Introduction to Natural Language Processing I [Statistické metody zpracování přirozených jazyků I] (NPFL067)

http://ufal.mff.cuni.cz/courses/npfl067

prof. RNDr. Jan Hajič, Dr. / doc. RNDr. Pavel Pecina, Ph.D.
ÚFAL MFF UK
{hajic,pecina}@ufal.mff.cuni.cz
http://ufal.mff.cuni.cz/jan-hajic
http://ufal.mff.cuni.cz/~pecina/index.html

Probability

Experiments & Sample Spaces

- Experiment, process, test, ...
- Set of possible basic outcomes: sample space Ω
  - coin toss (Ω = {head,tail}), die (Ω = {1..6})
  - yes/no opinion poll, quality test (bad/good) (Ω = {0,1})
  - lottery ($|\Omega| \cong 10^7 .. 10^{12}$)
  - # of traffic accidents somewhere per year (Ω = N)
  - spelling errors ($\Omega = Z^*$), where Z is an alphabet, and Z^* is a set of possible strings over such and alphabet
  - missing word ($|\Omega| \cong$ vocabulary size)

Events

- Event A is a set of basic outcomes
- Usually $A \subset \Omega$, and all $A \in 2^{\Omega}$ (the event space)
  - Ω is then the certain event, $\varnothing$ is the impossible event
- Example:
  - experiment: three times coin toss
    - **Ω = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}**
  - count cases with exactly two tails: then
    - **A = {HTT, THT, TTH}**
  - all heads:
    - **A = {HHH}**

Probability

- Repeat experiment many times, record how many times a given event A occurred ("count" c_1).
- Do this whole series many times; remember all c_is.
- Observation: if repeated really many times, the ratios of $\mathbf{c_i/T_i}$ (where T_i is the number of experiments run in the *i-th* series) are close to some (unknown but) **constant** value.
- Call this constant a ***probability of A***. Notation: **p(A)**

Estimating probability

- Remember: ... close to an *unknown* constant.
- We can only estimate it:
  - from a single series (typical case, as mostly the outcome of a series is given to us and we cannot repeat the experiment), set

$$p(A) = c_1/T_1.$$

  - otherwise, take the weighted average of all c_i/T_i (or, if the data allows, simply look at the set of series as if it is a single long series).
- This is the **best** estimate.

Example

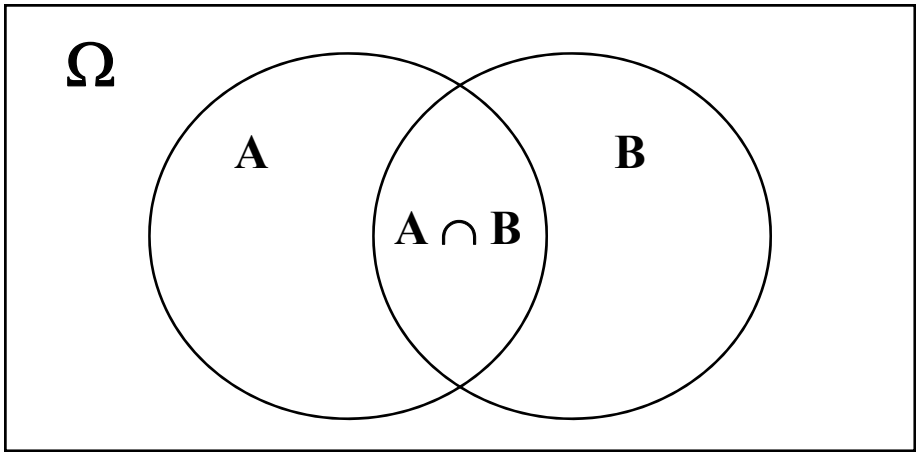
- Recall our example:
  - experiment: three times coin toss
    - **Ω = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}**
  - count cases with exactly two tails: A = **{HTT, THT, TTH}**
- Run an experiment 1000 times (i.e. 3000 tosses)
- Counted: 386 cases with two tails (**HTT**, **THT**, or **TTH**)
- estimate: p(A) = 386 / 1000 = .386
- Run again: 373, 399, 382, 355, 372, 406, 359
  - p(A) = .379 (weighted average) or simply 3032 / 8000
- *Uniform* distribution assumption: p(A) = 3/8 = .375

Basic Properties

- Basic properties:
  - $p: 2^{\Omega} \rightarrow [0,1]$
  - $p(\Omega) = 1$
  - Disjoint events: $p(\cup A_i) = \sum_i p(A_i)$
- [NB: *axiomatic definition* of probability: take the above three conditions as axioms]
- Immediate consequences:
  - $p(\varnothing) = 0, \quad p(\bar{A}) = 1 - p(A), \quad A \subseteq B \Rightarrow p(A) \leq p(B)$
  - $\sum_{a \in \Omega} p(a) = 1$

Joint and Conditional Probability

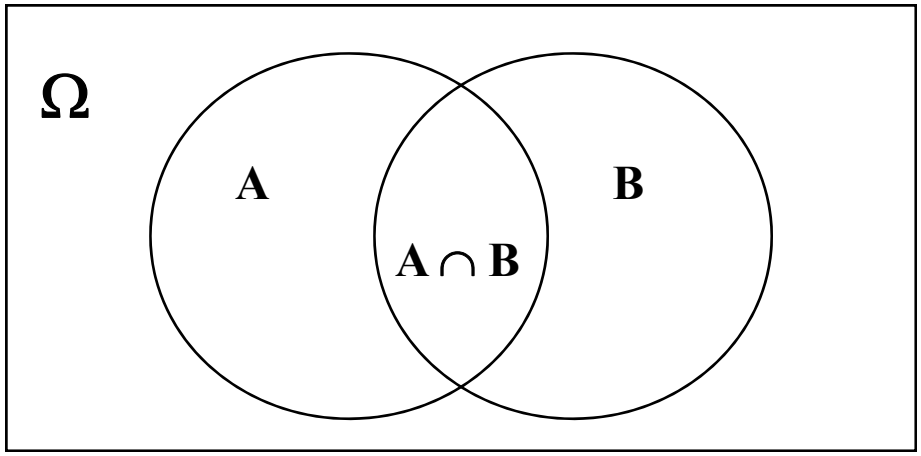
- $p(A,B) = p(A \cap B)$
- $p(A|B) = p(A,B) / p(B)$
  - Estimating form counts:
    - **$p(A|B) = p(A,B) / p(B) = (c(A \cap B) / T) / (c(B) / T) =$**
      $= c(A \cap B) / c(B)$



Bayes Rule

- $p(A,B) = p(B,A)$ since $p(A \cap B) = p(B \cap A)$
  - therefore: $p(A|B)\ p(B) = p(B|A)\ p(A)$, and therefore

$$p(A|B) = p(B|A)\ p(A) / p(B)$$ **!**



Independence

- Can we compute p(A,B) from p(A) and p(B)?
- Recall from previous foil:

$$p(A|B) = p(B|A)\ p(A) / p(B)$$

$$p(A|B)\ p(B) = p(B|A)\ p(A)$$

$$p(A,B) = p(B|A)\ p(A)$$

... we're almost there: how p(B|A) relates to p(B)?

  - p(B|A) = P(B) iff A and B are **independent**

- Example: two coin tosses, weather today and weather on March 4th 1789;
- Any two events for which p(B|A) = P(B)!

Chain Rule

$$p(A_1, A_2, A_3, A_4, ..., A_n) = \quad !$$

$$p(A_1|A_2,A_3,A_4,...,A_n) \times p(A_2|A_3,A_4,...,A_n) \times$$
$$\times p(A_3|A_4,...,A_n) \times \ ... \ p(A_{n-1}|A_n) \times p(A_n)$$

- this is a direct consequence of the Bayes rule.

The Golden Rule
(of Classic Statistical NLP)

- Interested in an event A given B (when it is not easy or practical or desirable to estimate p(A|B)):
- take Bayes rule, max over all As:
- $\text{argmax}_A\ p(A|B) = \text{argmax}_A\ p(B|A)\ .\ p(A)\ /\ \cancel{p(B)} =$

$$\text{argmax}_A\ p(B|A)\ p(A)$$!

- ... as p(B) is constant when changing As

Random Variable

- is a function $X: \Omega \rightarrow Q$
  - in general: $Q = R^n$, typically R
  - easier to handle real numbers than real-world events
- random variable is *discrete* if Q is countable (i.e. also if finite)
- Example: *die*: natural "numbering" [1,6], *coin*: {0,1}
- Probability distribution:
  - $p_X(x) = p(X=x) =_{df} p(A_x)$ where $A_x = \{a \in \Omega : X(a) = x\}$
  - often just p(x) if it is clear from context what X is

Expectation
Joint and Conditional Distributions

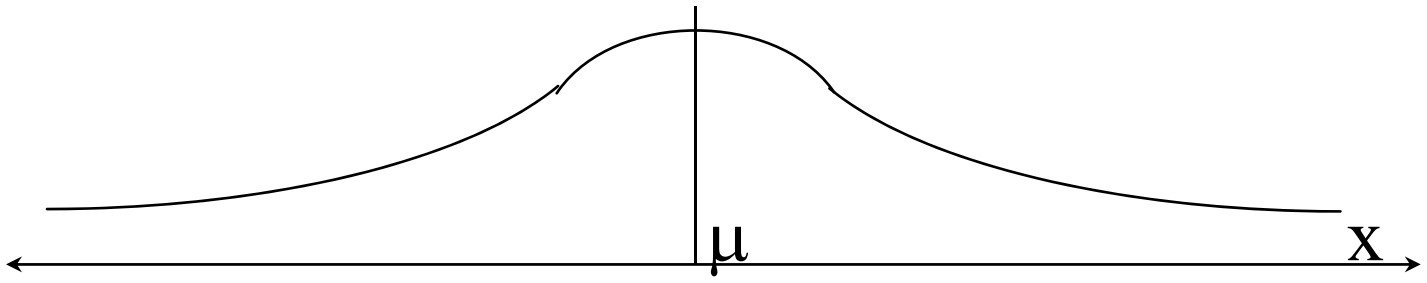
- is a mean of a random variable (weighted average)
  - $E(X) = \sum_{x \in X(\Omega)} x \,.\, p_X(x)$
- Example: one six-sided die: 3.5, two dice (sum) 7
- Joint and Conditional distribution rules:
  - analogous to probability of events
- Bayes: $p_{X|Y}(x,y) =_{\text{notation}} p_{XY}(x|y) =_{\text{even simpler notation}}$
  p(x|y) = p(y|x) . p(x) / p(y)
- Chain rule: **p(w,x,y,z) = p(z).p(y|z).p(x|y,z).p(w|x,y,z)**

Standard distributions

- Binomial (discrete)
  - outcome: 0 or 1 (thus: *bi*nomial)
  - make *n* trials
  - interested in the (probability of) number of successes *r*
- Must be careful: it's not uniform!
- $p_b(r|n) = \binom{n}{r} / 2^n$ (for equally likely outcome)
- $\binom{n}{r}$ counts how many possibilities there are for choosing r objects out of n; = n! / ((n-r)! r!)

Continuous Distributions

- The normal distribution ("Gaussian")
- $p_{norm}(x|\mu,\sigma) = e^{-(x-\mu)2/(2\sigma2)}/\sigma\sqrt{2\pi}$
- where:
  - μ is the mean (x-coordinate of the peak) (0)
  - σ is the standard deviation (1)



- other: hyperbolic, t