3	0.28	-0.75	0.45	-0.20	0.12	-0.33
4	0.00	0.00	0.58	0.00	-0.58	0.58
5	-0.53	0.29	0.63	0.19	0.41	-0.22

- One column per document, one row per $\min(M, N)$ where M is the number of terms and N is the number of documents.
- This is an orthonormal matrix: (i) Column vectors have unit length. (ii) Any two distinct column vectors are orthogonal to each other.
- These are again the semantic dimensions from matrices U and Σ that capture distinct topics like politics, sports, economics.
- Each number v_{ij} in the matrix indicates how strongly related document i is to the topic represented by semantic dimension j.

Example of $C = U\Sigma V^T$: All four matrices

C	d_1	d_2	d_3	d_4	d_5	d_6
ship	1	0	1	0	0	0
boat	0	1	0	0	0	0
ocean	1	1	0	0	0	0
wood	1	0	0	1	1	0
tree	0	0	0	1	0	1

=

U	1	2	3	4	5
ship	−0.44	−0.30	0.57	0.58	0.25
boat	−0.13	−0.33	−0.59	0.00	0.73
ocean	−0.48	−0.51	−0.37	0.00	−0.61
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×

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2	0.00	1.59	0.00	0.00	0.00
3	0.00	0.00	1.28	0.00	0.00
4	0.00	0.00	0.00	1.00	0.00
5	0.00	0.00	0.00	0.00	0.39

×

V^T	d_1	d_2	d_3	d_4	d_5	d_6
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2	−0.29	−0.53	−0.19	0.63	0.22	0.41
3	0.28	−0.75	0.45	−0.20	0.12	−0.33
4	0.00	0.00	0.58	0.00	−0.58	0.58
5	−0.53	0.29	0.63	0.19	0.41	−0.22

LSI is decomposition of C into a representation of the terms, a representation of the documents and a representation of the importance of the "semantic" dimensions.

LSI: Summary

- We've decomposed the term-document matrix C into a product of three matrices: $U\Sigma V^T$.
- The term matrix U – consists of one (row) vector for each term
- The document matrix V^T – consists of one (column) vector for each document
- The singular value matrix Σ – diagonal matrix with singular values, reflecting importance of each dimension
- Next: Why are we doing this?

Dimensionality reduction

How we use the SVD in LSI

- Key property: Each singular value tells us how important its dimension is.
- By setting less important dimensions to zero, we keep the important information, but get rid of the "details".
- These details may
  - be noise – the reduced LSI is a better representation because it is less noisy.
  - make things dissimilar that should be similar – the reduced LSI is a better representation because it represents similarity better.
- Analogy for "fewer details is better"
  - Image of a blue flower
  - Image of a yellow flower
  - Omitting color makes is easier to see the similarity

Reducing the dimensionality to 2

U	1	2	3	4	5
ship	−0.44	−0.30	0.00	0.00	0.00
boat	−0.13	−0.33	0.00	0.00	0.00
ocean	−0.48	−0.51	0.00	0.00	0.00
wood	−0.70	0.35	0.00	0.00	0.00
tree	−0.26	0.65	0.00	0.00	0.00

Σ_2	1	2	3	4	5
1	2.16	0.00	0.00	0.00	0.00
2	0.00	1.59	0.00	0.00	0.00
3	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	0.00	0.00	0.00
5	0.00	0.00	0.00	0.00	0.00

V^T	d_1	d_2	d_3	d_4	d_5	d_6
1	−0.75	−0.28	−0.20	−0.45	−0.33	−0.12
2	−0.29	−0.53	−0.19	0.63	0.22	0.41
3	0.00	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	0.00	0.00	0.00	0.00
5	0.00	0.00	0.00	0.00	0.00	0.00

Actually, we only zero out singular values in Σ. This has the effect of setting the corresponding dimensions in U and V^T to zero when computing the product $C = U\Sigma V^T$.

Reducing the dimensionality to 2

C_2	d_1	d_2	d_3	d_4	d_5	d_6
ship	0.85	0.52	0.28	0.13	0.21	−0.08
boat	0.36	0.36	0.16	−0.20	−0.02	−0.18
ocean	1.01	0.72	0.36	−0.04	0.16	−0.21
wood	0.97	0.12	0.20	1.03	0.62	0.41
tree	0.12	−0.39	−0.08	0.90	0.41	0.49

=

U	1	2	3	4	5
ship	−0.44	−0.30	0.57	0.58	0.25
boat	−0.13	−0.33	−0.59	0.00	0.73
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$\times$

Σ_2	1	2	3	4	5
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5	0.00	0.00	0.00	0.00	0.00

$\times$

V^T	d_1	d_2	d_3	d_4	d_5	d_6
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4	0.00	0.00	0.58	0.00	−0.58	0.58
5	−0.53	0.29	0.63	0.19	0.41	−0.22

Example of $C = U\Sigma V^T$: All four matrices

C	d_1	d_2	d_3	d_4	d_5	d_6
ship	1	0	1	0	0	0
boat	0	1	0	0	0	0
ocean	1	1	0	0	0	0
wood	1	0	0	1	1	0
tree	0	0	0	1	0	1

=

U	1	2	3	4	5
ship	−0.44	−0.30	0.57	0.58	0.25
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×

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3	0.00	0.00	1.28	0.00	0.00
4	0.00	0.00	0.00	1.00	0.00
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×

V^T	d_1	d_2	d_3	d_4	d_5	d_6
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LSI is decomposition of C into a representation of the terms, a representation of the documents and a representation of the importance of the "semantic" dimensions.

Original matrix C vs. reduced $C_2 = U\Sigma_2 V^T$

C	d_1	d_2	d_3	d_4	d_5	d_6
ship	1	0	1	0	0	0
boat	0	1	0	0	0	0
ocean	1	1	0	0	0	0
wood	1	0	0	1	1	0
tree	0	0	0	1	0	1

C_2	d_1	d_2	d_3	d_4	d_5	d_6
ship	0.85	0.52	0.28	0.13	0.21	−0.08
boat	0.36	0.36	0.16	−0.20	−0.02	−0.18
ocean	1.01	0.72	0.36	−0.04	0.16	−0.21
wood	0.97	0.12	0.20	1.03	0.62	0.41
tree	0.12	−0.39	−0.08	0.90	0.41	0.49

We can view C_2 as a two-dimensional representation of the matrix C. We have performed a dimensionality reduction to two dimensions.

Why the reduced matrix C_2 is better than C

C	d_1	d_2	d_3	d_4	d_5	d_6
ship	1	0	1	0	0	0
boat	0	1	0	0	0	0
ocean	1	1	0	0	0	0
wood	1	0	0	1	1	0
tree	0	0	0	1	0	1

C_2	d_1	d_2	d_3	d_4	d_5	d_6
ship	0.85	0.52	0.28	0.13	0.21	−0.08
boat	0.36	0.36	0.16	−0.20	−0.02	−0.18
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wood	0.97	0.12	0.20	1.03	0.62	0.41
tree	0.12	−0.39	−0.08	0.90	0.41	0.49

Similarity of d_2 and d_3 in the original space: 0.

Similarity of d_2 and d_3 in the reduced space: $0.52*0.28+0.36*0.16+0.72*0.36+0.12*0.20+-0.39*-0.08 \approx 0.52$

"boat" and "ship" are semantically similar. The "reduced" similarity measure reflects this.

What property of the SVD reduction is responsible for improved similarity?

LSI in information retrieval

Why we use LSI in information retrieval

- LSI takes documents that are semantically similar (= talk about the same topics),
- but are not similar in the vector space (because they use different words) ...
- and re-represents them in a reduced vector space
- in which they have higher similarity.
- Thus, LSI addresses the problems of synonymy and semantic relatedness.
- Standard vector space: Synonyms contribute nothing to document similarity.
- Desired effect of LSI: Synonyms contribute strongly to document similarity.

How LSI addresses synonymy and semantic relatedness

- The dimensionality reduction forces us to omit a lot of “detail”.
- We have to map differents words (= different dimensions of the full space) to the same dimension in the reduced space.
- The “cost” of mapping synonyms to the same dimension is much less than the cost of collapsing unrelated words.
- SVD selects the “least costly” mapping (see below).
- Thus, it will map synonyms to the same dimension.
- But it will avoid doing that for unrelated words.

LSI: Comparison to other approaches

- Recap: Relevance feedback and query expansion are used to increase recall in information retrieval – if query and documents have no terms in common. (or, more commonly, too few terms in common for a high similarity score)
- LSI increases recall and hurts precision.
- Thus, it addresses the same problems as (pseudo) relevance feedback and query expansion ...
- ...and it has the same problems.

Implementation

- Compute SVD of term-document matrix
- Reduce the space and compute reduced document representations
- Map the query into the reduced space $\vec{q}_k = \Sigma_k^{-1} U_k^T \vec{q}$.
- This follows from: $C_k = U\Sigma_k V^T \Rightarrow \Sigma_k^{-1} U^T C = V_k^T$
- Compute similarity of q_k with all reduced documents in V_k.
- Output ranked list of documents as usual
- Exercise: What is the fundamental problem with this approach?

Optimality

- SVD is optimal in the following sense.
- Keeping the k largest singular values and setting all others to zero gives you the optimal approximation of the original matrix C. Eckart-Young theorem
- Optimal: no other matrix of the same rank (= with the same underlying dimensionality) approximates C better.
- Measure of approximation is Frobenius norm: $||C||_F = \sqrt{\sum_i \sum_j c_{ij}^2}$
- So LSI uses the "best possible" matrix.
- There is only one best possible matrix – unique solution (modulo signs).

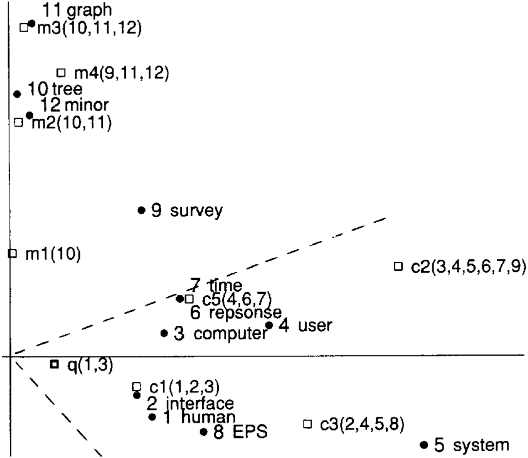
Data for graphical illustration of LSI

c_1	Human machine interface for lab abc computer applications
c_2	A survey of user opinion of computer system response time
c_3	The EPS user interface management system
c_4	System and human system engineering testing of EPS
c_5	Relation of user perceived response time to error measurement
m_1	The generation of random binary unordered trees
m_2	The intersection graph of paths in trees
m_3	Graph minors IV Widths of trees and well quasi ordering
m_4	Graph minors A survey

The matrix C

	c1	c2	c3	c4	c5	m1	m2	m3	m4
human	1	0	0	1	0	0	0	0	0
interface	1	0	1	0	0	0	0	0	0
computer	1	1	0	0	0	0	0	0	0
user	0	1	1	0	1	0	0	0	0
system	0	1	1	2	0	0	0	0	0
response	0	1	0	0	1	0	0	0	0
time	0	1	0	0	1	0	0	0	0
EPS	0	0	1	1	0	0	0	0	0
survey	0	1	0	0	0	0	0	0	1
trees	0	0	0	0	0	1	1	1	0
graph	0	0	0	0	0	0	1	1	1
minors	0	0	0	0	0	0	0	1	1

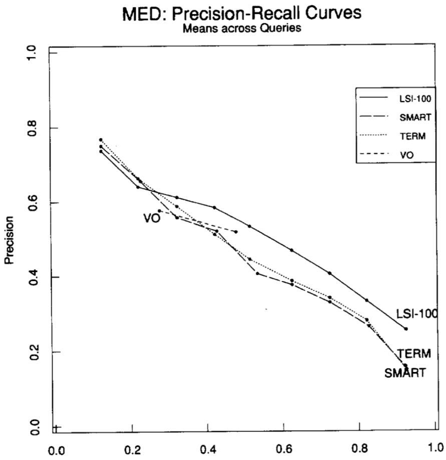
Graphical illustration of LSI: Plot of C_2



2-dimensional plot of C_2 (scaled dimensions). Circles = terms. Open squares = documents (component terms in parentheses). q = query "human computer interaction".

The dotted cone represents the region whose points are within a cosine of 0.9 from q . All documents about human-computer documents (c1-c5) are near q, even c3/c5 although they share no terms. None of the graph theory documents (m1-m4) are near q.

LSI performs better than vector space on MED collection



- LSI-100 = LSI reduced to 100 dimensions
- SMART = SMART implementation of vector space model

LSI as soft clustering

Example of $C = U\Sigma V^T$: All four matrices

C	d_1	d_2	d_3	d_4	d_5	d_6
ship	1	0	1	0	0	0
boat	0	1	0	0	0	0
ocean	1	1	0	0	0	0
wood	1	0	0	1	1	0
tree	0	0	0	1	0	1

=

U	1	2	3	4	5
ship	−0.44	−0.30	0.57	0.58	0.25
boat	−0.13	−0.33	−0.59	0.00	0.73
ocean	−0.48	−0.51	−0.37	0.00	−0.61
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tree	−0.26	0.65	−0.41	0.58	−0.09

×

Σ	1	2	3	4	5
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3	0.00	0.00	1.28	0.00	0.00
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V^T	d_1	d_2	d_3	d_4	d_5	d_6
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4	0.00	0.00	0.58	0.00	−0.58	0.58
5	−0.53	0.29	0.63	0.19	0.41	−0.22

LSI is decomposition of C into a representation of the terms, a representation of the documents and a representation of the importance of the “semantic” dimensions.

Why LSI can be viewed as soft clustering

- Each of the *k* dimensions of the reduced space is one cluster.
- If the value of the LSI representation of document *d* on dimension *k* is *x*, then *x* is the soft membership of *d* in topic *k*.
- This soft membership can be positive or negative.
- Example: Dimension 2 in our SVD decomposition
  - This dimension/cluster corresponds to the water/earth dichotomy.
  - "ship", "boat", "ocean" have negative values.
  - "wood", "tree" have positive values.
  - d_1, d_2, d_3 have negative values (most of their terms are water terms).
  - d_4, d_5, d_6 have positive values (all of their terms are earth terms).