NTIN066 - solutions 4

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1. Find a sequence of operations in $(a, 2a-1)$-trees in which almost every operations perform splits/joins up to the root.
Could you find similar sequence for $(a, 2a)$-trees?

For example, insert keys 1 to 14 into (2, 3)-tree and than alternately insert and delete number 15.

2. (2,4)-trees versus red-black trees
Red-black tree is a BST that is balanced by maintaining following invariants:

- Every node is either *black* or *red*
- Root and null pointers are *black*
- There are *no consecutive red nodes* (i.e. red node must have black parent).
- All root-leaf paths contain the same number of black nodes.

It can be useful to consider edge color instead: The color of an edge is the color of the lower end-point. I.e. parent-edge of a red node is red, parent-edge of a black node is black.

- Show, that every (2,4)-trees is in fact a red-black tree. That is, find a simple mapping that transforms given (2,4)-tree into a valid red-black tree. Note that we are not really looking for an algorithm but for mapping in the mathematical sense.
- What about the other way around? Can we turn any red-black tree into a (2,4)-tree?
- Left-leaning red-black tree (LLRBT) maintains additional invariant: If the node has a single red son, then it is the left son. Show, that there is a 1-1 correspondence between (2,4)-trees and left leaning red-black trees. That is, find a mapping between (2,4)-trees and LLRBT that assigns a unique LLRBT to any (2,4)-tree (or a unique (2,4)-tree to any LLRBT, which is the same thing).

See the example here: https://en.wikipedia.org/wiki/Red–black_tree.

3. Compare (a,b)-trees, 2-3-trees, 2-3-4-trees, B-trees, B$^+$-trees BB(α) trees, RB-trees, LLRB-trees, and AA-trees.

2-3-4-trees are (2,4)-trees; B-trees are ($\lceil b/2 \rceil, b$)-trees, B$^+$-trees store all keys on the last level and let the internal nodes contain copies (typically minima of subtrees); BB(α) trees are lazily balanced trees, where $size(children) \geq \alpha \cdot size(parent)$ *and* $\alpha < 0.5$*; RB-trees are Red-Black trees; LLRB-trees were defined in the previous exercise; AA-trees are Red-Black-trees in which each node has at most one red child and it is the right child.*

4. Top-down (a,b)-trees
Explain pre-emptive splitting in insert and delete operations.

See the lecture notes.