## 6. Cvičení z MA II. (31.3.09)

**1.** Nepříjemné substituce:

(a) $\int \frac{1}{x}\sqrt{\frac{x-1}{x+1}}\,dx$ (b) $\int \frac{1}{\sqrt{x^2+1}}\,dx$

(c) $\int \frac{1}{\sqrt{x^2-1}}\,dx$ (d) $\int \frac{1}{\sqrt{x^2+x+1}}\,dx$

(e) $\int \frac{1}{x+\sqrt{x^2+x+1}}\,dx$ (f) $\int \frac{1}{x\sqrt{x^2+x+1}}\,dx$

(g) $\int \frac{1}{x\sqrt{x^2+5x+1}}\,dx$ (h) $\int \frac{1}{\sqrt{2+x-x^2}}\,dx$

**2.** Příklady písemkového typu (doc. Kalenda):

(a) $\int \frac{\sin^2 x}{\sin x+cosx+2}\,dx$ (b) $\int \frac{x}{x^2+7+\sqrt{x^2+7}}\,dx$ (c) $\int \frac{x^2+1}{(x-1)(x^2-1)(x^2+x+1)}\,dx$

(d) $\int \frac{(\mathrm{tg}\ x+\mathrm{cotg}\ x)^2}{\sin^2 x-\cos^2 x}\,dx$ (e) $\int \frac{\sin x}{9\cos^2 x+2\sin^4 x}\,dx$ (f) $\int \frac{\sqrt{x+1}}{\sqrt{x+1}+\sqrt{x}}\,dx$

(g) $\int \frac{\cos^2 x}{\sin x(1-\cos x)}\,dx$

**3.** Hyperbolicke funkce:

(a) $\int x^2 e^x \sin x dx$

(b) $\int \frac{1}{\sqrt{1+x^2}}dx$ (Zkuste substituci $x = \sinh t$, kde $\sinh t = \frac{e^x-e^{-x}}{2}$)

**Řešení (až na $c$):**

**1a.** $\log|\frac{1+t}{1-t}| - 2\operatorname{arctg} t$, kde $t = \sqrt{\frac{x-1}{1+1}}$ **1b.** $\log|x+\sqrt{x^2+1}| = \operatorname{argsinh} x$ na $R$ **1c.** $\log|x+\sqrt{x^2-1}|$ na int. $(-\infty,-1)$ na $(1,\infty)$ **1d.** $\log|t+\sqrt{t^2+1}|$, kde $x+\frac{1}{2} = \sqrt{\frac{3}{4}}\cdot t$ na $R$ **1e.** $\log|t|+\frac{3}{1+2t}-\frac{3}{2}\log|1+2t|$ na $R$ **1f.** vede na $\int \frac{1}{\sqrt{x^2+x+1}}\,dx$ **1g.** $\log|\frac{t-1}{t+1}|$, kde $\sqrt{x^2+5x+1} = x+t$ , tedy $\log\frac{1}{3}\cdot\frac{\sqrt{7}}{\sqrt{7}-2}$ **1h.** $\arcsin(\frac{2}{3}(x-\frac{1}{2}))$, nebo $2\operatorname{arctg}\sqrt{\frac{x+1}{2-x}}$

**2a.** $\log\frac{t^2+2t+3}{t^2+1}+\frac{1}{\sqrt{2}}\operatorname{artcg}\frac{t+1}{\sqrt{2}}-\frac{1+t}{t^2+1}+k\frac{\pi}{\sqrt{2}}$ na int. $((2k-1)\pi,(2k+1)\pi)$, kde $t=\operatorname{tg}\frac{x}{2}$; lze "slepit" v krajních bodech (např. $\frac{\pi}{2\sqrt{2}}$ pro $x=\pi$) **2b.** $-\log t+\log(t^2+2t+7)$, kde $t=\sqrt{x^2+7}-x$ na $R$ **2c.** $-\frac{1}{6}\log|x-1|-\frac{1}{3(x-1)}+\frac{1}{2}\log|x+1|-\frac{1}{6}\log(x^2+x+1)-\frac{1}{\sqrt{3}}\operatorname{arctg}\frac{2x+1}{\sqrt{3}}$ na $(-\infty,-1)$ a na $(-1,1)$ a na $(1,\infty)$ **2d.** $\operatorname{tg} x+\operatorname{cotg} x+2\log|\operatorname{tg} x-1|-2\log|\operatorname{tg} x+1|$ na int. $(\frac{k\pi}{4},\frac{(k+1)\pi}{4})$ **2e.** $-\frac{\sqrt{2}}{3}\operatorname{arctg}(\sqrt{2}\cos x)+\frac{1}{3\sqrt{2}}\operatorname{arctg}\frac{\cos x}{\sqrt{2}}$ na $R$ **2f.** $\frac{1}{8}\frac{1}{(\sqrt{\frac{x}{x+1}}+1)^2}+\frac{1}{8}\frac{1}{(\sqrt{\frac{x}{x+1}}-1)^2}+\frac{3}{8}\frac{1}{\sqrt{\frac{x}{x+1}}-1}-\frac{3}{8}\frac{1}{\sqrt{\frac{x}{x+1}}+1}$ na $(0,+\infty)$ **2g.** $-\frac{1}{4}\log(1+\cos x)-\frac{3}{4}\log(1-\cos x)+\frac{1}{2(\cos x-1)}$ na $(k\pi,(k+1)\pi)$

**3a.** $\frac{1}{2}e^x((x^2-1)\sin x-(x-1)^2)\cos x$, na $R$ **3b.** $\log(x+\sqrt{x^2+1})$, na $R$