## Perplexity of n -gram and dependency language models

Martin Popel, David Mareček ÚFAL, Charles University in Prague


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## Outline

- Language Models (LM)
- basics
- design decisions
- Post-ngram LM
- Dependency LM
- Evaluation
- Conclusion \& future plans


## Language Models - basics

$$
P(s)=?
$$

$\mathrm{P}($ The dog barked again $)$

## Language Models - basics

$$
P(s)=?
$$

$\mathrm{P}($ The dog barked again $)>\mathrm{P}($ The dock barked again $)$

## Language Models - basics

$P(s)=P\left(w_{1}, w_{2}, \ldots w_{m}\right)$
$P($ The dog barked again $)=$
$P\left(w_{1}=\right.$ The,$w_{2}=$ dog,$w_{3}=$ barked,$w_{4}=$ again $)$

## Language Models - basics

$$
P(s)=P\left(w_{1}, w_{2}, \ldots w_{m}\right)=P\left(w_{1}\right) P\left(w_{2} \mid w_{1}\right) \ldots P\left(w_{m} \mid w_{1}, \ldots, w_{m-1}\right)
$$

## Chain rule

$P($ The dog barked again $)=$
$\mathrm{P}\left(\mathrm{w}_{1}=\right.$ The $)$.
$P\left(w_{2}=\operatorname{dog} \mid w_{1}=\right.$ The $)$.
$\mathrm{P}\left(\mathrm{w}_{3}=\right.$ barked $\mid \mathrm{w}_{1}=$ The, $\mathrm{w}_{2}=$ dog $)$.
$\mathrm{P}\left(\mathrm{W}_{4}=\right.$ again $\mid \mathrm{W}_{1}=$ The, $\mathrm{W}_{2}=$ dog, $\mathrm{W}_{3}=$ barked )

## Language Models - basics

$$
P(s)=P\left(w_{1}, w_{2}, \ldots w_{m}\right)=P\left(w_{1}\right) P\left(w_{2} \mid w_{1}\right) \ldots P\left(w_{m} \mid w_{1}, \ldots, w_{m-1}\right)
$$

## Changed notation

$P($ The dog barked again $)=$
$P\left(w_{i}=\right.$ The $\left.\quad i=1\right)$.
$\mathrm{P}\left(\mathrm{w}_{\mathrm{i}}=\operatorname{dog} \quad \mid \mathrm{i}=2, \mathrm{w}_{\mathrm{i}-1}=\right.$ The $)$.
$\mathrm{P}\left(\mathrm{w}_{\mathrm{i}}=\right.$ barked $\mid \mathrm{i}=3, \mathrm{w}_{\mathrm{i}-2}=$ The, $\left.\mathrm{w}_{\mathrm{i}-1}=\operatorname{dog}\right)$.
$\mathrm{P}\left(\mathrm{w}_{\mathrm{i}}=\right.$ again $\mid \mathrm{i}=4, \mathrm{w}_{\mathrm{i}-3}=$ The, $\mathrm{w}_{\mathrm{i}-2}=$ dog, $\mathrm{w}_{\mathrm{i}-1}=$ barked $)$

## Language Models - basics

$$
P(s)=P\left(w_{1}, w_{2}, \ldots w_{m}\right)=P\left(w_{1}\right) P\left(w_{2} \mid w_{1}\right) \ldots P\left(w_{m} \mid w_{1}, \ldots, w_{m-1}\right)
$$

Artificial start-of-sentence token
$P($ The dog barked again $)=$
$P\left(w_{i}=\right.$ The $\quad i=1, w_{i-1}=$ NONE $)$.
$P\left(w_{i}=\operatorname{dog} \quad i=2, w_{i-2}=\right.$ NONE,$w_{i-1}=$ The $)$.
$P\left(w_{i}=\right.$ barked $\mid i=3, w_{i-3}=$ NONE,$w_{i-2}=$ The,$\left.w_{i-1}=\operatorname{dog}\right)$.
$\mathrm{P}\left(\mathrm{w}_{\mathrm{i}}=\right.$ again $\mid \mathrm{i}=4, \mathrm{w}_{\mathrm{i}-4}=$ NONE, $\mathrm{w}_{\mathrm{i}-3}=$ The, $\mathrm{w}_{\mathrm{i}-2}=$ dog, $\mathrm{w}_{\mathrm{i}-1}=$ barked $)$

## Language Models - basics

$$
P(s)=P\left(w_{1}, w_{2}, \ldots w_{m}\right)=P\left(w_{1}\right) P\left(w_{2} \mid w_{1}\right) \ldots P\left(w_{m} \mid w_{1}, \ldots, w_{m-1}\right)
$$

## Position backoff

$\mathrm{P}($ The dog barked again $) \approx$
$P\left(w_{i}=\right.$ The $\quad w_{i-1}=$ NONE $)$.
$P\left(w_{i}=\operatorname{dog} \quad \mid \quad w_{i-2}=\right.$ NONE,$w_{i-1}=$ The $)$.
$P\left(w_{i}=\right.$ barked | $w_{i-3}=$ NONE,$w_{i-2}=$ The,$\left.w_{i-1}=\operatorname{dog}\right)$.
$\mathrm{P}\left(\mathrm{w}_{\mathrm{i}}=\right.$ again | $\mathrm{w}_{\mathrm{i}-4}=$ NONE, $\mathrm{w}_{\mathrm{i}-3}=$ The, $\mathrm{w}_{\mathrm{i}-2}=$ dog, $\mathrm{w}_{\mathrm{i}-1}=$ barked $)$

## Language Models - basics

$$
\mathrm{P}(\mathrm{~s})=\mathrm{P}\left(\mathrm{w}_{1}, \mathrm{w}_{2}, \ldots \mathrm{w}_{\mathrm{m}}\right) \approx \prod_{\mathrm{i}=1 . . \mathrm{m}} \mathrm{P}\left(\mathrm{w}_{\mathrm{i}} \mid \mathrm{w}_{\mathrm{i}-1}\right)
$$

## History backoff (bigram LM)

$\mathrm{P}($ The dog barked again $) \approx$
$P\left(w_{i}=\right.$ The $\quad w_{i-1}=$ NONE $)$.
$P\left(w_{i}=\operatorname{dog} \mid \quad w_{i-1}=\right.$ The $)$.
$P\left(w_{i}=\right.$ barked | $\left.\quad W_{i-1}=\operatorname{dog}\right)$.
$\mathrm{P}\left(\mathrm{w}_{\mathrm{i}}=\right.$ again $\mid \quad \mathrm{w}_{\mathrm{i}-1}=$ barked $)$

## Language Models - basics

$$
P(s)=P\left(w_{1}, w_{2}, \ldots w_{m}\right) \approx \prod_{i=1 . . m} P\left(w_{i} \mid w_{i-2}, w_{i-1}\right)
$$

## History backoff (trigram LM)

$\mathrm{P}($ The dog barked again $) \approx$
$P\left(w_{i}=\right.$ The $\quad W_{i-2}=$ NONE,$w_{i-1}=$ NONE $)$.
$P\left(w_{i}=\operatorname{dog} \quad \mid \quad w_{i-2}=\right.$ NONE,$w_{i-1}=$ The $)$.
$P\left(w_{i}=\right.$ barked | $\quad w_{i-2}=$ The,$\left.w_{i-1}=\operatorname{dog}\right)$.
$\mathrm{P}\left(\mathrm{w}_{\mathrm{i}}=\right.$ again $\mid \quad \mathrm{w}_{\mathrm{i}-2}=\operatorname{dog}, \mathrm{w}_{\mathrm{i}-1}=$ barked $)$

## Language Models - basics

$$
\begin{aligned}
& \mathrm{P}(\mathrm{~s})=\mathrm{P}\left(\mathrm{w}_{1}, \mathrm{w}_{2}, \ldots \mathrm{w}_{\mathrm{m}}\right) \approx \prod_{\mathrm{i}=1 . . \mathrm{m}} \mathrm{P}\left(\mathrm{w}_{\mathrm{i}} \mid \mathrm{w}_{\mathrm{i}-2}, \mathrm{w}_{\mathrm{i}-1}\right) \\
& \text { In general: } \\
& \Pi_{\mathrm{i}=1 . . \mathrm{m}} \mathrm{P}\left(\mathrm{w}_{\mathrm{i}} \mid \mathbf{h}_{\mathrm{i}}\right)
\end{aligned}
$$

$\mathbf{h}_{\mathrm{i}} \ldots$ context (history) of word $\mathrm{w}_{\mathrm{i}}$

## Language Models - design decisions

1) How to factorize $P\left(w_{1}, w_{2}, \ldots w_{m}\right)$ into $\prod_{i=1 . . m} P\left(w_{i} \mid h_{i}\right)$, i.e. what word-positions will be used as the context $\mathbf{h}_{\mathbf{i}}$ ?
2) What additional context information will be used (apart from word forms), e.g. stems, lemmata, POS tags, word classes,... ?
3) How to estimate $P\left(w_{i} \mid \mathbf{h}_{i}\right)$ from the training data? Which smoothing technique will be used? (Good-Turing, Jelinek-Mercer, Katz, Kneser-Ney,...) Generalized Parallel Backoff etc.

## Language Models - design decisions

1) How then $\left.{ }^{\prime} w_{1}, w_{2}, \ldots w_{m}\right)$ into $\prod_{i=1 \ldots m} P\left(w_{i} \mid \mathbf{h}_{i}\right)$, i.e. wha this itions will be used as the context $h_{i}$ ?
2) What add context information will be used (apart from d forms), e.g. stems, mmata, POS tags, word classes,... ?
3) How to estimate $P\left(w_{i} \mid \mathbf{h}_{i}\right)$ from the training data?

Linear interpolation que will be used? Weights trained by EM ercer, Katz, Kneser-Ney,...) Generalized Parallel Backoff etc.

## Language Models - design decisions

1) How t ${ }^{\text {r...... }} w_{1}, w_{2}$,
i.e. wha
this
work itions will
$\mathbf{h}_{\mathrm{i}}=\mathrm{w}_{\mathrm{i}-\mathrm{n}+1}, \ldots, \mathrm{w}_{\mathrm{i}-1}$
(n-gram-based LMs)
2) What add context information will ə used (apart from d forms), e.g. stems, mmata, POS tags, word es,...?
3) How to estimate $P\left(w_{i} \mid \mathbf{h}_{i}\right)$ from the ty other ata?

Linear interpolation que will be papers Weights trained by EM ercer, Katz

$$
y, \ldots)
$$

Generalized Parallel Backoff etc.

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## Post-ngram LM

In general:
$\mathrm{P}(\mathrm{s})=\mathrm{P}\left(\mathrm{w}_{1}, \mathrm{w}_{2}, \ldots \mathrm{w}_{\mathrm{m}}\right) \approx \Pi_{\mathrm{i}=1 . . \mathrm{m}} \mathrm{P}\left(\mathrm{w}_{\mathrm{i}} \mid \mathbf{h}_{\mathrm{i}}\right)$
$h_{i} \ldots$ context (history) of word $w_{i}$

## left-to-right factorization order

Bigram LM:
Trigram LM:

## Post-ngram LM

In general: $\quad P(s)=P\left(w_{1}, w_{2}, \ldots w_{m}\right) \approx \prod_{i=1 . . m} P\left(w_{i} \mid h_{i}\right)$
$\mathbf{h}_{\mathrm{i}} \ldots$ context (history) of word $\mathrm{w}_{\mathrm{i}}$
left-to-right factorization order
Bigram LM:
$h_{i}=w_{i-1}$
(one previous word)
Trigram LM:
$\mathbf{h}_{\mathrm{i}}=\mathrm{w}_{\mathrm{i}-2}, \mathrm{w}_{\mathrm{i}-1} \quad$ (two previous words)
right-to-left factorization order
Post-bigram LM: $\mathbf{h}_{\mathrm{i}}=\mathrm{w}_{\mathrm{i}+1} \quad$ (one following word)
Post-trigram LM: $\mathbf{h}_{\mathrm{i}}=\mathrm{w}_{\mathrm{i}+1}, \mathrm{w}_{\mathrm{i}+2} \quad$ (two following words)

## Post-ngram LM

In general: $\quad P(s)=P\left(w_{1}, w_{2}, \ldots w_{m}\right) \approx \prod_{i=1 . . m} P\left(w_{i} \mid h_{i}\right)$
$\mathbf{h}_{\mathrm{i}} \ldots$ context (history) of word $\mathrm{w}_{\mathrm{i}}$
left-to-right factorization order
Bigram LM:
$h_{i}=w_{i-1}$
(one previous word)
Trigram LM:
$\mathbf{h}_{\mathrm{i}}=\mathrm{w}_{\mathrm{i}-2}, \mathrm{w}_{\mathrm{i}-1} \quad$ (two previous words)
right-to-left factorization order
Post-bigram LM: $\mathbf{h}_{\mathrm{i}}=\mathrm{w}_{\mathrm{i}+1} \quad$ (one following word)
$\mathrm{P}($ The dog barked again $)=\mathrm{P}($ again $\mid$ NONE $) \cdot \mathrm{P}($ barked $\mid$ again $)$.

$$
P(\operatorname{dog} \mid \text { barked }) \cdot P(\text { The } \mid \text { dog })
$$

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## Dependency LM

- exploit the topology of dependency trees
The dog barked again


## Dependency LM

- exploit the topology of dependency trees



## Dependency LM

- exploit the topology of dependency trees


$$
\begin{aligned}
P(\text { The dog barked again })= & P(\text { The } \mid \text { dog }) \cdot P(\text { dog | barked }) \cdot \\
& P(\text { barked } \mid \text { NONE }) \cdot P(\text { again } \mid \text { barked })
\end{aligned}
$$

$\mathbf{h}_{\mathrm{i}}=\operatorname{parent}\left(\mathrm{w}_{\mathrm{i}}\right)$

## Dependency LM <br> Long distance dependencies

The dog I heard last night barked again


## Dependency LM <br> Motivation for usage

- How can we know the dependency structure without knowing the word-forms?


## Dependency LM <br> Motivation for usage

- How can we know the dependency structure without knowing the word-forms?
- For example in tree-to-tree machine translation.

ANALYSIS



## Dependency LM

## Examples

- Model wp word form of parent
$\mathrm{P}($ The dog barked again $)=$

| P ( The | \| dog |
| :---: | :---: |
| P ( dog | \| barked |
| $P($ barked | \| NONE |
| P ( again | \| barked |

## Dependency LM

## Examples

- Model wp, wg word form of parent, word form of grandparent



## Dependency LM

Examples

- Model E, wp
edge direction, word form of parent
$\mathrm{P}($ The dog barked again $)=$
$P($ The $\mid$ right, dog $)$.
$P($ dog | right, barked ).
$P($ barked l left, NONE $)$.
$P($ again | left, barked $)$


## Dependency LM

 Examples- Model C, wp number of children, word form of parent

```
P( The dog barked again ) =
\(\left.\begin{array}{llr}P(\text { The } \mid 0, & \text { dog }) \\ P(\text { dog } \mid 1, & \text { barked })\end{array}\right)\).
```



## Dependency LM

- Model N, wp the word is $\mathrm{N}^{\text {th }}$ child of its parent, word form of parent

| $\mathrm{P}($ The dog barked again | ) $=$ |
| :---: | :---: |
| P ( The \| 1, | dog ) |
| $\mathrm{P}(\mathrm{dog}$ \| 1, | barked ) |
| $\mathrm{P}($ barked \| 1, | NONE ) |
| $\mathrm{P}($ again \| 2 , | barked) |



## Dependency LM <br> Examples of additional context information

- Model tp, wp

POS tag of parent, word form of parent

| $\mathrm{P}($ The dog b | arked again | ) |
| :---: | :---: | :---: |
| P ( The | \| NN, | dog ) |
| P ( dog | \| VBD, | barked ) |
| $\mathrm{P}($ barked | NONE, | NONE ) |
| P ( again | \| VBD, | barked) |



## Dependency LM

## Examples of additional context information

- Model tp, wp

POS tag of parent, word form of parent


## naïve tagger assignes the most frequent tag a a given word frequent tag for a given word

## Dependency LM Examples of additional context information

- Model Tp, wp
coarse-grained POS tag of parent, word form of parent



## Dependency LM

## Examples of additional context information

- Model E, C, wp, N
edge direction, \# children, word form of parent, word is $\mathrm{N}^{\text {th }}$ child of its parent
$\mathrm{P}($ The dog barked again $)=$
$\mathrm{P}($ The $\mid$ right, $0, \operatorname{dog}, 1)$.
$P($ dog $\mid$ right, 1, barked , 1).
P( barked left, 2, NONE , 1) •
P( again |left, 0, barked, 2)



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## Evaluation

- Train and test data from CoNLL 2007 shared task
- 7 languages: Arabic, Catalan, Czech, English (450 000 tokens, 3 \% OOV), Hungarian, Italian (75000 tokens), and Turkish (26 \% OOV)
- Cross-entropy $=-(1 /|\mathrm{T}|) \Sigma_{\mathrm{i}=1 . .|\mathrm{T}|} \log _{2} \mathrm{P}\left(\mathrm{w}_{\mathrm{i}} \mid \mathbf{h}_{\mathrm{i}}\right)$, measured on the test data $T$
- Perplexity = $2^{\text {Cross-entropy }}$
- Lower perplexity ~ better LM
- Baseline ... trigram LM
- 4 experimental settings: PLAIN, TAGS, DEP, DEP+TAGS


## Evaluation



## Conclusion

Findings confirmed for all seven languages

Improvement over baseline for English

- Post-trigram better than trigram

PLAIN 8 \%
Post-bigram better than bigram

- Additional context (POS \& lemma) helps
- Dependency structure helps even more

TAGS 20 \%
DEP 24 \%

- The best perplexity achieved with DEP+TAGS $31 \%$


## Future plans

- Investigate the reason for better post-ngram LM perplexity
- Extrinsic evaluation
- Post-ngram LM in speech recognition
- Dependency LM in tree-to-tree machine translation
- Better smoothing using Generalized Parallel Backoff
- Bigger LM for real applications


## Thank you

