Decision Tree is a learning method suitable for both classification and regression tasks.

Example classification task: WSD
see the NPFL054 web page → Materials → wsd-attributes.pdf
A **decision tree** \( T = (V, E) \) is a rooted tree where \( V \) is composed of internal **decision nodes** and terminal **leaf nodes**.

- **Nodes**
  - **Root node**
  - Internal nodes with conditions on selected features
  - Leaf nodes with **TARGET OUTPUT VALUES**

- **Decisions**

\[
\begin{align*}
&\text{yes} \quad A_4 = \text{TRUE} \\
&\text{no} \\
&\phantom{\text{yes}} A_2 = \text{TRUE} \\
&\phantom{\text{yes}} \phantom{A_2 = \text{TRUE}} A_3 = \text{TRUE} \\
&\phantom{\text{yes}} \phantom{A_2 = \text{TRUE}} \phantom{A_3 = \text{TRUE}} A_9 = \text{TRUE}
\end{align*}
\]
Decision tree learning

- Building a decision tree $T_D = (V, E)$ is based on a training data set $D = \{\langle x, y \rangle : x \in X, y \in Y \}$.
- Each node is associated with a set $t$, $t \subseteq D$. The root node is associated with $t = D$.
- Each leaf node is associated with a fixed output value.
A very basic idea: Assume binary decisions.

- **Step 1** Create a root node.

- **Step 2** Select decision $d$ and add child nodes to an existing node.

Step 2 is then applied recursively.
Building a decision tree from training data

The learning process, i.e. building the tree starts from the root node and continues top-down. The root node is associated with the whole training set.

Example

1. Assume decision if $A_4 = TRUE$.
2. Split the training set $t$ according to this decision into two subsets – “yellow” and “blue”.

| SENSE    | ... | A4  | ...
|----------|-----|-----|-----
| FORMATION|     | TRUE|     
| FORMATION|     | FALSE|    
| PHONE    |     | TRUE|     
| CORD     |     | TRUE|     
| DIVISION |     | FALSE|    
| ...      | ... | ... | ... |
3. Add two child nodes, “yellow” and “blue”, to the root. Associate each of them with the corresponding subsets $t_L$ and $t_R$, respectively. The subsets are always disjoint.

<table>
<thead>
<tr>
<th>SENSE</th>
<th>...</th>
<th>A4</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>FORMATION</td>
<td>TRUE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CORD</td>
<td>TRUE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PHONE</td>
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</table>
• **Step 4** Repeat recursively steps (2) and (3) for both child nodes and their associated training subsets.

• **Step 5** Stop recursion for a node if a stopping criterion is fulfilled. Then create a leaf node with an output value.
Learning decision tree – example training data

Two continuous features $A_1$ and $A_2$, and three target classes
First split divides the training data set into two partitions by condition $A_2 \geq a_2$
Second split is defined by $A_1 \geq a_1$ and applies only if $A_2 \geq a_2$
Two splits in the example produce a tree with two inner nodes and three leaves.
Prediction on test data

Once a decision tree predictor is built, an unseen instance is predicted by starting at the root node and moving down the tree branch corresponding to the feature values asked in decisions.
Decision tree predictor for the WSD-line task
According to existing feature values in a given test instance you can use the decision tree as a predictor to get the classification of the instance.
Decision trees for classification and for regression

Decision trees can be used both for classification and regression tasks

### Classification trees
- Categorical output value

![Classification tree diagram](image)

**Figure**: Tree for predicting the sense of *line* based on binary features.

### Regression trees
- Numerical output value

![Regression tree diagram](image)

**Figure**: Tree for predicting the salary of a baseball player based on the number of years that he has played in the major leagues (*Year*) and the number of hits that he made in the previous year (*Hits*). See the ISLR Hitters data set.
Each terminal node in the decision tree is associated with one of the regions in the feature space. Then

**Classification trees**
- **output value**: the most common class in the data associated with the terminal node

**Regression trees**
- **output value**: the mean output value of the training instances associated with the terminal node
Building a decision tree is in fact a recursive partitioning process

Tree growing
The growing process is based on subdividing the feature space recursively into non-overlapping regions.
Recursive data partitioning – regression case

![Diagram showing recursive data partitioning for regression case. The tree structure is shown with nodes and branches, indicating decision points based on hits and years.]
Historical excursion

Decision trees concept (Hunt, 1962)

ID3 (Quinlan, 1979)

C4.5 (Quinlan, 1993)

AID (Morgan, 1964)

CART (Breiman, 1984)

- ID3 $\sim$ Iterative Dichotomiser
- AID $\sim$ Automatic Interaction Detection
- CART $\sim$ Classification and Regression Trees

Probably most well-known is the “C 5.0” algorithm developed by Quinlan for commercial use, which has also become the industry standard. C 5.0 is an improved extension of C 4.5. Single-threaded version is distributed under the terms of the GNU General Public License.
Learning a decision tree – key problems

Building a decision tree means to make a hierarchical sequence of splits. Each practical algorithm must be able to efficiently decide the following key questions:

1. How to choose a suitable splitting condition?

2. When to stop the splitting process?

A practical answer to problem (1) is to employ entropy or another similar measure. Each node is defined by an associated subset of examples with a specific distribution of target values. After a split, the entropy in child nodes should decrease in comparison with entropy in the parent node.

The splitting process should be duly stopped just to not produce model that overfits the training data. To avoid overfitting, practical implementations usually use pruning after building a relatively deep tree.
Practical implementations of decision tree learning usually work in two main phases:

1. **Tree growing**
2. **Tree pruning**

**Basic underlying idea**

- First, grow a large tree that *fits the training data* quite well.
- Second, prune this tree to *avoid overfitting*. 
Building a decision tree — how to avoid overfitting

Generally, overfitting can be avoided by

• applying a stopping criterion that prevents some sets of training instances from being subdivided,
• removing some of the structure of the decision tree after it has been produced.

Practically preferred strategy

• Grow a large tree $T_0$, stop the splitting process when only some minimum node size (say 5) is reached.
• Then prune $T_0$ using some pruning criteria.
• **data splitting**
  — deeper nodes can learn only from small data portions

• **sensitivity to training data set (unstable algorithm)**
  — learning algorithm is called unstable if small changes in the training set cause large differences in generated models
There are two widely used packages in R

- `rpart`
- `tree`

The algorithms used are very similar.

References

- An Introduction to Recursive Partitioning Using the RPART Routines by Terry M. Therneau, Elizabeth J. Atkinson, and Mayo Foundation (available online)
- *An Introduction to Statistical Learning with Application in R* Chapters 8.1, 8.3.1, and 8.3.2 by Gareth James, Daniela Witten, Trevor Hastie and Rob Tibshirani (available online)
- R packages documentation — `rpart`, `tree` (available online)
Examination requirements

- You should understand the basic ideas of building and using Decision Trees for classification task.

- You should be able to practically use `rpart()` package in R.

- Later, we will revisit Decision Trees and go into more details in connection with the ensemble method called Random Forests, which is an important and more effective extension of simple Decision Trees.