



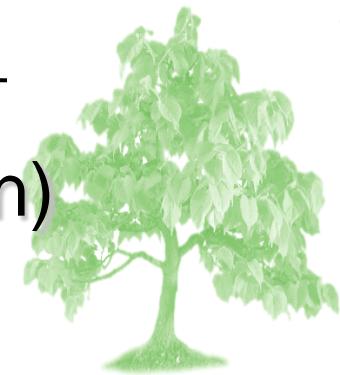
# Prague Dependency Treebank: Introduction – (Non-)Projectivity

Markéta Lopatková, Jiří Mírovský

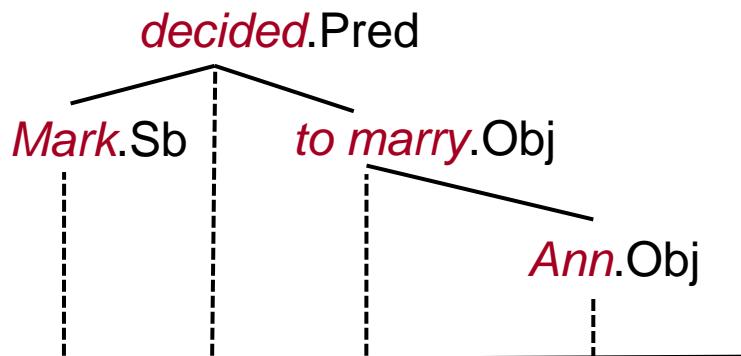
Institute of Formal and Applied Linguistics, MFF UK  
[lopatkova@ufal.mff.cuni.cz](mailto:lopatkova@ufal.mff.cuni.cz)

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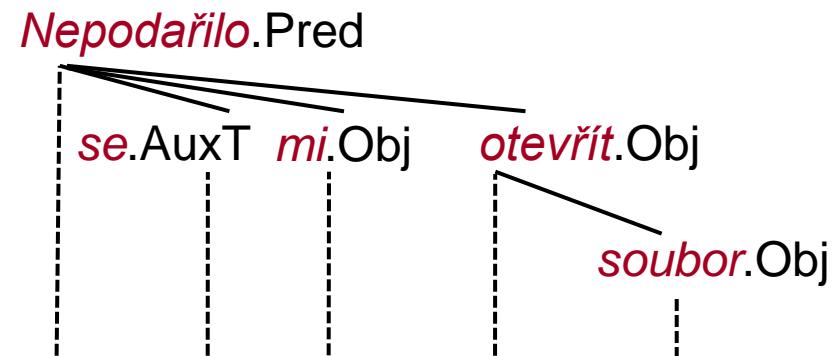
# Projectivity and non-projectivity (definition)



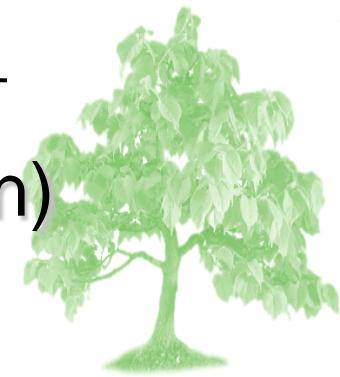
Mark decided to marry Ann.



*Nepodařilo se mi otevřít soubor.*



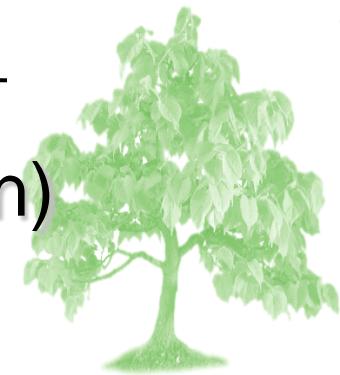
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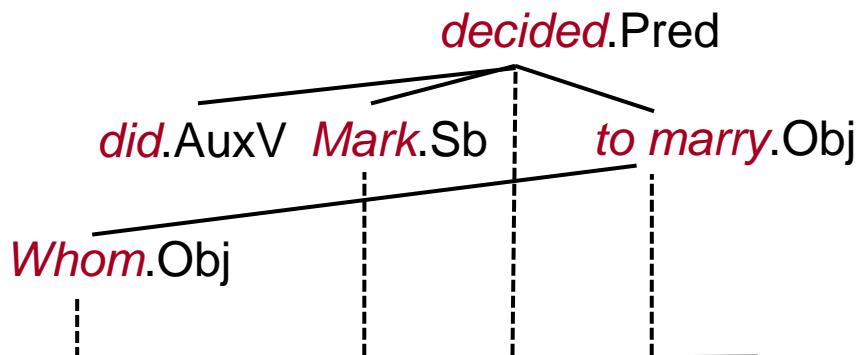
Whom did Mark decided to marry?

*Soubor se mi nepodařilo otevřít.* (Oliva)

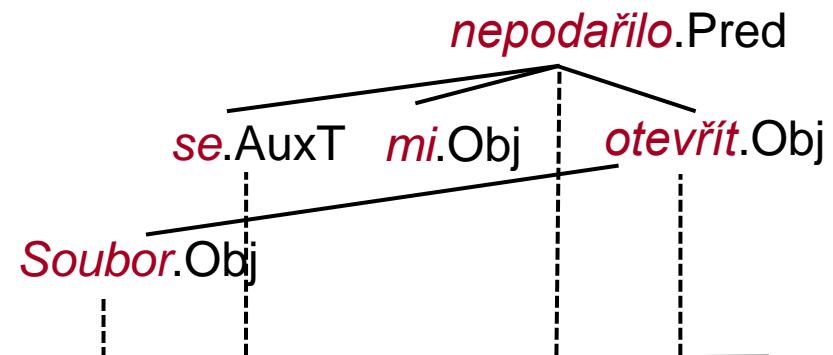
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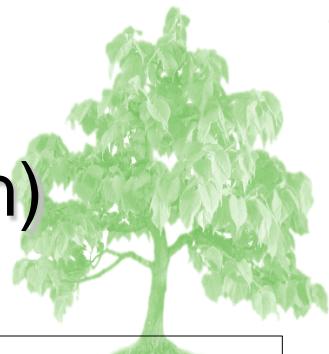


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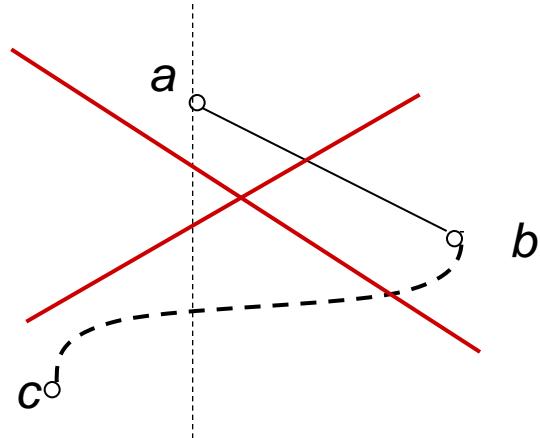
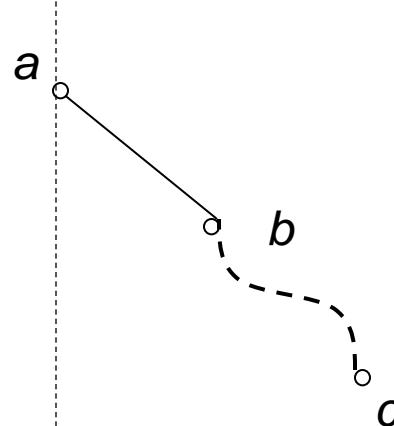
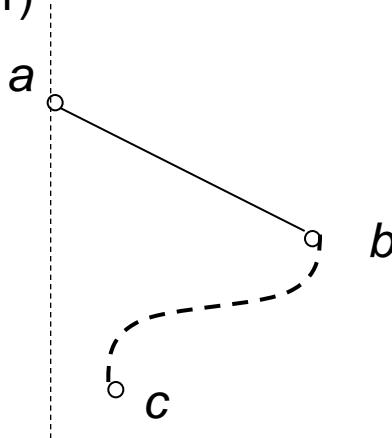
A subtree  $S$  of a rooted dependency tree  $T$  is *projective* iff for all nodes  $a$ ,  $b$  and  $c$  of the subtree  $S$  the condition holds:

$$(1) \quad (a \leq_D b) \ \& \ (a <_{\text{WO}} b) \ \& \ (b \leq_D^* c) \Rightarrow (a <_{\text{WO}} c)$$

and

$$(2) \quad (a \leq_D b) \ \& \ (b <_{\text{WO}} a) \ \& \ (b \leq_D^* c) \Rightarrow (c <_{\text{WO}} a)$$

(1)



# Projectivity and free word order

free word order:

- freedom of word order of dependents within a continuous ‘head domain’ (i.e., substring of head + its dependents)





# Projectivity and free word order

## free word order:

- freedom of word order of dependents within a continuous ‘head domain’ (i.e., substring of head + its dependents)
- relaxation of continuity of a head domain

German:

*Maria hat einen Mann kennengelernt der Schmetterlinge sammelt.*

Mary - has - a man - met - the butterflies - collects

Mary has met a man who collects butterflies

# Projectivity and free word order



English: long-distance unbounded dependency

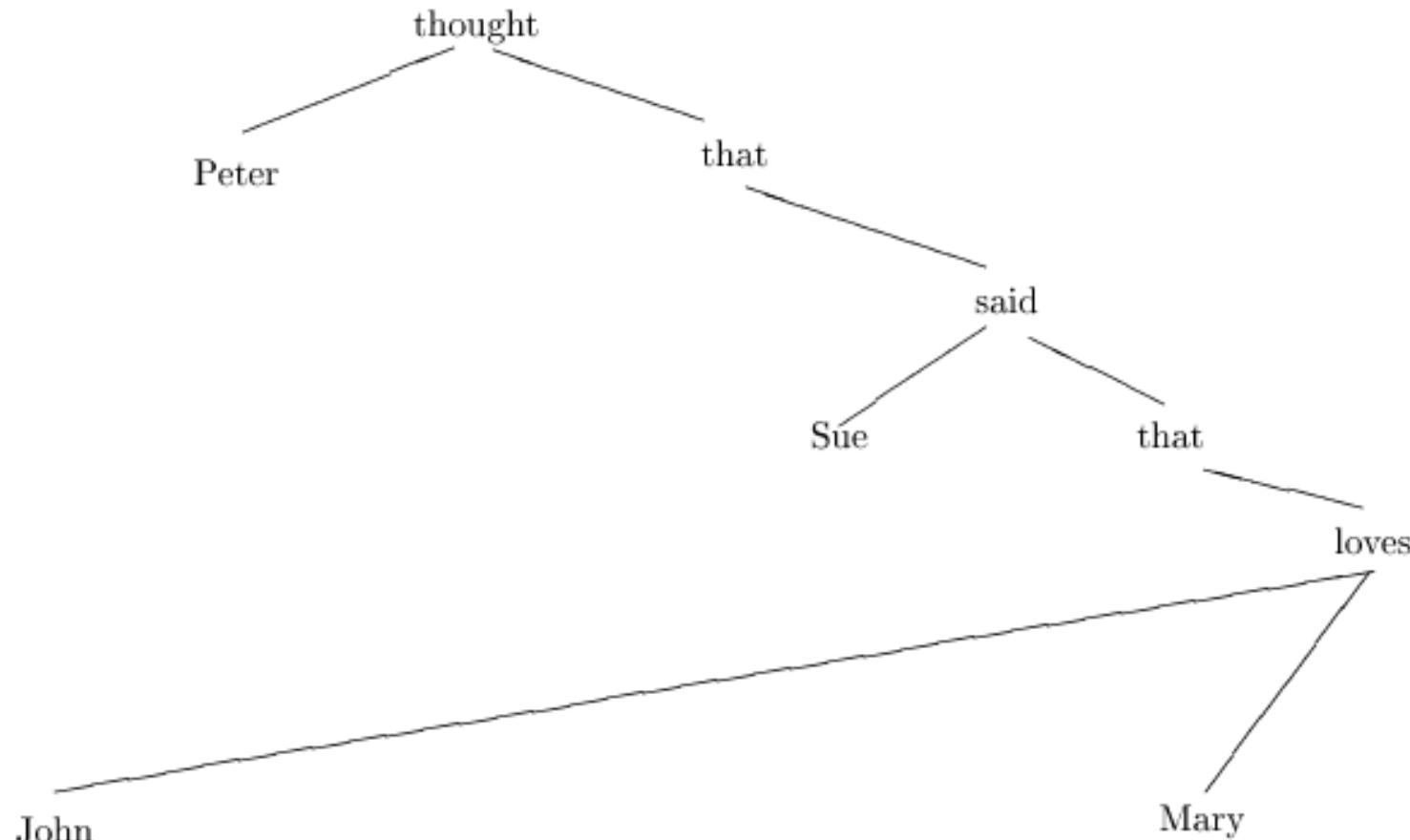
*John, Peter thought that Sue said that Mary loves.*

# Projectivity and free word order



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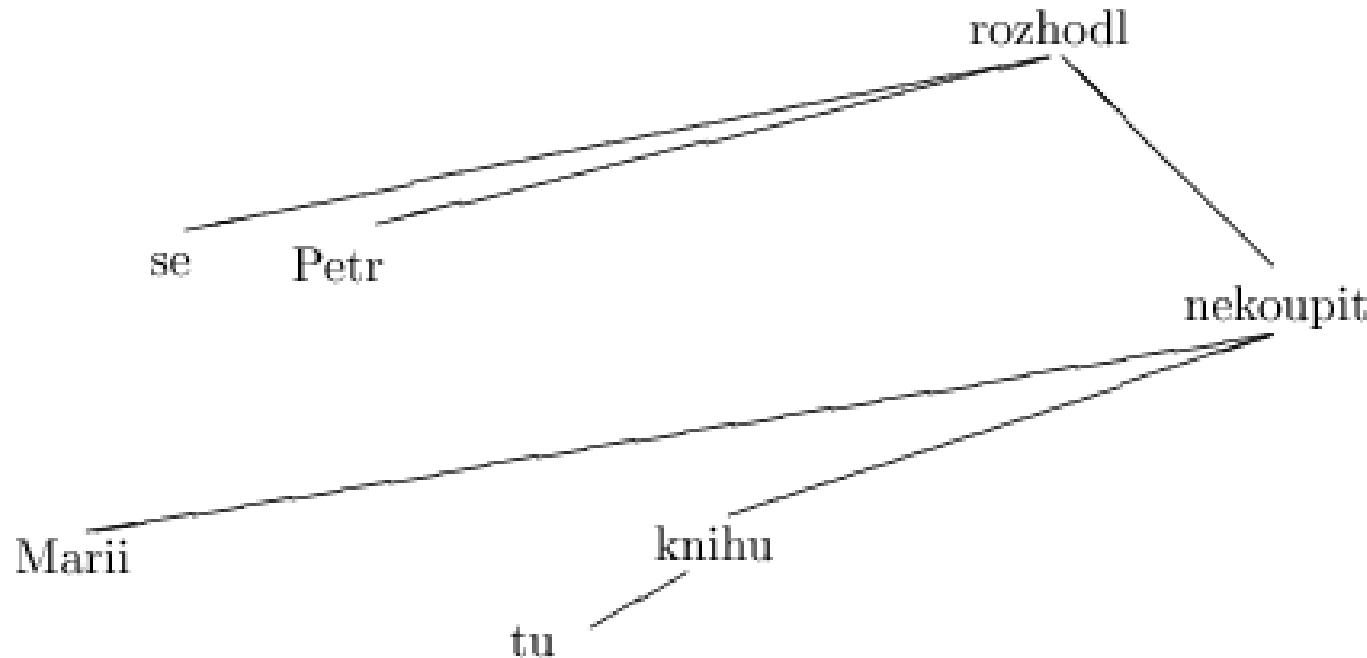


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Czech:

*Marii se Petr tu knihu rozhodl nekoupit.*  
to-Mary PART Peter that book decided not-buy  
[Peter decided not to buy that book to Mary.]

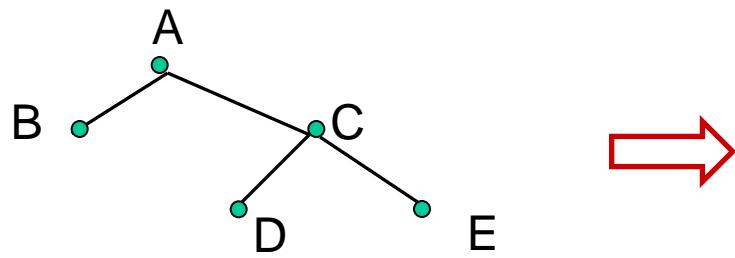




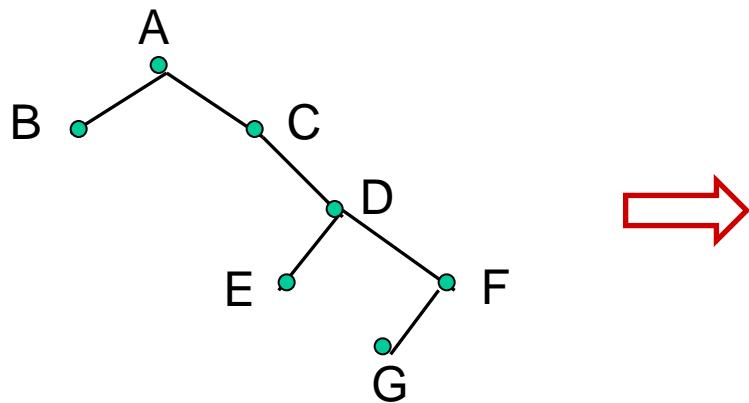
# Projectivity and non-projectivity

Projective dependency trees can be encoded by  
*linearization*:

- string of nodes, edges ~ brackets



A ( B C ( D ) )    without WO ordering  
( B ) A ( ( D ) C ( E ) )    with WO

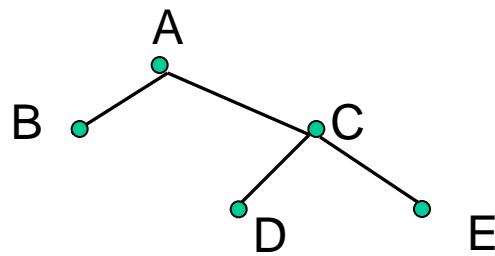




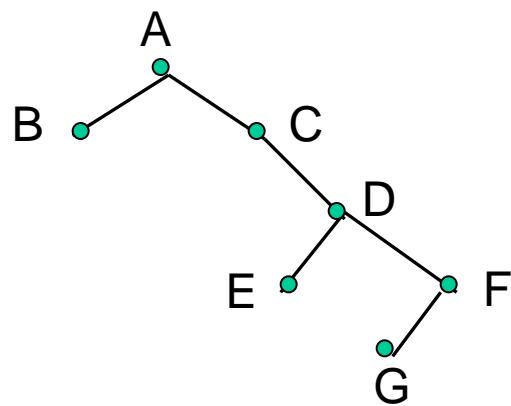
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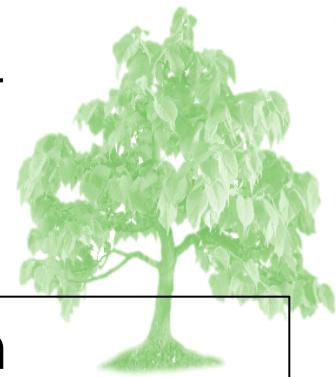


A ( B C ( D E ) )    without WO ordering  
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A ( B C ( D ( E F ( G ) ) ) )    without WO  
( B ) A ( C ( ( E ) D ( ( G ) F ) ) )    with WO

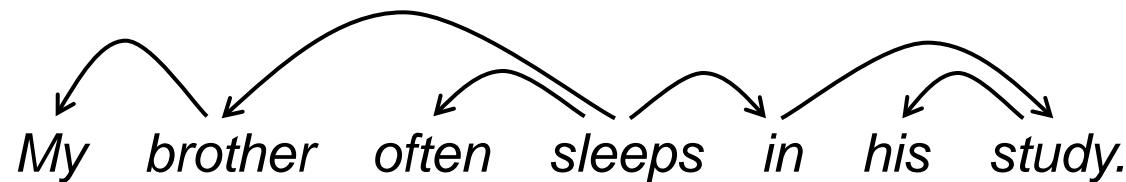
# Planarity



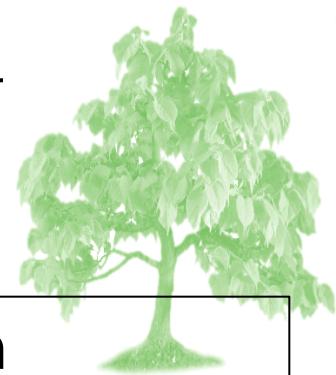
A dependency graph  $T$  is *planar*, if it does not contain nodes  $a, b, c, d$  such that:

$\text{linked}(a,c) \ \& \ \text{linked}(b,d) \ \& \ a <_{\text{wo}} b <_{\text{wo}} c <_{\text{wo}} d$

*linked(i,j)* ... ‘there is an edge in  $T$  from  $i$  to  $j$ , or vice versa’



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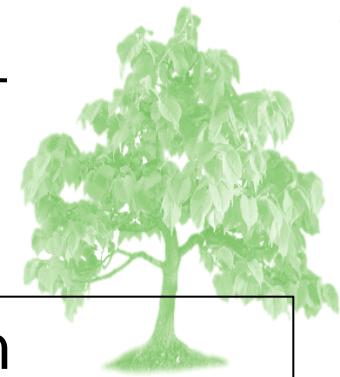
$\text{linked}(a,c) \ \& \ \text{linked}(b,d) \ \& \ a <_{\text{wo}} b <_{\text{wo}} c <_{\text{wo}} d$

**linked( $i,j$ )** ... ‘there is an edge in  $T$  from  $i$  to  $j$ , or vice versa’

My brother often sleeps in his study.

Jan viděl větší město než Praha.  
Jan - saw - bigger - city - than – Prague  
Jan saw a city bigger than Prague.

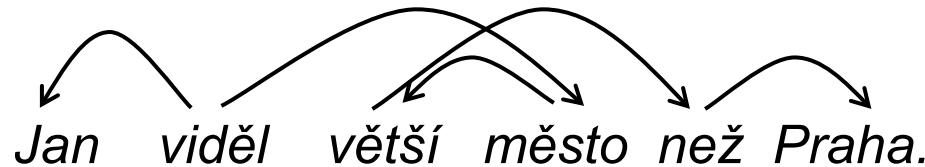
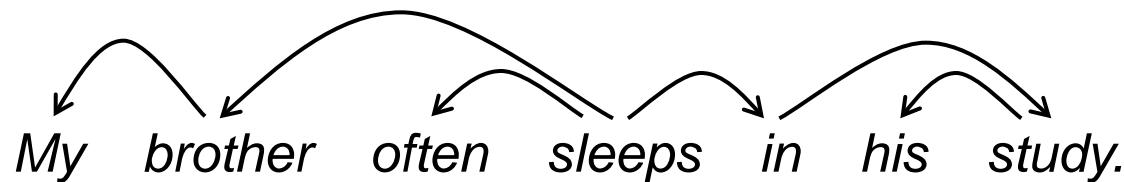
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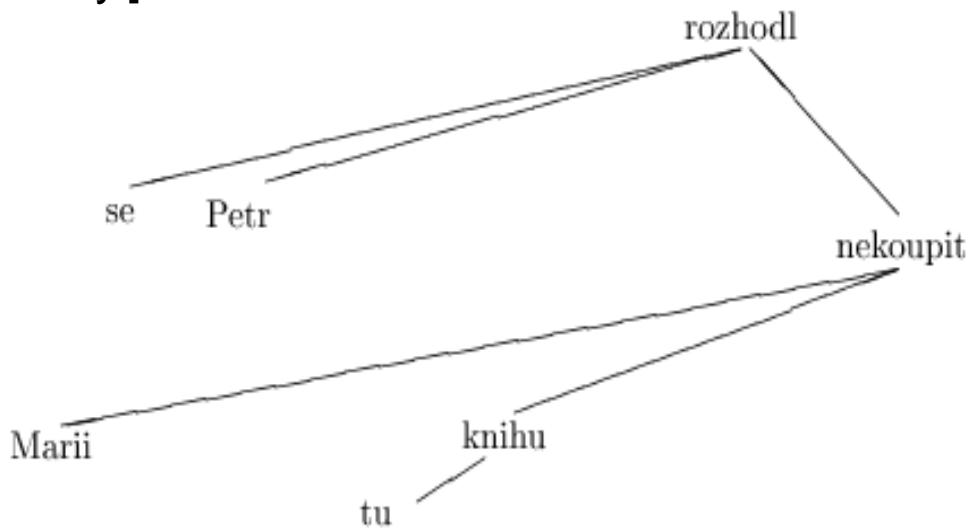
Informally, a dependency graph is planar, if its edges can be drawn above the sentence without crossing.

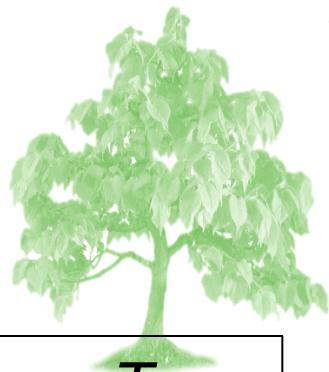
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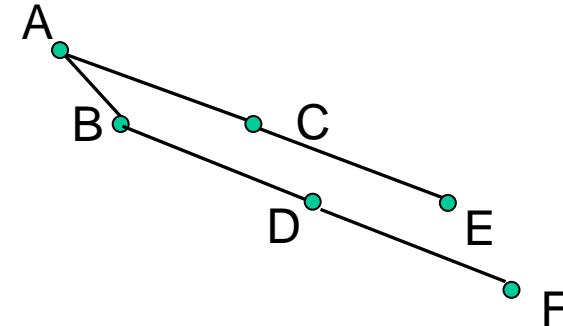
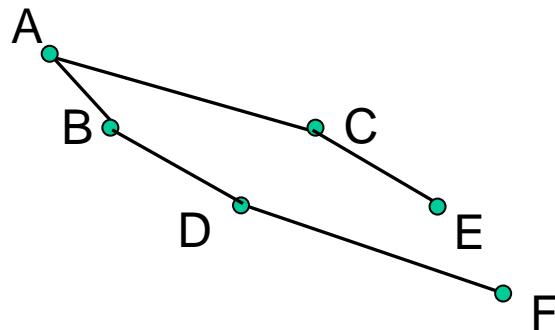


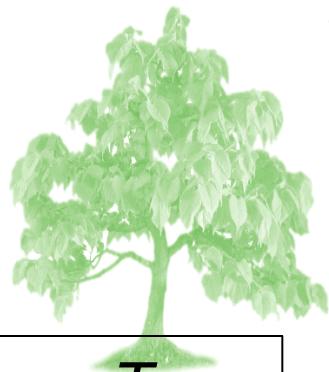
# 'Well-Nestedness'

Two subtrees  $T_1, T_2$  **interleave**, if there are nodes  $l_1, r_1 \in T_1$  and  $l_2, r_2 \in T_2$  such that

$$l_1 <_{wo} l_2 <_{wo} r_1 <_{wo} r_2$$

A dependency graph is **well-nested**, if no two of its disjoint subtrees interleave.'



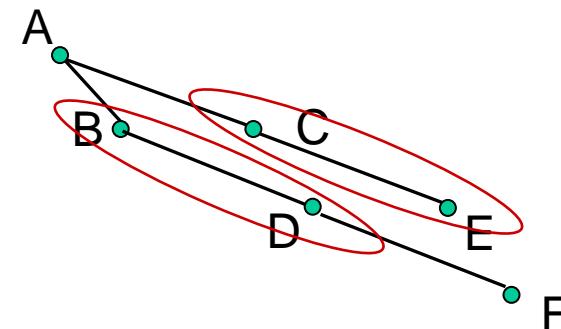
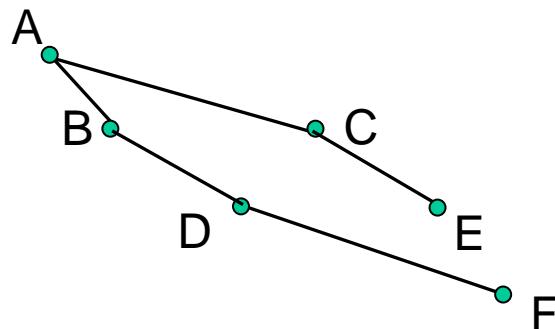


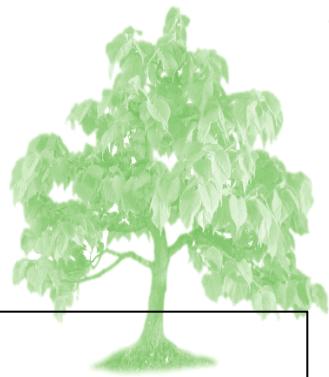
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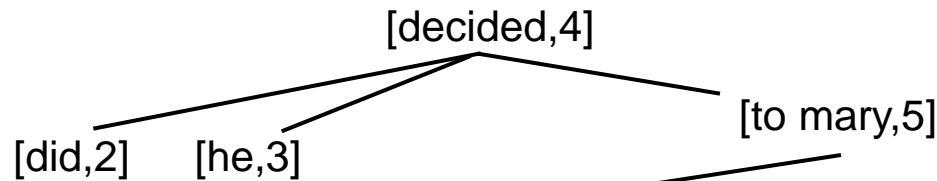


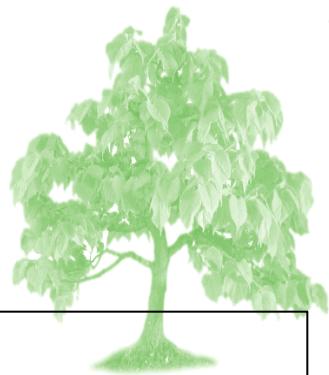
# Gap Degree $dNh(T)$

**Coverage** of a node  $u \in T$

$\text{Cov}(u, T) = \{ i \mid i - \text{word order position of } v \in T \text{ such that, } u \leq_D v \}$

$\text{Cov}(u_1, T) = \{1\}; \text{ } \text{Cov}(u_2, T) = \{2\}; \text{ } \text{Cov}(u_3, T) = \{3\}; \text{ } \text{Cov}(u_4, T) = \{1, 2, 3, 4, 5\}; \text{ } \text{Cov}(u_5, T) = \{1, 5\}$



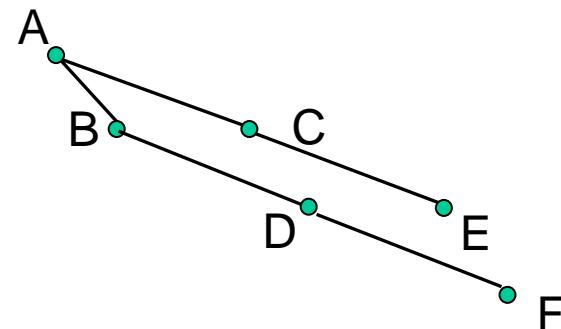
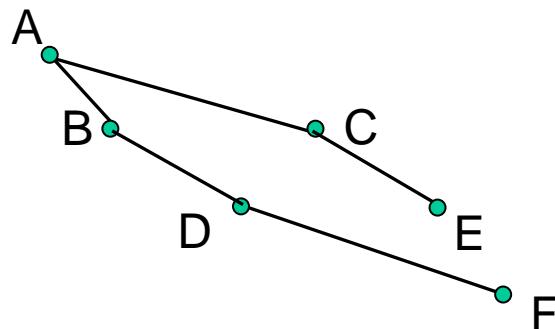


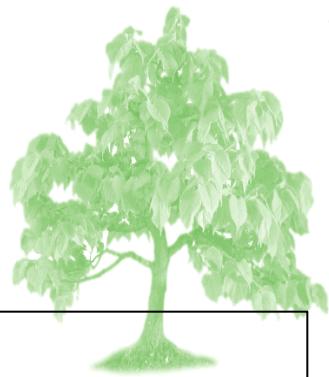
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**Gap in Coverage** of a node  $u \in T \Leftrightarrow_{\text{def}} \text{Cov}(u, T)$  is not an interval



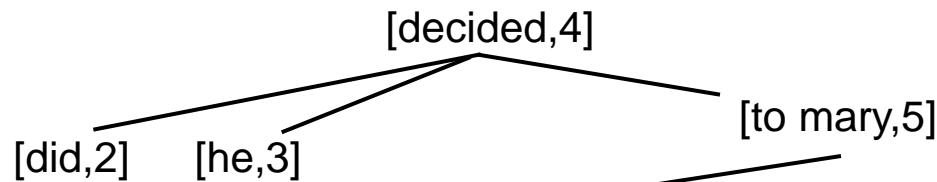


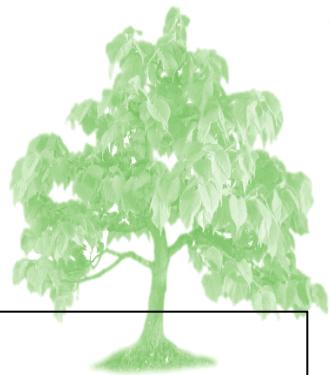
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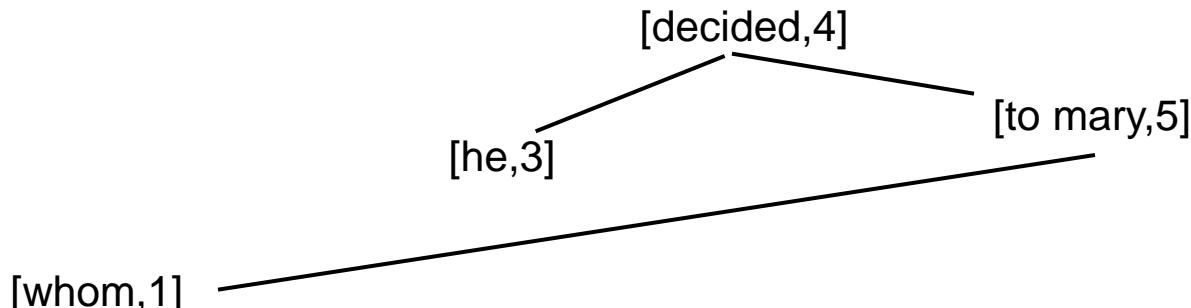
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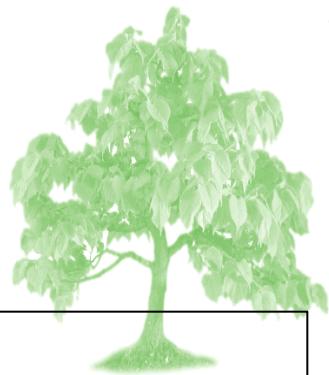
**Gap in Coverage** of a node  $u \in T \Leftrightarrow_{\text{def}} \text{Cov}(u, T)$  is not an interval

$dNh(u, T)$  ... **number of Gaps** in  $\text{Cov}(u, T)$

**Tree Degree**  $dNh(T) = \max \{dNh(u, T) \mid u \in T\}$

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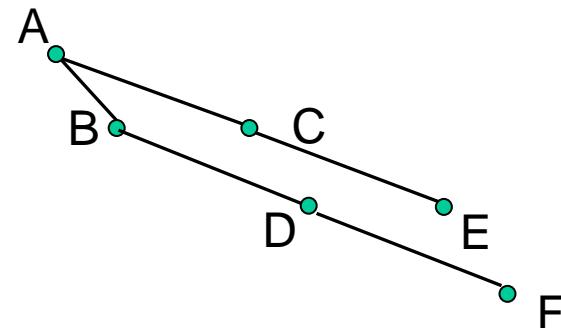
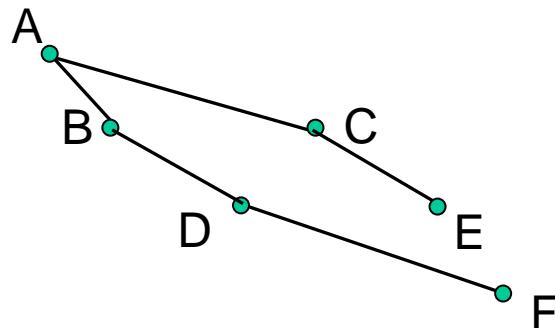
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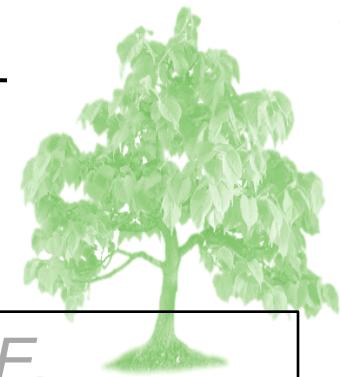
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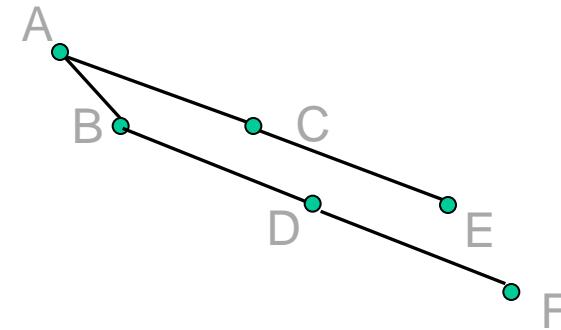
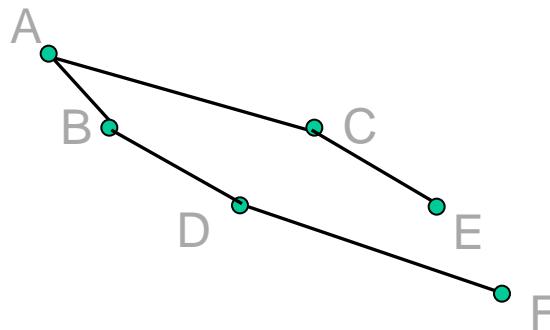
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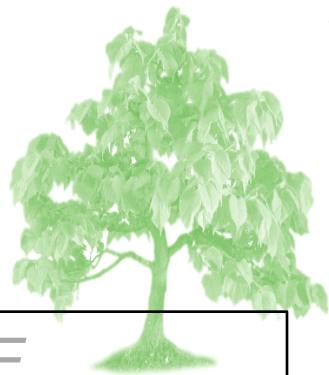


Let  $T = (N, E)$  dependency tree,  $e = [i, j]$  an edge in  $E$ ,  $T_e$  the subgraph of  $T$  induced by the nodes contained in the span of  $e$ .

**Degree of an edge**  $e \in E$ ,  $\text{ed}(e)$ , is the number of connected components  $c$  in  $T_e$  such that the root of  $c$  is not dominated by the head of  $e$ .

**Edge degree of  $T$** ,  $\text{ed}(T) \dots \max \{\text{ed}(e) | e \in T\}$



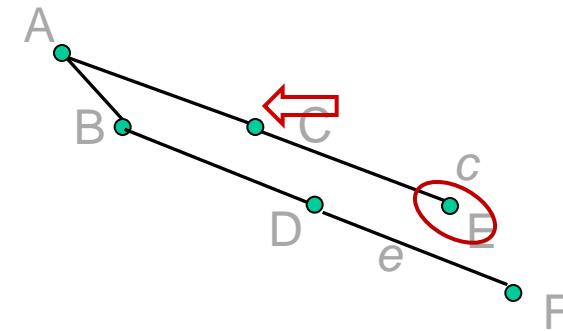
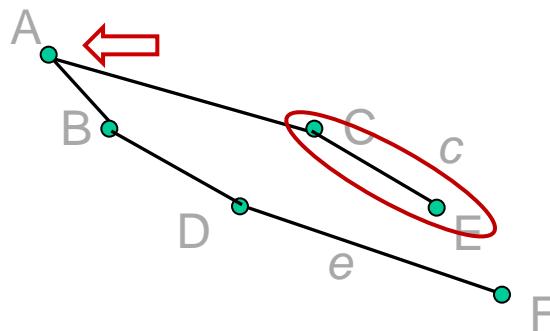


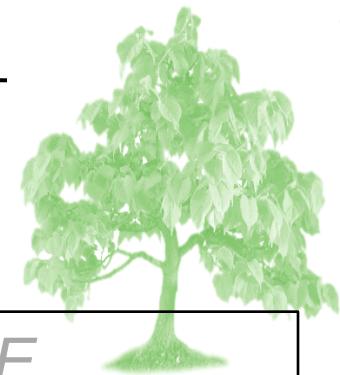
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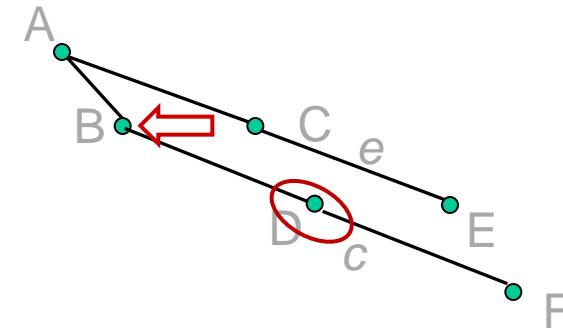
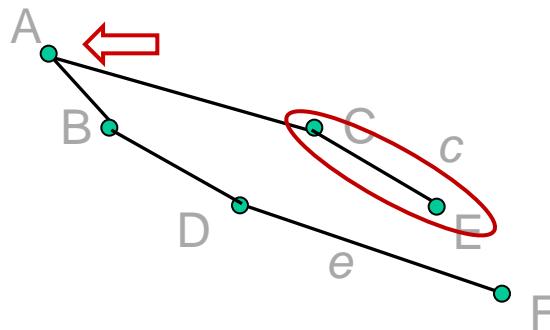


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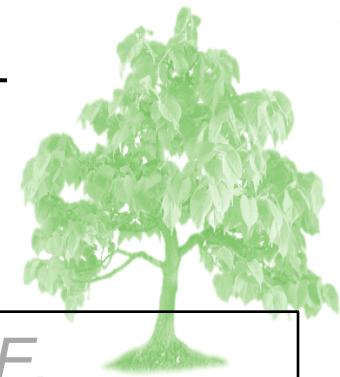
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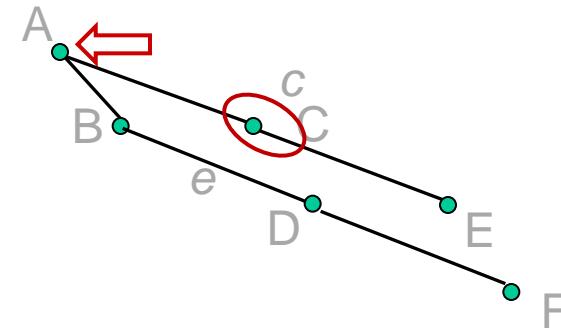
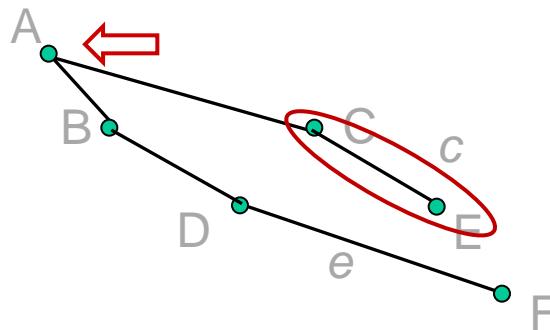
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property	DDT		PDT	
<i>all structures</i>	<i>n</i> = 4393		<i>n</i> = 73088	
gap degree 0	3732	84.95%	56168	76.85%
gap degree 1	654	14.89%	16608	22.72%
gap degree 2	7	0.16%	307	0.42%
gap degree 3	—	—	4	0.01%
gap degree 4	—	—	1	< 0.01%
edge degree 0	3732	84.95%	56168	76.85%
edge degree 1	584	13.29%	16585	22.69%
edge degree 2	58	1.32%	259	0.35%
edge degree 3	17	0.39%	63	0.09%
edge degree 4	2	0.05%	10	0.01%
edge degree 5	—	—	2	< 0.01%
edge degree 6	—	—	1	< 0.01%
projective	3732	84.95%	56168	76.85%
planar	3796	86.41%	60048	82.16%
well-nested	4388	99.89%	73010	99.89%
<i>non-projective structures only</i>	<i>n</i> = 661		<i>n</i> = 16920	
planar	64	9.68%	3880	22.93%
well-nested	656	99.24%	16842	99.54%





# References

- Hajičová, E., Havelka, J., Sgall, P., Veselá, K., Zeman, D. (2004) Issues of Projectivity in the Prague Dependency Treebank. *PBML*, vol. 81
- Holan, T., Kuboň, V., Oliva, K., Plátek, M. (2000) On Complexity of Word Order. *Les grammaires de dépendance – Traitement automatique des langues*, vol. 41, no. 1, 273-300
- Kuhlmann, M., Nivre, J. (2006) Mildly Non-Projective Dependency Structures. In COLING/ACL Main Conference Poster Sessions, 507–514.
- Mel'čuk, I. (1988) *Dependency Syntax: Theory and Practice*. State University of New York Press, Albany
- Partee, B. H.; ter Meulen, A.; Wall, R. E. (1990) *Mathematical Methods in Linguistics*. Kluwer Academic Publishers
- Petkevič, V. (1995) A New Formal Specification of Underlying Structure. *Theoretical Linguistics*, vol. 21, No.1
- Sgall, P., Hajičová, E., Panevová, J. (1986) *The Meaning of the Sentence in Its Semantic and Pragmatic Aspects*. D. Reidel Publishing Company, Dordrecht/Academia, Prague
- Štěpánek, J. (2006) Závislostní zachycení větné struktury v anotovaném syntaktickém korpusu. PhD Thesis, MFF UK