More Theories, Formal semantics

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Parts are based on slides by Carl Pollard

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Optimality Theory

- Universal set of violable constraints:
  - Faithfulness constraints: surface forms should be as close as to underlying forms
  - Markedness constraints: work on output (along the lines: CV structure is preferred, voiceless final sounds are preferred)

- Language differ in constraint rankings
- Language acquisition = discovering the ranking
- Mostly in phonology
HPSG

- The most widely used grammar framework in computational linguistics
- Fully formalized
- Model theoretic approach
- Objects: Typed feature structures - directed graph with labeled edges and nodes
- Grammar: set of constraints (a la Prolog + types + negation)
- Constraints can be expressed as AVMs
Expression, Meaning, and Reference

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- We say an expression **expresses** its meaning, and **refers to**, or **denotes**, its reference.
- In general, the reference of an expression can be **contingent** (depend on how things are), while the meaning is independent of how things are (examples coming soon).
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- In general, the reference of an expression can be contingent (depend on how things are), while the meaning is independent of how things are (examples coming soon).

Note: Here, we are ignoring the distinction between an expression and an utterance of an expression.
Examples

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- Names are controversial! Vastly oversimplifying:
  - **Descriptivism** (Frege, Russell) the meaning of a name is a **description** associated with the name by speakers; the reference is what satisfies the description.
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- Names are controversial! Vastly oversimplifying:
  - **Descriptivism** (Frege, Russell) the meaning of a name is a **description** associated with the name by speakers; the reference is what satisfies the description.
  - **Direct Reference Theory** (Mill, Kripke) the meaning of a name is its reference, so names are **rigid** (their reference is independent of how things are.)
Grammar and Meaning

The grammar of a language specifies meanings of expressions.
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- The grammar of a language specifies meanings of expressions.
- Grammar says nothing about reference.
The Principle of Compositionality: The meaning of an expression is a function of the meanings of its parts and of the way they are syntactically combined.
Compositionality

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A grammar specifies

- the meaning of words (or morphemes)
- how to derive a meaning of a complex expression from its components
Entailment

- *John ate a cake.*
- *A cake was eaten.*
- *There was a cake.*
Entailment

- John ate a cake.
- A cake was eaten.
- There was a cake.

**Entailment**: $\phi \models \psi$ iff (if $\phi$ is true then $\psi$ must be true)
Our theory will use the following sets as building blocks:

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A Theory of Meanings and Extensions

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- **Bool**: The *truth values* (extensions of propositions)
- **Ind**: The *individuals* (meanings of names).
- **World**: The *worlds* (ultrafilters of propositions)
- **One**: The *unit set* \{0\}.

It’s conventional to call the member of this set *, rather than 0, since the important thing about it is that it is a singleton and not what its member is.
Propositions

- Propositions are primitive notions.
Propositions

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- We are agnostic only about their formal nature not about their properties.
The set of propositions (Prop) forms a pre-lattice:
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- They are related by entailment:
  \[ \models : \text{Prop} \times \text{Prop} \rightarrow \text{Bool} \]
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- Entailment is constrained to be a preorder (i.e., reflexive, transitive, but not antisymmetric)
  The absence of antisymmetry allows two propositions to entail each other and still be distinct objects. Equality implies equivalence but not vice versa.
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- There are the usual glb/lub, top/bottom, complement, residual operations.
(Hyper)intensional types

Meaning, the kind of hyperintensional types is defined as follows:

- Prop, Ind, One ∈ Meaning.
- If $A, B \in \text{Meaning}$ then
  - $A \times B \in \text{Meaning}$.
  - $A \to B \in \text{Meaning}$
- Nothing else is a hyperintensional type.
Examples of word meanings

<table>
<thead>
<tr>
<th>Syntax</th>
<th>Semantics</th>
</tr>
</thead>
<tbody>
<tr>
<td>proper name Chiquita</td>
<td>Chiquita’ : Ind</td>
</tr>
<tr>
<td>common noun donkey</td>
<td>donkey’ : Ind → Prop</td>
</tr>
<tr>
<td>sentential adverb obviously</td>
<td>obvious’ : Prop → Prop</td>
</tr>
<tr>
<td>dummy pronoun itₐ</td>
<td>* ∈ One</td>
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It is obvious that ...
Meanings can be mapped to extensions (references).
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</tr>
<tr>
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<td>Bool</td>
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<td>Prop → Bool</td>
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<td>etc.</td>
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<tr>
<td>donkey’: Ind → Prop</td>
<td>f : Ind → Bool; f(i) ⇔ (i is a donkey)</td>
</tr>
<tr>
<td>obviously : Prop → Prop</td>
<td>g : Prop → Bool; g(p) ⇔ (p is obvious)</td>
</tr>
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Categorial Grammar

- logical formalism, several implications (usually written as / and \)
- small number of language independent rules (e.g., modus ponens = function application), the rest of grammar is in the lexicon (radical lexicalism)
- syntactic structure is an equivalence set of proofs
- Usually, surface form and semantics are derived in parallel.
we can say that verbs are functions taking noun phrases as their arguments

\[
\text{John}:\text{NP} \quad \text{sleeps}:\text{NP}\backslash \text{S} \\
\text{John sleeps: S}
\]

application

If there is an object John of type NP and an object sleeps of type NP\backslash S then there is an object John sleeps of type S. Note that John sleeps is a syntactical object, not the actual surface form. We could have written x123 for John.
Recall

**currying** – transformation of a function with multiple parameters into a function taking a single argument (the first of the arguments of the original function) and returning a new function which takes the remainder of the arguments.
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uncurried
plus : (Int × Int) → Int
plus(3, 4)

curried
plus’ : Int → (Int → Int)
plus’(3)(4)
**Recall**

**currying** – transformation of a function with multiple parameters into a function taking a single argument (the first of the arguments of the original function) and returning a new function which takes the remainder of the arguments.

uncurried

\[ \text{plus} : (\text{Int} \times \text{Int}) \to \text{Int} \]
\[ \text{plus}(3, 4) \]

curried

\[ \text{plus'} : \text{Int} \to (\text{Int} \to \text{Int}) \]
\[ \text{plus'}(3)(4) \]

\[ \text{inc} = \text{plus'}(1) \]
All together now

- Words, phrases and sentences are modeled as signs.
- Signs are the combinations of phonological, syntactic, and semantic objects that makes sense.
- The phonological component of a sign is the pronunciation of the syntactic component and the semantic component is the meaning of it.
- The type Sign is a subtype of the following tuple type
  $\left[ \begin{array}{ll}
  \text{phon} & \text{Phon}^* \\
  \text{syn} & \text{Syn} \\
  \text{sem} & \text{Meaning}
  \end{array} \right]$
- Syn is the kind of syntactic types, i.e., set of basic types (NP, N, S, ...) closed under syntactic constructors ($\times$, $\Rightarrow$, ...).
All together now – Lexicon

Lexicon consists of axioms of the form:

\[
\begin{align*}
\vdash & \left[ \begin{array}{l}
\text{phon} \ [\text{boI}] : \text{Phon}^* \\
\text{syn} \quad \text{boy} : \text{N} \\
\text{sem} \quad \text{boy'} : \text{Ind} \Rightarrow \text{Prop}
\end{array} \right]
\end{align*}
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- In abbreviated form:
  
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  \text{[boī]} : \text{Phon}^* & \quad \text{boy} : \text{N} & \quad \text{boy'} : \text{Ind} \Rightarrow \text{Prop}
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\end{bmatrix}
\end{array}
\]

In abbreviated form:

- [boi]: Phon*  boy: N  boy’: Ind ⇒ Prop
- [sno:rz]: Phon*  snores: NP ⇒ S  snore’: Ind ⇒ Prop
- [sli:ps]: Phon*  sleeps: NP ⇒ S  sleep’: Ind ⇒ Prop
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[\text{sno:rz}] : \text{Phon}^* & \quad \text{snores} : \text{NP} \Rightarrow \text{S} & \quad \text{snore}' : \text{Ind} \Rightarrow \text{Prop} \\
[\text{sli:ps}] : \text{Phon}^* & \quad \text{sleeps} : \text{NP} \Rightarrow \text{S} & \quad \text{sleep}' : \text{Ind} \Rightarrow \text{Prop} \\
[\text{lau:dl}] : \text{Phon}^* & \quad \text{loudly} : \text{VP} \Rightarrow \text{VP} & \quad \text{loud'} : (\text{Ind} \Rightarrow \text{Prop}) \Rightarrow (\text{Ind} \Rightarrow \text{Prop})
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\vdash & \phi \ \
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[\text{boy}] : \text{Phon}^* \\
[\text{syn}] \ 
[\text{boy}] : \text{N} \\
[\text{sem}] \ 
[\text{boy}'] : \text{Ind} \Rightarrow \text{Prop}
\end{array}
\end{aligned}
\]

In abbreviated form:

- \([\text{boI}] : \text{Phon}^* \) \quad \text{boy: N} \quad \text{boy': Ind} \Rightarrow \text{Prop}
- \([\text{sno:rz}] : \text{Phon}^* \) \quad \text{snores: NP} \Rightarrow \text{S} \quad \text{snore': Ind} \Rightarrow \text{Prop}
- \([\text{sli:ps}] : \text{Phon}^* \) \quad \text{sleeps: NP} \Rightarrow \text{S} \quad \text{sleep': Ind} \Rightarrow \text{Prop}
- \([\text{laudli}] : \text{Phon}^* \) \quad \text{loudly: VP} \Rightarrow \text{VP} \quad \text{loud': (Ind} \Rightarrow \text{Prop} \Rightarrow (\text{Ind} \Rightarrow \text{Prop})
- \([\text{evroi}] : \text{Phon}^* \) \quad \text{every: N} \Rightarrow \text{NP} \quad \text{every'} = \lambda q, p.\lambda x.(q(x) \Rightarrow p(x)) : \quad (\text{Ind} \Rightarrow \text{Prop}) \times (\text{Ind} \Rightarrow \text{Prop}) \Rightarrow \text{Prop}
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- In abbreviated form:

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  [\text{sno}:\text{rz}] : \text{Phon}^* \quad \text{snores} : \text{NP} \Rightarrow \text{S} \quad \text{snore'} : \text{Ind} \Rightarrow \text{Prop} \\
  [\text{sli}:\text{ps}] : \text{Phon}^* \quad \text{sleeps} : \text{NP} \Rightarrow \text{S} \quad \text{sleep'} : \text{Ind} \Rightarrow \text{Prop} \\
  [\text{laʊdli}] : \text{Phon}^* \quad \text{loudly} : \text{VP} \Rightarrow \text{VP} \quad \text{loud'} : (\text{Ind} \Rightarrow \text{Prop}) \Rightarrow (\text{Ind} \Rightarrow \text{Prop}) \\
  [\text{evrɪ}] : \text{Phon}^* \quad \text{every} : \text{N} \Rightarrow \text{NP} \quad \text{every'} = \lambda q, p. \lambda x. (q(x) \Rightarrow p(x)) : (\text{Ind} \Rightarrow \text{Prop}) \times (\text{Ind} \Rightarrow \text{Prop}) \Rightarrow \text{Prop} \\
  [\text{ænd}] : \text{Phon}^* \quad \text{and} : \forall A. A \times A \Rightarrow A \quad \text{and'} : \forall A. A \times A \Rightarrow A
  \end{array}
  \]
All together now – Grammar

- function application in syntax corresponds to:
  - function application in semantics
  - concatenation in phonology
- Possibly other rules in individual sub-grammars. For example, phonotactic constraints in phonology.
every boy:
every boy:

Relevant lexicon:

\[
\Gamma \vdash \begin{bmatrix}
\text{phon} & \text{[boI]} : \text{Phon}^* \\
\text{syn} & \text{boy} : \text{N} \\
\text{sem} & \text{boy'} : \text{Ind} \Rightarrow \text{Prop}
\end{bmatrix}
\quad \Pi \vdash \begin{bmatrix}
\text{phon} & \text{[evri]} : \text{Phon}^* \\
\text{syn} & \text{every} : \text{N} \Rightarrow \text{NP} \\
\text{sem} & \text{every'} = \lambda q, p. \lambda x. (q(x) \Rightarrow p(x)) : \\
& (\text{Ind} \Rightarrow \text{Prop}) \times (\text{Ind} \Rightarrow \text{Prop}) \Rightarrow \text{Prop}
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\end{bmatrix} \\
\vdash & \begin{bmatrix}
\text{phon} & \text{[evri boi]} : \text{Phon}^* \\
\text{syn} & \text{every(boy)} : \text{NP} \\
\text{sem} & \text{every'}(\text{boy'}) : (\text{Ind} \Rightarrow \text{Prop}) \Rightarrow \text{Prop}
\end{bmatrix}
\end{align*}
\]
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Relevant lexicon:

\[ \vdash \left[ \begin{array}{ll} phon & \text{[boi]} : \text{Phon}^* \\ syn & \text{boy} : \text{N} \\ sem & \text{boy'} : \text{Ind} \Rightarrow \text{Prop} \end{array} \right] \quad \vdash \left[ \begin{array}{ll} phon & \text{[evri boi]} : \text{Phon}^* \\ syn & \text{every} : \text{N} \Rightarrow \text{NP} \\ sem & \text{every'} = \lambda q, p. \lambda x. (q(x) \Rightarrow p(x)) : \text{(Ind} \Rightarrow \text{Prop}) \times (\text{Ind} \Rightarrow \text{Prop}) \Rightarrow \text{Prop} \end{array} \right] \]

\[ \vdash \left[ \begin{array}{ll} phon & \text{[evri boi]} : \text{Phon}^* \\ syn & \text{every(boy)} : \text{NP} \\ sem & \text{every'(boy')} : (\text{Ind} \Rightarrow \text{Prop}) \Rightarrow \text{Prop} \end{array} \right] \]

\[ \text{every'(boy')} = [\lambda q, p . \lambda x. (q(x) \Rightarrow p(x))] (\lambda x . \text{boy'}(x)) \]
every boy:

Relevant lexicon:

\[ 
\Gamma \vdash \begin{cases} 
\text{phon} & [\text{boi}] : \text{Phon}^* \\
\text{syn} & \text{boy} : \text{N} \\
\text{sem} & \text{boy'} : \text{Ind} \Rightarrow \text{Prop} 
\end{cases} 
\]

\[ 
\Gamma \vdash \begin{cases} 
\text{phon} & [\text{evri} \text{ boi}] : \text{Phon}^* \\
\text{syn} & \text{every} : \text{N} \Rightarrow \text{NP} \\
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\text{every'(boy')} = [\lambda q, p. \lambda x. (q(x) \Rightarrow p(x))] (\lambda x. \text{boy'}(x)) \\
\lambda p. \lambda x (\text{boy'}(x) \Rightarrow p(x)) 
\]
every boy sleeps and snores loudly
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\[\begin{align*}
&\text{phon} & \varepsilon\text{vri boi slrips ænd sno:rz laudli} : \text{Phon}^* \\
&\text{syn} & \text{and}(\text{sleeps, loud(\text{snore}) (every(boy))) : \text{S} \\
&\text{sem} & \text{and'}(\text{sleep’, loud’(\text{snore’}) (every’(boy’)) : \text{Prop}}
\end{align*}\]
every boy sleeps and snores loudly

\[\begin{array}{c|c}
phon & [\text{evri boi sli:ps ænd sno:rz laudlɪ}] : \text{Phon}^* \\
\hline
syn & \text{and(sleeps, loud(snore))(every(boy))} : S \\
sem & \text{and’(sleep’, loud’(snore’))(every’(boy’))} : \text{Prop} \\
\end{array}\]

\[
\text{and’(sleep’, loud’(snore’))(every’(boy’))} = \\
[\lambda p. \lambda x (\text{boy’}(x) \Rightarrow p(x))] (\lambda s. \text{and’(sleep’, loudly’(snore’))}(s)) =
\]
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\begin{array}{l}
\vdash \left[ \begin{array}{c}
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\text{sem} & \text{and’(sleep’, loud’(snore’))(every’(boy’))} : \text{Prop}
\end{array} \right]
\end{array}
\]

\[\text{and’(sleep’, loud’(snore’))(every’(boy’))} = \]
\[\left[ \lambda p. \lambda x (\text{boy’}(x) \Rightarrow p(x)) \right] (\lambda s. \text{and’(sleep’, loudly’(snore’))}(s)) = \]
\[\lambda x. (\text{boy’}(x) \Rightarrow \text{and’(sleep’, loud’(snore’))}(x))\]