Introduction to Natural Language Processing I [Statistické metody zpracování přirozených jazyků I] (NPFL067)

http://ufal.mff.cuni.cz/courses/npfl067

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Word Classes:
Programming Tips & Tricks

The Algorithm (review)

- Define merge(r,k,l) = (r',C') such that
  - **C' = C - {k,l} ∪ {m (a new class)}**
  - **r'(w) = r(w) except for k,l member words for which it is m.**
- 1. Start with each word in its own class (C = V), r = id.
- 2. Merge two classes k,l into one, m, such that

$$(k,l) = \text{argmax}_{k,,l}\ I_{merge(r,k,l)}(D,E).$$

- 3. Set new (r,C) = merge(r,k,l).
- 4. Repeat 2 and 3 until |C| reaches a predetermined size.

Complexity Issues

- Still too complex:
  - $|V|$ iterations of the steps 2 and 3.
  - $|V|^2$ steps to maximize $\text{argmax}_{k,l}$ (selecting k,l freely from $|C|$, which is in the order of $|V|^2$)
  - $|V|^2$ steps to compute I(D,E) (sum within sum, all classes, also: includes log)
  - $\Rightarrow$ total: $|V|^5$
  - i.e., for $|V| = 100$, about 10^{10} steps; ~ several hours!
  - but $|V| \sim 50{,}000$ or more

Trick #1: Recomputing The MI the Smart Way: Subtracting...

- Bigram count table:

l \ r	c_1	c_2	c_3	c_4
c_1	10	2	0	1
c_2	0	0	5	2
c_3	0	2	0	3
c_4	2	3	0	0

- Test-merging c_2 and c_4: recompute only rows/cols 2 & 4:
  - subtract column/row (2 & 4) from the MI sum (intersect.!)
  - add sums of merged counts (row & column)

...and Adding

- Add the merged counts:

l \ r	c_1	c_2'	c_3
c_1	10	3	0
c_2'	2	5	5
c_3	0	5	0

- Be careful at intersections:
  - (don't forget to add this:)

	c_2	c_3	c_4
c_2	0	5	2
c_3	2	0	3
c_4	3	0	0

Trick #2: Precompute the Counts-to-be-Subtracted

- Summing loop goes through i,j
- ...but the single row/column sums do not depend on the (resulting sums after the) merge
- $\Rightarrow$ can be precomputed
  - **only 2k logs to compute at each algorithm iteration, instead of k^2**
- Then for each "merge-to-be" compute only add-on sums, plus "intersection adjustment"

Formulas for Tricks #1 and #2

- Let's have k classes at a certain iteration. Define:

$$q_k(l,r) = p_k(l,r) \log(p_k(l,r) / (p_{kl}(l)\, p_{kr}(r)))$$

now the same, but using counts:

$$q_k(l,r) = c_k(l,r)/N \log(N\, c_k(l,r)/(c_{kl}(l)\, c_{kr}(r)))$$

- Define further (row+column i sum):

intersection adjustment

precomputed

$$s_k(a) = \Sigma_{l=1..k} q_k(l,a) + \Sigma_{r=1..k} q_k(a,r) - q_k(a,a)$$

- Then, the subtraction part of Trick #1 amounts to

$$sub_k(a,b) = s_k(a) + s_k(b) - q_k(a,b) - q_k(b,a)$$

remaining intersect. adj.

Formulas - cont.

- After-merge add-on:

  $add_k(a,b) = \sum_{l=1..k, l\neq a,b} q_k(l,a+b) + \sum_{r=1..k, r\neq a,b} q_k(a+b,r) + q_k(a+b,a+b)$
- What is it <u>a+b</u>? Answer: the <u>new (merged) class</u>.
- Hint: use the definition of q_k as a "macro", and then

  $p_k(a+b,r) = p_k(a,r) + p_k(b,r)$ (same for other sums, equivalent)
- The above sums cannot be precomputed
- After-merge Mutual Information (I_k is the "old" MI, kept from previous iteration of the algorithm):

  $I_k(a,b)$ (MI after merge of cl. a,b) $= I_k - sub_k(a,b) + add_k(a,b)$

Trick #3: Ignore Zero Counts

- Many bigrams are 0
  - (see the paper: Canadian Hansards, < .1 % of bigrams are non-zero)
- Create linked lists of non-zero counts in columns and rows (similar effect: use perl's hashes)
- Update links after merge (after step 3)

Trick #4: Use Updated Loss of MI

- We are now down to $|V|^4$: $|V|$ merges, each merge takes $|V|^2$ "test-merges", each test-merge involves order-of-$|V|$ operations ($add_k(i,j)$ term, foil #8)
- *Observation*: many numbers (s_k, q_k) needed to compute the mutual information loss due to a merge of i+j ***do not change***: namely, those which are not in the vicinity of neither i nor j.
- *Idea*: keep the MI loss matrix for all pairs of classes, and (after a merge) update only those cells which have been influenced by the merge.

Formulas for Trick #4 (s_{k-1},L_{k-1})

- Keep a matrix of "losses" $L_k(d,e)$.[1]
- Init: $L_k(d,e) = sub_k(d,e) - add_k(d,e)$ [then $I_k(d,e) = I_k - L_k(d,e)$]
- Suppose a,b are now the two classes merged into a:
- Update (k-1: index used for the *next* iteration; $i,j \neq a,b$):
  - $s_{k-1}(i) = s_k(i) - q_k(i,a) - q_k(a,i) - q_k(i,b) - q_k(b,i) + q_{k-1}(a,i) + q_{k-1}(i,a)$
  - [2]$L_{k-1}(i,j) = L_k(i,j) - s_k(i) + s_{k-1}(i) - s_k(j) + s_{k-1}(j) +$

    $+ q_k(i+j,a) + q_k(a,i+j) + q_k(i+j,b) + q_k(b,i+j) -$

    $- q_{k-1}(i+j,a) - q_{k-1}(a,i+j)$ *[NB: may substitute even for s_k , s_{k-1}]*

NB [1] L_k is symmetrical $L_k(d,e) = L_k(e,d)$ (q_k is something different!)

[2]The update formula $L_{k-1}(l,m)$ is wrong in the Brown et. al paper

Completing Trick #4

- $s_{k-1}(a)$ must be computed using the "Init" sum.
- $L_{k-1}(a,i) = L_{k-1}(i,a)$ must be computed in a similar way, for all $i \neq a,b$.
- $s_{k-1}(b)$, $L_{k-1}(b,i)$, $L_{k-1}(i,b)$ are not needed anymore (keep track of such data, i.e. mark every class already merged into some other class and do not use it anymore).
- Keep track of the minimal loss during the $L_k(i,j)$ update process (so that the next merge to be taken is obvious immediately after finishing the update step).

Efficient Implementation

- Data Structures: (N - # of bigrams in data [fixed])
  - Hist(k) history of merges
    - **Hist(k) = (a,b) merged when the remaining number of classes was k**
  - $c_k(i,j)$ bigram class counts [updated]
  - $c_{kl}(i)$, $c_{kr}(i)$ unigram (marginal) counts [updated]
  - $L_k(a,b)$ table of losses; upper-right trianlge [updated]
  - $s_k(a)$ "subtraction" subterms [optionally updated]
  - $q_k(i,j)$ subterms involving a log [opt. updated]
    - **The optionally updated data structures will give linear improvement only in the subsequent steps, but at least $s_k(i)$ is necessary in the initialization phase (1st iteration)**

Implementation: the Initialization Phase

- 1 Read data in, init counts $c_k(l,r)$; then $\forall l,r,a,b;\ a < b$:
- 2 Init unigram counts:

$$c_{kl}(l) = \Sigma_{r=1..k} c_k(l,r), \qquad c_{kr}(r) = \Sigma_{l=1..k} c_k(l,r)$$

  – complicated? remember, must take care of start & end of data!
- 3 Init $q_k(l,r)$: use the 2^{nd} formula (count-based) on foil 7,

$$q_k(l,r) = c_k(l,r)/N \log(N\, c_k(l,r)/(c_{kl}(l)\, c_{kr}(r)))$$

- 4 Init $s_k(a) = \Sigma_{l=1..k} q_k(l,a) + \Sigma_{r=1..k} q_k(a,r) - q_k(a,a)$
- 5 Init $L_k(a,b) = s_k(a)+s_k(b)-q_k(a,b)-q_k(b,a)-q_k(a+b,a+b)+$

$$- \Sigma_{l=1..k,l\neq a,b} q_k(l,a+b) - \Sigma_{r=1..k,r\neq a,b} q_k(a+b,r)$$

Implementation: Select & Update

- 6 Select the best pair (a,b) to merge into a (watch the candidates when computing $L_k(a,b)$); save to Hist(k)
- 7 Optionally, update $q_k(i,j)$ for all $i,j \neq b$, get $q_{k-1}(i,j)$
  - remember those $q_k(i,j)$ values needed for the updates below
- 8 Optionally, update $s_k(i)$ for all $i \neq b$, to get $s_{k-1}(i)$
  - again, remember the $s_k(i)$ values for the "loss table" update
- 9 Update the loss table, $L_k(i,j)$, to $L_{k-1}(i,j)$, using the tabulated q_k, q_{k-1}, s_k and s_{k-1} values, or compute the needed $q_k(i,j)$ and $q_{k-1}(i,j)$ values dynamically from the counts: $c_k(i+j,b) = c_k(i,b) + c_k(j,b)$; $c_{k-1}(a,i) = c_k(a+b,i)$

Towards the Next Iteration

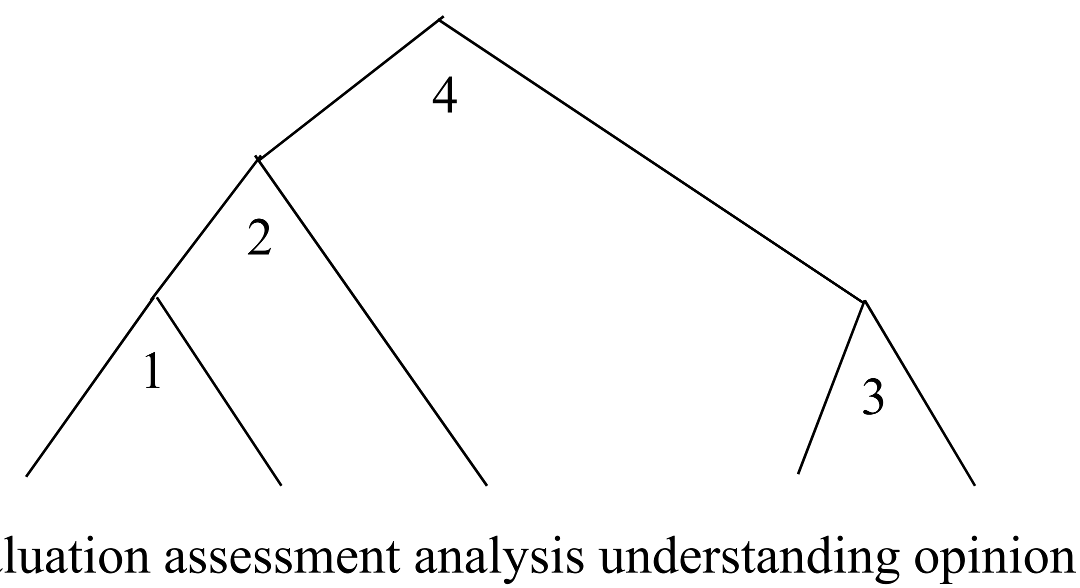
- 10 During the $L_k(i,j)$ update, keep track of the minimal loss of MI, and the two classes which caused it.
- 11 Remember such best merge in Hist(k).
- 12 Get rid of all s_k, q_k, L_k values.
- 13 Set k = k -1; stop if k == 1.
- 14 Start the next iteration
  - either by the optional updates (steps 7 and 8), or
  - directly updating $L_k(i,j)$ again (step 9).

Moving Words Around

- Improving Mutual Information
  - take a word from one class, move it to another (i.e., two classes change: the moved-from and the moved-to), compute $I_{new}(D,E)$; keep change permanent if
    $$I_{new}(D,E) > I(D,E)$$
  - keep moving words until no move improves I(D,E)
- Do it at every iteration, or at every <u>m</u> iterations
- Use similar "smart" methods as for merging

Using the Hierarchy

- Natural Form of Classes
  - follows from the sequence of merges:



Numbering the Classes (within the Hierarchy)

- Binary branching
- Assign 0/1 to the left/right branch at every node: