Montague Semantics

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Historical Background

Montague semantics emerged around 1970, and constituted a fundamentally new approach to the semantics of natural language. In order to understand the importance of this approach, it is useful to consider the situation in that period of some neighboring disciplines of semantics: philosophy of language, philosophical logic, and linguistics. These subjects are considered below, a more extended picture of the historical background is given by Partee and Hendriks (1997).

One of the subjects studied in philosophy of language was meanings of natural language, and these were (before 1970) sometimes represented in some or other logic. The mapping between a sentence and its logical representation was made in an *ad hoc* way; it was more or less stipulated which formula was the correct meaning representation of a given sentence. The situation could be characterized as follows: a 'bilingual logician,' who knew logic and who knew a natural language, provided the formula. It was noticed that there could be a large difference between the sentence and the formula, and a dominant view of these matters at that time was the so-called 'misleading form thesis,' saying that there is a sharp distinction between the grammatical and the logical form of sentences, and that the grammatical form disguises the logical form to such an extent that they cannot be related in a systematic way. It was sometimes even proposed to design for certain purposes an improved version of natural language in which the form does not obscure the meaning.

In philosophical logic there is a tradition to study words of natural language that are philosophically interesting. Examples are 'necessarily' and 'possibly.' Axiom systems for these notions were designed which expressed their properties. A jungle of systems of modal logics arose, motivated by different properties of embeddings of these notions. Kripke brought about, in the mid-1960s, an enormous change in this field. He introduced semantic models for modal logics, thereby making it possible to conceive modal logic in the same way as mathematical logic: as a formal language with a model-theoretic semantics. The variety of systems could be structured by conceiving them as expressing relations between possible worlds in a model.

Around 1960, Chomsky brought about great changes in linguistics by introducing mathematical standards of explicitness. He developed the tools for this (context-free grammars and transformations), and syntax became a flourishing branch of linguistics. There was some attention to semantic issues, such as whether transformations were meaning-preserving, or what would be the input for a semantic component, but the theory was a theory about syntax that did not deal explicitly with semantics.

These three lines were brought together by Richard Montague. He was a mathematical logician who had made important contributions to the foundations of set theory. He was attracted by Chomsky's formal treatment of natural language, but unsatisfied by its (lack of) treatment of semantics. Therefore he developed an alternative to the Chomskyan approach that satisfied his (logical) standards. He presented a fragment of English and provided it with a rigorous model-theoretic interpretation. Most important is the fact that the relation between a sentence and its meaning is defined in a systematic way. It became possible, for the first time in history, to calculate which meaning is associated with any given sentence, hence to make predictions concerning semantics.

By developing his grammar model, Montague provided evidence for his opinion that there is no important theoretical difference between natural languages and the languages studied by logicians: both can be dealt with using the same methods and with the same mathematical rigor (Montague, 1970a: 189; Montague, 1970b: 313; Thomason, 1974: 188, 222). The title of one of Montague's first publications on this subject provides clear evidence of his position: 'English as a formal language' (Montague, 1970a).

Aims

The aim of Montague semantics is to describe, predict, and explain semantic relations between natural language utterances, an aim it shares with other theories. In the present section, some important semantic relations between natural language utterances will be introduced by means of examples, viz. entailment, valid reasoning, synonymy, and ambiguity. The examples given here are realistic in the sense that they are treated within the field of Montague semantics, thus giving an impression of the variety of phenomena that are studied. The examples that occur without reference are within the fragment of Montague (1973), or are variants of such examples.

An important semantic relation is the 'entailment' relation between sentences, say A and B. This means that whenever A is accepted as being true, B must also be accepted as being true, on grounds of meaning properties. Sentence (1) entails sentence (2):

(1) Mary is singing and dancing.

(2) Mary is singing.

This entailment, however, does not hold for all grammatical subjects: witness (3) and (4), where in fact the inverse relationship holds.

(3) No-one is singing and dancing.

(4) No-one is singing.

This means that the noun phrases have to be divided into two classes, one for which the entailment holds, and one for which it does not. Then one would like to have an explanation of why precisely *two girls* and *both girls* are in different classes (*Both/precisely two girls were singing and dancing*), and a prediction concerning compound terms like *few girls and many boys*. For an overview of properties of quantified noun phrases, see Keenan and Westerstahl (1997) *and* the article **Quantifiers: Semantics**.

An example of a different nature is the following. (5) entails (6) (see Dowty, 1979):

(5) John cools the soup.

(6) The soup cools.

Here, one would like to have a description of the relation between the meaning of *cool* as a transitive causative verb and *cool* as an intransitive verb. And one would like to know why the implication holds for *boil* but not for *stir*.

A 'valid reasoning' is a generalization of entailment that involves more sentences than two. If someone accepts (7) and (8), they may conclude correctly (9):

(7) John sings.

(8) John is a man.

(9) A man sings.

A more intricate example is (Groenendijk and Stokhof, 1982):

(10) John knows whether Mary comes.

(11) Mary comes.

(12) John knows that Mary comes.

Here, one sees that there is a relation between the meaning of the *whether* clause and the *that* clause. It seems that the latter is weaker. However, the relation is more complicated: if one has the negated version (11a), then (12a) follows from (10) and (11a)

(10) John knows whether Mary comes.

(11a) Mary does not come.

(12a) John knows that Mary does not come.

Hence the relation between the *whether* clause and the *that* clause depends on the factual situation.

A special case of entailment is 'synonymy.' Sentences are called synonymous just in case they entail each other. For example, sentences (13) and (14) are synonymous:

(13) John or Mary comes.

(14) John comes or Mary comes.

Another example of synonymy is (Partee and Bach, 1981):

(15) Mary admires herself, and Sue does too.

(16) Mary admires herself, and Sue admires herself.

It may seem that the meaning of the *does too* clause can be found by the substitution of a phrase that occurs elsewhere in the sentence. This is, however, not always the case. It is, for instance, possible to conclude from (17) to (18).

(17) Mary believes that she is ill, and Sue does too.

(18) Mary believes that she is ill, and Sue believes that Mary is ill.

This illustrates the important phenomenon of 'ambiguity.' Sometimes a sentence can be understood in two or more ways corresponding with distinct consequences. From (17), one may conclude either that

(19) Sue believes that Mary is ill.

or that

(20) Sue believes that she (= Sue) is ill.

Another example of ambiguity is (e.g., Janssen, 1983, 1986b):

(21) Ten years ago, John met the president of the USA.

(22) The president of the USA is Bush.

On the one reading of (21), one may conclude that ten years ago John met Mr. Bush. On the other reading, this does not follow since John met the person who was president ten years ago. This ambiguity clearly concerns the functioning of tense operators.

The decision whether a sentence is ambiguous or vague is not always clear. Consider (23):

(23) Two girls are five sandwiches.

One may ask what counts as a source of semantic ambiguity; if it makes a difference whether they shared all five sandwiches; whether the sandwiches were distributed between the girls; or if it makes a difference whether they ate together or whether each girl ate on her own. Maybe the issue would be more exciting if the sentence was *Two girls shot five men* and one had to judge the girls. Verkuyl and Van der Does (1996) argue that (23) has one meaning, whereas they refer to other authors who argue for a fourfold or even ninefold ambiguity.

So far, only examples of relations between declarative sentences have been cited. Other types of sentences take part in 'other semantic relations,' as between the questions (24) and (25):

(24) Which girls came to the party?

(25) Did Mary come to the party?

In this example, there is nothing like the acceptance of a premise or of a conclusion. Nevertheless, there is a relation: every answer to the first implies an answer to the second question. The meanings assigned to these questions should account for this (Groenendijk and Stokhof, 1989).

Sequences of sentences likewise have logical properties. Consider (26) and (27).

(26) A man walks in the park. He whistles.

(27) A man in the park whistles.

This example puts a requirement on the treatment of texts. The meaning of the two sentences together should be equivalent to the meaning of the single sentence.

All examples discussed in this section have the common feature that they do not depend on specific details of the meanings of the words involved (except for logically crucial words such as *not* or *and*). It is, for example, possible to account for the inference from (1) *Mary is singing and dancing* to (2) *Mary is dancing* without describing the differences between *dancing, jogging*, and *walking*. Each of the examples can be replaced by another which exhibits the same pattern but uses other words. The examples illustrate that in Montague semantics one is mainly interested in structural aspects of the semantic relations between sentences, that is, in the systematic aspects of meaning. The formalization of what meanings are, does not need to go further than is required for an adequate account of these structural aspects.

The Compositional Approach

The most salient aspect of Montague semantics is the systematic way in which natural language expressions are connected with their respective meanings. This relation is characterized by the principle of compositionality of meaning. Such a principle can, in several formulations, be found in many disciplines that deal with semantics, such as linguistics, philosophy, and computer science. In philosophy of language, it has a long tradition and is often called 'Frege's principle.' The version of this principle that describes the method of Montague grammar most explicitly is (Partee *et al*., 1990: 318):

> The meaning of a compound expression is a function of the meanings of its parts and of the syntactic rules by which they are combined.

The formulation of the principle contains several vague terms, and a proper application of the principle requires a more formal interpretation (for a discussion of the principle, its formalization, consequences, and its status, *see* **Compositionality, Aspects of**). The main points of its formalization are summarized here.

The syntax of the grammar consists of rules which express how new expressions can be formed from already available ones. The rules are, therefore, operations which act on inputs and yield an output. If an expression E is the output of the application of a rule R, then the inputs that form E are defined as being the parts of E in that derivation. The semantic component is organized in a parallel way: there are semantic rules that operate on input meanings and yield an output meaning. The crucial factor for obeying compositionality is that there is a strict correspondence between syntax and semantics. For each syntactic rule, there should be a corresponding semantic rule expressing the semantic effect of that syntactic rule. Compositionality is taken to be not an empirically verifiable property of natural language but a methodological principle: it constrains the organization of the grammar.

Consider example (28):

(28) Penguins do not fly.

A very simple grammar will be considered, naively as regards its meaning. For example, it will be assumed that the sentence says something about all penguins, whereas plurals without article usually have a more subtle meaning. The intention of this example is, however, only to illustrate the *method* of compositionality.

Suppose the grammar has as basic expressions the plural noun phrase *penguins* and the verb *fly*. A rule (say R1) forms the verb phrase *do not fly* from this verb. Furthermore, there is a rule (R2) combining a noun phrase with a verb phrase to form a sentence, by concatenating them and performing the required changes in the verb phrase for agreement and similar trimmings. Then sentence (28) has, according to this grammar, two parts: *penguins* and *do not fly*, and the latter phrase has in turn one part: *fly*.

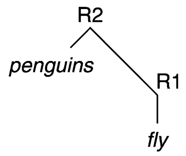

Figure 1 The derivation tree of 'Penguins do not fly.'

This derivation might be represented in the form of a tree, as in Figure 1. Note that this tree does not depict the constituent structure of the sentence; for example, there are no separate nodes for *do* and *not*. The tree shows how the sentence is formed; it is a construction tree or derivation tree. There is no *a priori* reason why the derivation would be identical to the constituent structure of the result (one might impose this as an additional requirement).

Of course, there might be good arguments for preferring a different grammar. Thus, one might construct (28) out of the positive sentence *Penguins fly*, or, alternatively, from *penguins, fly*, and *not*. In the former case the rule has one part, and in the latter case three parts. Compositionality as such provides no criterion for such issues. The best choice is probably to be discovered by considering more examples and larger fragments.

The principle of compositionality states that the meaning of the sentence is a function of the meanings of its parts, hence (according to the given grammar) of *penguins* and *do not fly*. Of course, the meaning of the latter is, in turn, a function of the meaning of its part *fly*. So, in the end, the meanings of the basic expressions are attained. Adopting for the moment a very simple conception of meaning that will be revised in the next section, one can take the meaning of *fly* to be the set of individuals who fly, and the meaning of *penguins* to be the set of individuals who are penguins.

According to the rules, the verb phrase *do not fly* has only *fly* as a part, and therefore the meaning of this verb phrase has to be formed out of the meaning of *fly*. So there has to be a semantic operation that expresses the meaning of negating a verb. For the moment, the meaning of *do not fly* is, in analogy of that of *fly*, the set of individuals who do not *fly*. The operation that forms this meaning from the meaning of *fly* is the formation of a set complement. In line with the above assumptions about meaning, one may say that the sentence means that the set of penguins is included in the set of nonflying individuals. This meaning is to be obtained from the meanings of its parts: from the set of penguins, and from the set of individuals who do not fly. This can indeed be done.

The situation is as follows. Two syntactic rules (R1 and R2) are each accompanied by a semantic interpretation (M1 and M2, respectively):

R1: negating a verb
M1: complement formation
R2: concatenating a noun phrase with a verb phrase, performing agreement changes
M2: set inclusion

In this section, the method of compositionality has been exemplified. The crucial aspect is the correspondence between syntax and semantics. One might change the concept of meaning used above, or change the syntax used; but as long as the correspondence remains infact, the grammar can be seen as an instance of Montague grammar. This characterization of Montague's method is given in a formal mathematical terminology in his paper 'Universal grammar' (Montague, 1970b).

Interpretation in a Model

In Montague grammar, as well as in all other formal theories of semantics, the natural language expressions are interpreted in a class of abstract models. For example, a name like *John* is associated with an individual in such models, and an adjective like *brave* is a property. Each model is constructed out of a number of basic sets by means of standard constructions, and the result can be restricted by 'meaning postulates.' The most characteristic feature of the models in Montague grammar is the distinction made between the 'extension' of an expression and its 'intension,' a distinction that will be the topic of the next section. In the present section, the status of the model and its connection with natural language will be considered.

The model in which humans interpret natural languages has, of course, a certain resemblance to the real world, but it should not be conceived of as a model of the real world. There are two differences. First, in language, one speaks not only about the real world, past, present, and future, but also about situations that might be the case. Even though unicorns do not exist, one can speak about them, and the sentences used have semantic properties that should be dealt with. The model thus embraces much more than reality.

Second, as far as the model is connected with reality, it is a model of how natural language 'conceives' of it. This conception might be different from the real situation. Examples are mass nouns like *water* or *air*. In natural language, they are used in a different way from count nouns such as *chair* or *flower*. The mass noun *water* is used as if every part of a quantity of water is again water; as if it had no minimal parts. The same holds for *air*. Although in reality water has minimal molecules, and air is a mixture, the model will not reflect that fact (Pelletier and Schubert, 2003).

Although the model does not reflect reality, one can be interested in the relation which it has with reality. Examples are the relation between *blue* and the frequencies of light, or the number of exceptions accepted when it is said that *all ravens are black*. This kind of research is rare in Montague grammar, however, because it amounts to the analysis of specific words, whereas in Montague grammar one is mainly interested in the more structural aspects of the semantics of natural language.

The model, not being a model of reality, might be regarded as a model of how the human mind conceives reality. Although psychological reality is not one of the claims of Montague grammar (it is so in some other theories), the issue has received some attention (e.g., Dowty, 1979: ch. 8).

The connection between natural language and the model can be made in two ways. One method, the direct one, was followed in the section 'The Compositional Approach': for a word, some element (set, function) in the model is given as the meaning of that word, and for each syntactic rule a corresponding meaning operation is given. This method is used in Montague's first publication (Montague, 1970a), and in a few other publications as well (e.g., Keenan and Faltz, 1985). The other method is the indirect one: natural language is translated into some logical language, which is interpreted in a model. If this translation is compositional, and the interpretation of the logical language is compositional, then the combination of the two processes is a compositional process of meaning assignment. Care has to be taken that the logic is used as an auxiliary language only, so that this intermediate language can in principle be eliminated. This implies that every operation that is performed in the logic should have an interpretation in the model. This indirect method is the standard method in Montague grammar; usually, (variants of) intensional logic are used as the intermediate language.

Extension and Intension

An important decision concerns the question of how to model meaning. Previously, the meaning of *penguins* was defined as the set of penguins, and of *fly* as the set of flying individuals. Applying the same approach to *the president of the USA* [example (22)], would yield as meaning the individual Bush, and applied to *unicorn* (assuming there are none) the empty set. This approach would, however, give results that are intuitively incorrect. It would have the consequence that in case neither unicorns nor centaurs exist, the meaning of *unicorn* would be equal to the meaning of *centaur*. As for *the president of the USA*, it would have the undesirable consequence that its meaning changes after each election. Examples like these have led to the distinction between two kinds of interpretation: extension and intension.

At the time of writing, the president of the USA is Mr Bush, but at other moments in time a different person will be president, and in another course of events Mr Bush could have lost the election and Mr Kerry would be president. The model in which natural language is interpreted has components dealing with this. It has a collection of time points for dealing with changes in the course of time, and it has a collection of so-called 'possible worlds.' These possible worlds represent the possibility that Kerry has won. They also represent the possibility that unicorns exist. Intensions are functions with possible worlds and time points as domain. The intension of *the president of the USA* is a function from time points and possible worlds that yields an individual (the president at that moment in that possible world), and the intension of *unicorn* is the function that yields for each possible world and time point a set of individuals (the unicorns). The extension of an expression is the value of the intension function with respect to a given world and time point, for example, the moment now in the actual world. Then the extension of *the president* is the president now (Mr Bush), and the corresponding extension of *unicorn* is the actual set of unicorns. The extension of a sentence is traditionally identified with its truth value. Thus, the extension of *John kisses Mary* is true just in case John kisses Mary. The intension is the function which says, for each possible world, in which moments John kisses Mary.

Since there are possible worlds in which there are unicorns but no centaurs, the words *unicorn* and *centaur* have different intensions. As a consequence, sentences (29) and (30) will have different intensions too.

(29) John seeks a unicorn.

(30) John seeks a centaur.

Thus, using intensions, the nonsynonymy of (29) and (30) can be accounted for. For this purpose, no further information concerning relations between different possible worlds is needed, nor any information concerning relations between time points. This holds for all examples mentioned in the section 'Aims,' and therefore the set of possible worlds and the set of time points are usually introduced as just sets without further specification. This is, however, not always sufficient.

If one is interested in tense phenomena in natural language, then more has to be said about the moments of time, for example that they are linearly ordered, and whether they are indivisible points or intervals with a duration. If one is interested in causatives (*John broke the glass*) or counterfactuals (*If Mary did not come, John would fly*), then the set of possible worlds needs more structure. For dealing with these phenomena, it is crucial to know how the world was just before the breaking of the glass, or which world resembles the present one except for the coming of Mary.

The above discussion shows that the formalization of the intuitive notion of meaning as intension is much better than as extension. However, intensions only deal with those aspects of meaning that have to do with truth and denotation, and neglect aspects such as style, new information versus old information, etc. Therefore, they can only be regarded as a restricted approximation of meaning. Even accepting this limitation, however, intensions are still not completely satisfactory. At important shortcoming concerns tautologies. Consider (31):

(31) Bill comes or does not come.

The intension of this sentence is the function that always yields 'true.' Hence (32) and (33) have the same intension.

(32) John comes.

(33) John comes and Bill comes or does not come.

This causes problems with embedded clauses. Sentences (34) and (35) will have the same intension, whereas they should not be equivalent since Mary's beliefs in (34) do not concern Bill; she may not even know about his existence:

(34) Mary believes that John comes.

(35) Mary believes that John comes and that Bill comes or does not come.

The conclusion is that intensions are not fine-grained enough to distinguish the meanings of sentences like (34) and (35). Several improvements have been proposed, such as structured intensions (Lewis, 1970), and an approach based on partial functions (Muskens, 1989).

A Small Fragment

This section presents as examples three sentences and their treatment. The treatment given in Montague (1973) will be followed, except for one detail (see next section). In the course of the presentation, some important features of intensional logic will be explained.

The sentences are:

(36a) John walks.

(37a) A man walks.

(38a) Every man walks.

These sentences are very simple, and it is easy to present their interpretation in traditional predicate logic:

(36b) walk(john)

(37b) $\exists x[\text{man}(x) \wedge \text{walk}(x)]$

(38b) $\forall x[\text{man}(x) \rightarrow \text{walk}(x)]$

One immediately sees that these three sentences are syntactically much alike (a subject and a verb), but the formulas in predicate logic are rather different: two with a quantifier (different ones!), and in each case another main operator ($\rightarrow$, $\wedge$, and predicate application). What makes it interesting to present these examples here is not the meaning assigned to them, but the fact that these very different meanings can be derived compositionally from the corresponding sentences.

All three sentences are built from a singular noun phrase and the verb *walks*, and for this reason one can design a single syntactic rule that combines a noun phrase and a verb. Since in Montague grammar there is a correspondence between syntactic rules and semantic rules, one also has to design a rule that combines the meaning of the verb with the meaning of the noun phrase. This in turn requires that there be meanings for verbs and for noun phrases. These meanings will be discussed first, then the semantic rule will be designed.

As explained in the previous section, predicates are given intensions as meanings: functions from possible worlds and moments of time to sets of individuals. Thus, the intension, or meaning, of the verb *walk* is a function from possible worlds and moments of time to sets: for each possible world and each moment of time, there is a set (possibly empty) of individuals who walk. Such an intension is called a 'property.' For the noun phrases, it is more difficult to select a format that allows these meanings to be rendered uniformly. In order to keep the calculus going in a maximally uniform way, all noun phrase meanings should preferably be of the same type. This requirement is easily seen to be nontrivial, as, for example, the expression *every man* extends over sets of possibly many individuals, where *John* seems to refer to one individual only.

Montague proposed (1973) to model the meaning of a noun phrase as a set ('bundle') of properties. An individual is, in this approach, characterized by the set of all its properties. This is possible since no two individuals share all their properties at the same time, if only because the two cannot be at the same place at the same time. This approach permits generalization over all noun phrases referring to (sets of) individuals (see also Lewis, 1970).

Meanings of linguistic expressions (functions from world–time pairs to specific world extensions) are denoted by formulas in the language of intensional logic. The meaning of *John* will be denoted by:

(39) $\lambda P[{}^{\vee}P(\text{john})]$

To explain this formula, the variable

(40) P

is a variable over properties of individuals: P may be replaced, for example, by the property 'walking.' The expression

(41) ${}^{\vee}P$

denotes the extension of the property expressed by any predicate P in any given world at a given moment of time. Thus, for the actual world now.

(42) ${}^{\vee}P(\text{john})$

is true if the predicate ${}^{\vee}P$ holds for John now in this world, and false otherwise. Any property denoted by P can be abstracted by means of the lambda operator: λP. Lambda abstraction of (42) gives:

(43) $\lambda P[{}^{\vee}P(\text{john})]$

which is the same as (39) above. This expression denotes a function which says, for any property α, given a world–time pair <w, t>, whether John belongs to the extension of that property in <w, t>. Let this function be called X_j. Some properties of this function will now be investigated.

According to the definition of X_j:

$X_j(\alpha)$ is true if and only if α now holds for the individual John, i.e., ${}^{\vee}\alpha$ (*john*) is true, and false otherwise (I)

X_j is, therefore, the characteristic function of the set of properties that can be predicated of John. As usual in logic, this characteristic function can be identified with the set of properties that John has X_j can therefore be seen as a formalization of the idea that the meaning of *John* is a set of properties.

This function X_j can be evaluated with as argument the property of being a man, that is, the function that yields for each index (i.e., for each world and time) the extension of the predicate *man*. The notation for this argument is:

(44) ${}^{\wedge}$man

The symbol ${}^{\wedge}$ translates the predicate *man* into the language of intensional logic, where ${}^{\wedge}$*man* denotes the intension associated with *man*. X_j is now applied to this argument to obtain, using result (I).

$X_j({}^{\wedge}man)$ is true if and only if ${}^{\wedge}$*man* now holds for the individual John, i.e., if and only if ${}^{\vee\wedge}$*man*(*john*) is true, and false otherwise (II)

In the expression $X_j({}^{\wedge}\text{man})$, X_j can be replaced by its original definition (viz. (43)). Then, (45) is obtained.

(45) $\lambda P[{}^{\vee}P(\text{john})]\ ({}^{\wedge}\text{man})$

Result (II) now states that this is equivalent with

(46) ${}^{\vee\wedge}\text{man}(\text{john})$.

Thus, it has been shown, using semantic considerations, that it is allowed to substitute the argument ${}^{\wedge}$*man* for the variable P, an operation known as 'lambda conversion.' According to the definitions given with (41) and (44), ${}^{\vee\wedge}$*man* denotes the present value of the property of being a man. Hence (46) and (45) are equivalent with (47)

(47) man(john)

Summarizing, the variable P in (45) has been replaced with the property ${}^{\wedge}$*man*, which is in the range of P, and ${}^{\vee\wedge}$*man* has been reduced to just *man*. As the operations concerned are admissible independently of the particular choice of the predicate *man*, the procedure can be generalized to all predicates of the same class (type).

To revert to the treatment of the simple sentence *John walks*, the syntax has, as stipulated earlier, a rule that combines a noun phrase with a verb to form a sentence. What is still needed is a semantic operation that matches the syntactic rule with its semantic consequences. The operation that does this is so-called 'function application': one of the two syntactic constituents acts as a function, while the other acts as argument (input). In this case, the verb meaning is allowed to act as the argument, and the noun phrase meaning as the function. The result of function application, in this case, is a function from world–time pairs to truth-values, or the set of world–time pairs where John walks.

According to the rule just given, the meaning of *John walks* is found by application of the meaning of *John*, that is (43), to the meaning of *walk*. This is denoted by:

(48) $\lambda P[{}^{\vee}P(\text{john})]({}^{\wedge}\text{walk})$

This, as seen above, can be reduced, in two steps, to:

(49) walk(john)

which now gives the meaning representation aimed at.

For the other sentences, one proceeds analogously. The noun phrase *a man* is likewise translated as a set of properties:

(50) $\lambda P[\exists x[man(x) \wedge {}^{\vee}P(x)]]$

This denotes the characteristic function of the set of properties such that for each property in the set there is a man that has that property. The sentence *A man walks* is then represented as:

(51) $\lambda P[\exists x[man(x) \wedge {}^{\vee}P(x)]]({}^{\wedge}walk)$

which reduces to:

(52) $\exists x[man(x) \wedge walk(x)]$

Analogously, every man walks is represented as.

(53) $\lambda P[\forall x[man(x) \rightarrow {}^{\vee}P(john)]]({}^{\wedge}walk)$

or equivalently

(54) $\forall x[man(x) \rightarrow walk(x)]$

This treatment illustrates some of the power of lambda abstraction and lambda conversion. The meaning of the verb is 'plugged into' the meaning of the noun phrase in the right position. Lambda calculus is frequently used in Montague grammar. Without the lambda operator, it would be impossible to maintain compositionality. Impressed by the power of lambdas, Barbara Partee once said: 'Lambdas really changed my life.' What has been, in the end, obtained as meaning representations for the three sentences discussed is nothing more than the formulas usually associated with them in elementary logic courses. There, however, they are found on intuitive grounds, whereas in Montague grammar they are the result of a formal system which relates syntax and semantics in a systematic way.

Some PTQ Phenomena

Montague worked out his ideas in a number of papers, the most influential of which is 'The proper treatment of quantification in ordinary English,' henceforth PTQ (Montague, 1973). This article deals with some semantic phenomena, all connected with quantifiers. Three such phenomena will be discussed here: identity and scope ambiguities (both presented here because they have been the subject of a great deal of discussion since the publication of PTQ), and the 'de dicto–de re' ambiguity, central to Montague grammar, and the origin of its trade mark, the unicorn.

The first phenomenon concerns problems with identity. Consider:

(55) The price of one barrel is $1.00.

(56) The price of one barrel is rising.

(57) $1.00 is rising.

It is obvious that (57) must not be allowed to follow logically from (55) and (56), as $1.00 will remain $1.00 and will neither rise nor fall. The same phenomenon occurs with temperatures, names, telephone numbers, percentages, and in general with all nouns which may denote values. The PTQ treatment of such cases is as follows. Prices, numbers, etc. are treated as basic entities, just as persons and objects. The expression *the price of one barrel* is semantically a function that assigns to each world–time index a particular price. Such a function is called an 'individual concept.' In (55) an assertion is made about the present value of this function, but in (56) an assertion is made about a property of the function. Thus, the expression *the price of one barrel* is considered ambiguous between a value reading and a function reading. The difference in readings blocks the false conclusion (57).

In spite of criticisms, mainly related to the treatment of such nouns as basic entities (e.g., Bennett, 1976), the notion of individual concepts also seems useful to account for cases like the following (Janssen, 1984).

(58) The treasurer of the charity organization is the chairman of the hospital board.

(59) The treasurer of the charity organization has resigned.

(60) The chairman of the hospital board has resigned.

Here, substitution *salva veritate* of terms under an identity statement seems to be running into difficulty. Again, one can say that (58) is a statement about the identity of the values of two different functions at a given world–time index, whereas (59) is a statement about the function (in Frege's terms, about the *Wertverlauf*). Hence the nonvalidity of (60) as a conclusion is explained.

The second phenomenon is scope ambiguity. Consider

(61) Every man loves a woman.

In PTQ, this sentence is considered to be ambiguous (though many linguists would disagree; see below). In the one reading, one particular woman is loved by all men, and in the other every man has at least one woman whom he loves. The first reading is given in

(62), and is called the specific reading, or the wide scope reading for *a woman*. The second reading is given in (63); it is called the 'narrow scope reading.'

(62) $\exists x[\text{woman}(x) \wedge \forall y[\text{man}(y) \rightarrow \text{love}(y, x)]]$

(63) $\forall y[\text{man}(y) \rightarrow [\exists x\ \text{woman}(x) \wedge \text{love}(y, x)]]$

Many linguists consider (61) to be unambiguous. Well-known is the principle which states that the most plausible reading is given by the surface order of the quantified NPs (and of the negation). Following this principle, (61) has only one reading, viz. (63). Note that the reading expressed by (62) is a special case of (63). The principle has not remained unchallenged. Witness (64), where the most plausible reading is not given by the surface order.

At the finish, a medal is available for all participants in the race.

There are other linguistic theories which also assign one reading to (61). But whether (61), the PTQ example, really is ambiguous is less relevant as long as there are sentences that do show scope ambiguities, which seems beyond doubt. For instance, previously an example involving tense was given (*Ten years ago, John met the president*). The machinery of PTQ for dealing with scope ambiguities is presented below.

Since the scope ambiguity does not seem to have a lexical source, the principle of compositionality of meaning requires the construction of two different derivations for the two readings. In PTQ, this is done as follows. First, the basic sentence (65) is formed, in which neither of the two noun phrases occurs but which contains indexed variables (he_n) instead:

(65) he_1 loves he_2.

Next, noun phrases are substituted for the variables. This can be done in two different orders. The noun phrase that is substituted last gets the widest reading. Thus, the specific reading (wide scope for *a woman*) is obtained from (65) by first forming (66).

(66) Every man loves he_2.

and then

(67) Every man loves a woman.

These rules are called 'quantification rules.' The corresponding semantic rule leads to an interpretation equivalent to that of (62). For the other reading, the reverse order of quantifier substitution is used.

These quantification rules have met with some resistance from linguistic quarters, where they are looked upon as unusual creatures. Other solutions have been attempted, where the disambiguation is not done in the syntax. Cooper (1979) deals with scope phenomena in a separate component, a 'storage' mechanism. Hendriks (1993) uses rules which change the meaning of a noun phrase from a narrow scope reading to a wider scope reading.

The third phenomenon is the ambiguity of 'de dicto' versus 'de re' readings. To see the difference, first consider (68) and (69):

(68) John finds a unicorn.

(69) There is a unicorn that John finds.

Sentence (69) follows from (68). Yet (71) does not follow from (70):

(70) John seeks a unicorn.

(71) There is a unicorn that John seeks.

In fact, (70) is ambiguous between a specific reading in which there is a specific unicorn that John seeks, and an intensional reading where John is said to engage in the activity of seeking a unicorn and nothing is implied about the real existence of such animals. The latter reading is usually called the 'de dicto' reading ('de dicto': Latin for 'about what is said'). The former reading is the 'de re' reading ('de re': Latin for 'about the thing'). The ambiguity is accounted for, in principle, in the following way. In the 'de re' reading, a relation is asserted to hold between two individuals. The 'de dicto' reading establishes a relation between John and the set of properties of a unicorn (i.e., the interpretation of the noun phrase *a unicorn*). Whether this way of analyzing and accounting for the 'de dicto' reading is satisfactory in all respects is a question still widely debated (see e.g., Gamut, 1991).

The two readings of *John seeks a unicorn* are obtained in the following way. For the 'de re' reading, a quantification rule introduces the expression *a unicorn*. And if the expression *a unicorn* is introduced directly, that is, without the intervention of a quantification rule, the result is the 'de dicto' reading. In principle, this method is analogous to the method used to express scope differences. One may, in fact, regard the distinction between 'de dicto' and 'de re' readings as a distinction due to scope differences of the intensional operator *seek*.

Developments

Montague (1973) (henceforth PTQ) signaled the start of a great amount of semantic research. Nowadays there are many linguistic theories that deal in a formal way with semantics, and certainly not all of them obey Montague's compositional framework, but in one respect they are all indebted to him: lambdas are used frequently (see section 'A Small Fragment' for the power of lambdas). This section reviews some of the developments that have emanated since PTQ and remain more or less within his compositional framework.

In the first period one kept close to the style of PTQ: one widened the original fragment with new phenomena. A characteristic example is the title ‘A variation and extension of a Montague fragment of English’ (Bennett, 1976). In such articles one finds treatments of relative clauses, passives, scope, control, numbers, plurals, generics, complement structures, and deictic pronouns: most deal with English, some with other languages. For references to these early works see Partee (1976), Gamut (1991), and Dowty *et al.* (1981). Nowadays the habit of providing explicit fragments is abandoned: one focuses on some semantic phenomenon and provides rules for that phenomenon only.

The first broadening of the scope of Montague semantics was toward lexical semantics. In PTQ the words are regarded as basic and remain unanalyzed except for a few logically interesting words such as the verb *be*: often, however, it is linguistically interesting to carry the analysis of words further. Examples (5) and (6) featured the word *cool*, which can be an adjective, an intransitive verb phrase, and a causative verb, with related meanings. Such lexical analyses are given in Dowty (1979).

Montague used a primitive kind of syntax. On the one hand, this appears from the fact that the syntactic rules are not subject to any formal restriction; on the other hand, from the absence of grammatical features and a well-motivated constituent structure and grammatical functions. Soon after the publication of PTQ, proposals were made to incorporate syntactic know-how as developed in transformational grammar into Montague grammar (see, e.g., Partee, 1975 and Dowty, 1982). Partee’s ideas for combining Montague grammar with transformational grammar are worked out in the large grammar used in the machine translation project Rosetta (Rosetta, 1994).

Since the earliest years of Montague semantics there have been attempts to make the syntactic rules as restricted as possible by using only concatenation rules. As a consequence, the constituent structure of the natural language expression becomes isomorphic to its compositional analysis. Because of this feature, such approaches are said to obey ‘surface compositionality.’ The approach leads to (variants of) categorical grammar. An early advocate of this approach was Bach (see, e.g., Oehrle *et al*., 1988); a recent monograph within the framework of Montague is Morill (1994). Surface compositionality requires a treatment of scope phenomena different than Montague’s. An important idea is the use of type shifting rules: such rules make the relation between a word and its semantic type less rigid than in PTQ, and therefore this approach is called ‘flexible Montague grammar.’ The method is useful for several phenomena (including scope); see Partee and Rooth (1983), Hendriks (1993), and Groenendijk and Stokhof (1989).

As explained previously, noun phrases are treated in PTQ as sets of properties. Consequently the determiner ‘every’ is a function from sets to sets: it can be conceived of as a relation between two sets (e.g., in ‘Every man dances’ as a relation between ‘man’ and ‘dance’). This abstract analysis of noun phrases opened the way to a new field: the theory of ‘generalized quantifiers,’ in which many phenomena concerning noun phrases are studied. The pioneering article is Barwise and Cooper (1981); monographs are Keenan and Faltz (1985) and Winter (2001), (*see* **Quantifiers: Semantics**).

The largest linguistic units analyzed in PTQ are sentences. The task of extending Montague’s method to discourses looked unrewarding at first, particularly in view of cross-sentential anaphora phenomena. The first step from sentence to discourse (Kamp, 1981) was, on certain details, not compositional. Later a compositional treatment was given (Groenendijk and Stokhof, 1991). The key feature of their approach was that the meaning of a sentence is not a truth value, but its information change potential: a function that changes the information state. This led to dynamic semantics, an approach in which several other ‘information’ phenomena can be analyzed (*see* **Dynamic Semantics**).

Also, the ontology used by Montague has been challenged. His ontology was based upon a hierarchy of typed sets, but several phenomena suggest a change. Mass nouns seem to ask for sets that do not have elements (Pelletier and Schubert, 2004). Nominalizations (as in *I love loving*) seem to ask for an alternative in which functions can have themselves as arguments; this leads to property theory (see, e.g., Chierchia, 1982). Perception verbs challenge the basic role of possible worlds. They are abandoned by Barwise and Perry (1983), who replace them by ‘situations,’ but a classical model with partial functions can also be used (Muskens, 1989). PTQ deals with only one type of language utterances, declarative sentences, and one may think that other utterances require other types. But questions can be treated without a new ontology, as is shown by the work on the semanties of questions and wh-complements (Groenendijk and Stokhof, 1982, 1989).

Further Reading

A well-known introduction to Montague grammar (especially written for linguists) is Dowty *et al.*

(1981). An introduction with much attention paid to logical and philosophical motivation is Gamut (1991). An introductory article on 'Universal grammar' (Montague, 1970b) is Halvorsen and Ladusaw (1979), its mathematical background is investigated in Janssen (1986a) and discussed in Janssen (1997). Publications on Montague grammar often appear in *Linguistics and Philosophy*. Many references are given in Partee and Hendriks (1997), recent collections of papers about Montague grammar are Partee (2004), and Portner and Partee (2002).

See also: Compositionality: Semantic Aspects; Dynamic Semantics; Extensionality and Intensionality; Formal Semantics; Montague, Richard (1931–1971); Quantifiers: Semantics.

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Montague, Richard (1931–1971)

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Richard Montague was a logician and philosopher whose seminal works on language (Montague, 1970a, 1970b, 1973) were the basis for the theory known after his death as Montague grammar, one of the main starting points for the field of formal semantics.

Montague was born September 20, 1930 in Stockton, California, and died March 7, 1971 in Los Angeles. At St Mary's High School in Stockton, he studied Latin and Ancient Greek. After a year at Stockton Junior College studying journalism, he entered the University of California, Berkeley in 1948, and studied mathematics, philosophy, and Semitic languages, graduating with an A.B. in philosophy in 1950. He continued graduate work at Berkeley in all three areas, especially with Walter Joseph Fischel in classical Arabic, with Paul Marhenke and Benson Mates in philosophy, and with Alfred Tarski in mathematics and philosophy, receiving an M.A. in mathematics in 1953 and his Ph.D. in philosophy in 1957. Tarski, one of the pioneers, with Frege and Carnap, in the model–theoretic semantics of logic, was Montague's main influence and directed his dissertation (Montague, 1957). Montague taught in the UCLA philosophy department from 1955 until his death.

Montague quickly became a major figure in mathematical logic, with contributions to proof theory, model theory, axiomatic set theory, and recursion theory. He applied logical methods to a number of problems in philosophy, including the philosophy of language, and co-authored a logic textbook (Kalish and Montague, 1964). He directed three UCLA Ph.D. dissertations (Cocchiarella, 1966; Grewe, 1965; Kamp, 1968). A fourth, by Gallin (1972, revised and published as Gallin, 1975), was completed at Berkeley after Montague's death. Michael Bennett would also have been Montague's dissertation student; his dissertation on Montague grammar (Bennett, 1974) was supervised by David Kaplan and Barbara Partee.

Of most significance for linguistics was Montague's work on semantics. Building on his development of a higher-order typed intensional logic with a possible-worlds model–theoretic semantics, and a formal pragmatics incorporating indexical pronouns and tenses (Montague, 1968, 1970c), Montague turned in the late 1960s to the project of 'universal grammar.' For him that meant developing a philosophically satisfactory and logically precise account of syntax, semantics, and pragmatics, encompassing both formal and natural languages. Montague's idea that a natural language like English could be formally described using logicians' techniques was a radical one at the time. Most logicians believed that natural languages were not amenable to precise formalization, while most linguists doubted the appropriateness